2022 SAJC Prelim P1

Q1

1 The equation of a curve C is given by $4(x+y)^2+(x-y)^2=20$.

(i) Show that the gradient of C at the point (x, y) is given by

$$\frac{dy}{dx}=-\frac{5x+3y}{3x+5y}.$$ [3]

(ii) Find the equation(s) of the tangent(s) to the curve C which are perpendicular to the line $y = x$. [6]

2 The curves C_1 and C_2 are defined by the equations $y=\frac{6}{4-x^2}$ and $y=6-x^2$ respectively.

(i) On the same axes, sketch the graphs of the curves C_1 and C_2, stating the equations of any asymptotes, the exact coordinates of the turning point(s) and any points where the curve crosses the x- and y-axes. [5]

(ii) Solve the inequality $\frac{6}{4-x^2} < 6 - x^2$. [2m]

(iii) The transformations A and B are given as follows:

A : Reflection about the y-axis;

B : Translation of 4 units in the negative x-direction.

The graphs C_1 and C_2 undergo in sequence, the transformations A and B. The resulting equations of the transformed graphs of C_1 and C_2 are $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ respectively.

Deduce the solution set of the inequality $\mathrm{f}(x) < \mathrm{g}(x)$. [2]

3 The function f is defined by $f: x \mapsto ax^3 + bx^2 + cx + d$, where $x \in \mathbb{R}$ and a, b, c and d are constants.

The graph of f intersects the y-axis at $y = -3$ and passes through the points $(-1, 0)$ and $(2, 0)$.

(i) Explain why f does not have an inverse. [1]

(ii) Given also that the tangent to the graph of f at $x = 1$ is a horizontal line, find f(x). [3]

(iii) Sketch the graph of $y = f(x)$, giving the coordinates of the turning points and the points which the graph intersects the axes. [2]

(iv) Given that the function f has an inverse if its domain is restricted to $x \geq k$, state the smallest possible value of k. [1]

For the rest of the question, use the domain given and value of k found in part (iv).

(v) Describe the relationship between the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$. [1]

(vi) Show that the solution of the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$ satisfies the equation $3x^3 - 11x - 6 = 0$. Hence, find the solution of the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [3]

(vii) It is given that $\mathrm{g}(x) = \ln(x+5)$, where $x > -5$. A student attempts to find the composite function gf.
The student's solution is shown below:

$\mathrm{g}(x) = \ln(x+5)$
$\mathrm{D}_{gf} = \mathrm{D}_f = [k, \infty)$
$\therefore \mathrm{gf}(x) = \ln\left(ax^3 + bx^2 + cx + d + 5\right),\ x \in \mathbb{R},\ x \geq k.$

Comment on the validity of the student's solution. [1]

4 (i) By sketching the graph of $y = \frac{x+1}{2x-1}$, find the range of values of x for which $\frac{x+1}{2x-1} \geq 0$.

[4m]

(ii) Hence, without the use of a calculator, show that $\int_{-2}^{0} \left| \frac{x+1}{2x-1} \right| \, dx = \frac{3}{2}\ln 3 - \frac{3}{4}\ln 5$.

[4]

5 The curve C is defined by the equation $y = \frac{1}{2}\tan^{-1}(2x)$ and the line L is defined by the equation $y = \frac{1}{2}x + \left(\frac{1}{4} - \frac{\pi}{8}\right)$. It is given that the line L intersects the y-axis at the point Q and is a tangent to the curve C at the point P where $x = -\frac{1}{2}$.

(i) Find the y-coordinates of P and Q. [2]

The region R is bounded by the line L, the curve C and the y-axis.

(ii) Find the exact volume of the solid generated when R is rotated through 2π radians about the y-axis, giving your answer in the form $\frac{\pi}{8}(a-b)$ where a and b are positive constants to be found. [6]

6 With reference to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are perpendicular. A point P lies on AB between A and B such that $AP : PB = \lambda : 1-\lambda$, $0 < \lambda < 1$.

(i) Show that $\cos(\angle AOP) = \dfrac{(1-\lambda)|\mathbf{a}|}{|(1-\lambda)\mathbf{a} + \lambda\mathbf{b}|}$. [4]

(ii) Prove that $\left[(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\right]\bullet\left[(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\right]=(1-\lambda)^2|\mathbf{a}|^2+\lambda^2|\mathbf{b}|^2$. Hence, given also that OP bisects $\angle AOB$, find the ratio of $\dfrac{|\mathbf{a}|}{|\mathbf{b}|}$, leaving your answer in terms of λ. [6]

7 The rate of temperature loss of an animal corpse can be estimated using Newton's Law of Cooling, which states that the rate of change of temperature θ°C, t hours after death of an animal is proportional to the difference between its body temperature θ°C and the surrounding temperature θ_0°C, where $\theta > \theta_0$.

(i) Write down a differential equation for this situation. Solve this differential equation and show that the general solution of the above differential equation is given by $\theta = \theta_0 + Ae^{-kt}$, where A and k are positive constants. [3]

It is given that $\theta_0 = 24$, the initial value of θ is 36 and the initial rate of temperature loss is 2.5 °C per hour.

(ii) Calculate the exact values of A and k. [3]

(iii) Hence, sketch the graph of θ against t. [2]

(iv) Explain why the rate of change of temperature of an animal corpse cannot be modelled by a constant rate of decrease of 1,5°C. [1]

8 **(a)** It is given that the equation $3z^3 + az^2 + bz + c = 0$, where a, b and c are real numbers, has roots $\frac{5}{3} - \frac{\sqrt{11}}{3}\mathrm{i}$ and -2. Find the integer values of a, b and c. [4]

(b) It is known that a complex number $w = \dfrac{e^{i\theta} + e^{i\phi}}{e^{i\theta} - e^{i\phi}}$, where $\theta - \phi \neq 2n\pi$ and $\theta > \phi$ for any integer n.

(i) Show that $w = e^{-i\frac{\pi}{2}}\left(\cot\dfrac{1}{2}(\theta - \phi)\right)$. [3]

(ii) Hence, find $|w|$ and $\arg(w)$. **[2m]**

9 The coach of Besto running club designed a training programme such that runners begin with a 400 m run on the first training session. On each subsequent session, the distance covered is 250 m more than the distance covered on the previous session.

(i) Find the minimum number of sessions required for runners from Besto on the training programme to run at least 20 km in a training session. [3]

For another group of runners in Besto, a circuit training exercise was designed to build up their stamina.

In this exercise, this group of runners from Besto run from a starting point O to and from a series of points, $P_1, P_2, P_3, \cdots$, increasingly far away in a straight line. In the exercise, they start at O and run stage 1 from O to P_1, and back to O, then stage 2 from O to P_2, and back to O, and so on.

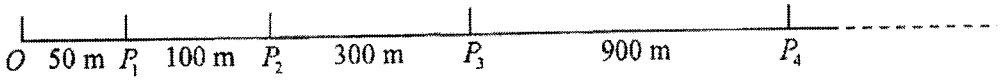


Fig. 1

The distances between the points are such that $OP_1 = 50\,\text{m}$, $P_1P_2 = 100\,\text{m}$, $P_2P_3 = 300\,\text{m}$ and $P_nP_{n+1} = 3P_{n-1}P_n$ (see Fig. 1).

(ii) Find an expression for the distance run by a runner from Besto who completes n stages of the circuit training exercise. [3]

(iii) Hence, find the distance from O and the direction of travel, of a runner from Besto undergoing the circuit training exercise after he has run exactly 42 km. [3]

Another running club, Choco, designed a different training programme. The runners in Choco began with running 400 m on the 1st session. On each subsequent session, the distance covered was increased by 10% of the distance covered on the previous session. From the 11th session onwards and for all subsequent sessions, the distance covered was increased by r% of the distance covered on the previous session.

(iv) Given that the runners from Choco club covered at least 20 km on the 70th training session, find the range of values of r. [4]

10 The diagram below shows the floorplan of a square lawn with a circular pond with its centre coinciding with the centre of the square lawn (see Fig. 2).

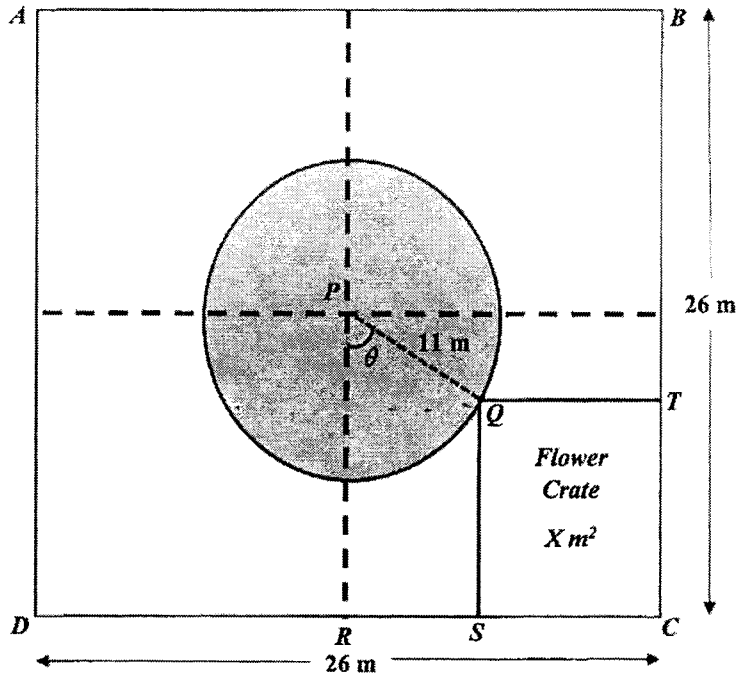


Fig. 2

The floorplan consists of a square lawn $ABCD$ of side 26 m with a circular pond of radius 11 m built at the centre P of the square lawn. The owner intends to build a rectangular flower crate with base $QTCS$, with one corner of the base at C, and the opposite corner Q of the base, touching the circular pond, where angle $RPQ = \theta$ radians, measured from RP and $0 \le \theta \le \frac{\pi}{4}$. The base of the flower crate has area X m^2.

(i) By finding an expression of X in terms of θ, show that $\frac{dX}{d\theta} = 11(\sin\theta - \cos\theta)(13 - 11\sin\theta - 11\cos\theta)$.

[3m]

(ii) For stationary values of X, show that the corresponding values of θ, given by θ_1 and θ_2 satisfy the equations $\tan\theta_1 = 1$ and $\sin(\theta_2 + \alpha) = k$ respectively, where k and α are constants in exact form. Hence, find the values of θ_1 and θ_2. **[4m]**

(iii) Determine which of the values of θ found in part **(ii)** give a minimum value of X and which give a maximum value of X, and find these values. [3]

(iv) The owner wishes to build 4 identical flower crates with corners at A, B, C and D respectively, and he intends to cover the rest with grass. Find the smallest area of the square lawn to be covered by grass, giving your answers to 3 significant figures. [3]

SAJC 2022 P2

Q1

Section A: Pure Mathematics (40 marks)

A sequence $u_1, u_2, u_3, \ldots$ is such that $u_n = \frac{1}{n!}$ for $n \in \mathbb{Z}^+$ and $u_n = u_{n-1} + \frac{1}{n^2 - 1}$ for all $n \geq 2$.

(i) Find $\sum_{n=2}^{N} \frac{1}{(n+1)(n-1)}$. [3]

(ii) Give a reason why the series in (i) is convergent and state the sum to infinity.

[2m]

(iii) Hence find $\sum_{n=8}^{N+5} \frac{1}{n(n-2)}$ in terms of N.

[3m]

2 The equations of three planes Π_1, Π_2 and Π_3 are

$$x - py + z = 9, \quad 3x - y - 2z = 10 \quad \text{and} \quad x - ay - z = 5$$

respectively, where a and p are constants.

The line l_1 has equation $\mathbf{r} = 2\mathbf{i} - 4\mathbf{j} + 3\mathbf{k} + \mu(\mathbf{i} - \mathbf{j} + 3\mathbf{k})$, where $\mu \in \mathbb{R}$.

(i) Given that l_1 does not intersect with Π_1, show that $p = -4$ and find the shortest distance between the line l_1 and Π_1. [3]

(ii) The line l_2 is the reflection of the line l_1 in Π_1. Hence, or otherwise, find the Cartesian equation of the plane that contains the line l_2 and is parallel to Π_1.

[2m]

(iii) Given that the line l_3 lies on both Π_2 and Π_3, find a vector equation of l_3, leaving your answer in terms of a. [3]

(iv) Let θ be the acute angle between l_3 and Π_1. Find the value(s) of a if $\sin\theta = \frac{\sqrt{3}}{18}$. [3]

3 In this question you may use expansions from the List of Formulae (MF26).

(i) Find the Maclaurin expansion of $\ln(1+\cos 3x)$ in ascending powers of x, up to and including the term in x^4, for $0 \leq x < \frac{\pi}{3}$. [4]

(ii) Use your expansion from part **(i)** and to find an approximate value for $\int_0^{0.5} x\ln(1+\cos 3x)\,dx$, giving your answer to 5 decimal places. [2]

(iii) Use your calculator to find the value of $\int_0^{0.5} x \ln(1+\cos 3x)\, dx$ up to 5 decimal places. [1m]

(iv) With the aid of a suitable diagram, comparing your answers in (ii) and (iii), comment on the accuracy of your approximations. [2]

4 A curve C has parametric equations

$$x = t^2 - 9 \ , \ y = t^3 - 12t \ ,$$

where $t \in [-3, 3]$.

(i) Sketch the graph of C, indicating the coordinates of the end points. [2]

The normal to the curve C at the point P is given by $9y + 2x = 83$.

(ii) By finding the gradient of the curve C at the point with parameter t, calculate the value of t at the point P. [3]

Given that the normal to curve C at P intersects the curve C at another point Q with parameter q.

(iii) Show that $9q^3+2q^2-108q-101=0$ at Q. Hence, find the coordinates of Q. [4]

(iv) Find the area bounded by the curve C and the normal to the curve at point P, giving your answers correct to 3 significant figures. [3]

Section B: Probability and Statistics (60 marks)

5 A group of 10 people consists of 9 men and 1 woman. Find the number of ways which the group can be seated at a round table with 10 identical chairs if

(i) 2 particular men, Caleb and James are not seated beside the woman, but are seated next to each other. [3]

The 10 identical chairs at the table are replaced with 10 chairs of different colours.

(ii) Find the number of ways in which the group can be seated at the round table if Caleb, James and the woman are not all seated next to one another. [3]

6 On average, 30% of the students in Saints Senior Institute could solve the differentiation question in Paper One of the Preliminary Examinations. The Head of Department randomly selects a class to analyse the results. You may assume that the number of students who could solve the differentiation question follows a binomial distribution.

Given that there are 30 students in the class, find

(i) the probability that at least 6 students in that class could solve that question. [2]

(ii) the probability that only 2 students among the first 8 selected students in that class could solve the question given that at least 6 students could solve that question. [3m]

Another class with n students is then randomly selected.

(iii) It is known that the probability of no more than 5 students could solve the question in a randomly selected class exceeds 0.9. Find the largest possible number of students in that class. [3]

(iii) The relationship between M and P can also be modelled by an equation of the form $\ln P = aM + b$, where a and b are constants. Using the scatter diagram in **(ii)**, explain whether a is positive or negative. Find the product moment correlation coefficient between M and $\ln P$. [2]

(iv) Using **(ii)** and **(iii)**, explain which of $P = 0.92588M + 69.804$ or $\ln P = aM + b$ is the better model. [2]

(v) John is 50 years old. His BMI is 35 kg/m^2. Comment on whether it is reliable to estimate his DBP using the better model in **(iv)**. [1]

8 In a game, two boxes have the following contents.

Box A contains 3 green balls and 4 white balls.

Box B contains 6 green balls and 5 white balls.

A fair die is thrown once. Two balls are then drawn in sequence in the following manner:

1. If the number that appears on the top face of the die is less than 3, the first ball is drawn from Box A and transferred to Box B. Otherwise, the first ball is drawn from Box B and transferred to Box A.
2. The second ball is then randomly picked from the box that received the transferred ball in step 1.

(i) Draw a probability tree diagram to represent the above information. [2]

(ii) If the second ball drawn is white, the player wins the game. Otherwise, the player loses the game. Find the probability that the first ball drawn is from Box A, given that the player wins the game. [5]

9 A random variable X has the probability distribution given as

$$\mathrm{P}(X=x)=\begin{cases} p, & x=2,5 \\ q, & x=3,4 \\ 0, & \text{otherwise} \end{cases}$$

(i) Given that $\mathrm{E}\left(X^2\right)=13.3$, find the values of p and q. Hence, without the use of the calculator, find $\mathrm{Var}(X)$. [5]

(ii) Thirty independent observations of X are taken. Using a suitable approximation, find the probability that the mean value of these observations exceeds 3.8. [2]

10 In this question you should state clearly the parameters of any distributions that you use.

A supermarket sells honeydews and watermelons. The masses, in kilograms, of the honeydews and the watermelons each follow a normal distribution. The means and standard deviations of these distributions are shown in the following table:

	Mean (kg)	Standard deviation (kg)
Honeydew	1.5	0.2
Watermelon	8.5	0.3

You may assume that the masses of the fruits (watermelon and honeydew) are independent of one another.

(i) Find the probability that for 3 randomly chosen honeydews, two of the honeydews each has mass less than 1.8 kg and one of the honeydews has mass more than 1.8 kg.

[2m]

(ii) Find the probability that the total mass of 5 randomly chosen honeydews is less than the mass of one randomly chosen watermelon. [3]

The supermarket wants to pack fruits into gift packs to be donated to needy families. Each gift pack consists of one randomly chosen honeydew and one randomly chosen watermelon.

(iii) 90% of the gift packs have masses differ from the mean mass of gift packs by less than m kg, find the value of m. You may assume that the packing material has negligible mass. [3]

The selling price of honeydews is \$3.50 per kilogram and the selling price of watermelons is \$0.70 per kilogram. Lam has a budget of \$10.

(iv) Lam intends to buy one honeydew and one watermelon. Find the probability that Lam is able to pay for his purchase. [3]

Let the probability that a honeydew and watermelon cost at most \$5 **each** be k.

(v) Explain, without any further calculation, why the probability in (iv) is at least k. [1]

11 The branch manager of a bank would like to find out about the satisfactory level of the customer services provided by the branch. He would like to survey 80 customers of the branch to find out about their opinion on the wait time.

(i) Describe how the branch manager could obtain a random sample of 80 customers to conduct his survey. [2]

The mean amount of time a customer needs to wait in the queue until they were served was known to be at least 15 minutes. After the survey was conducted, he realised that the waiting time for each customer before they were served at the counter was of a major concern. A change in processes at the branch was implemented. After a month, the branch manager of the bank decided to record the waiting time, t minutes, for a customer at the branch for 50 different customers to evaluate if the changes were effective. The results are summarised by

$$\sum(t-15)=-60\ ,\ \sum(t-15)^2=1168\ .$$

(ii) Find the unbiased estimates of the population mean and variance. [2]

(iii) Test, at the 5% level of significance, whether the change in processes have been effective. [4]

(iv) Explain what is meant by the p-value obtained in **(iii)** in the context of the question. [1]

(v) The quality service manager claimed that that the mean waiting time before there were any changes in processes was in fact k minutes. With the same data collected from the same 50 customers, find the range of values of k if there is insufficient evidence to conclude that there was a change in the mean waiting time at 2% level of significance. [3]

St Andrew's Junior College
2022 Preliminary Examination
H2 Mathematics Paper 1 Examiner's Comments (9758/01)

General Comments

For students who failed to score or did not pass the paper, they face with the following issues:

1) Poor grasp of O level and basic knowledge and skills – Trigonometry knowledge (Q4, 10), fundamentals of graphing, recognising number patterns (Q9) and solving simple equations. This included mensuration knowledge such as volume of cone (Q4).
2) This paper reflected an extremely weak foundation in trigonometry – special angles, solving of trigonometric equations and inverse trigonometry.
3) There were also instances of poor attention paid to calculations and algebraic manipulations.
4) With the number of questions reduced to 10 compared to Block Test, many students are still struggling in completing the paper, leaving too little time to solve the Application Questions! All students are reminded to improve your time management skills and use the most efficient method for each question to be able to complete the paper in time.
5) Both Application Questions were poorly attempted and done. It is unwise to think that you only need to complete the first few questions well. AQs have the highest weightage for the entire paper.
6) Q1, 9 and 10 exposed the weakness of students' reading and comprehension skills.
7) Questions such as Q2, 4, 6, 9 and 10 showed that students lack the ability to recognise that parts within the question are actually linked to one another.
8) For some questions, you would also need to identify key words and instructions and answer to the question. Many students failed to adhere to the requirement stated resulting in loss of marks.
9) There was a lack of well presented and elegant responses. Some scripts were found to be very disorganised.

For the upcoming A level examination preparation, students are reminded of the following:

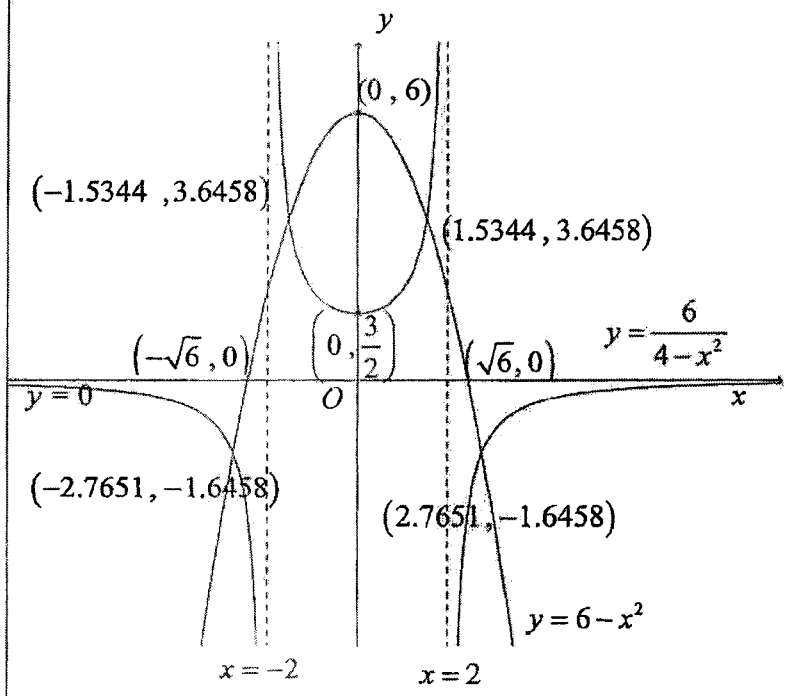
1) Importance of using precise mathematical language and notations. For eg, the use of 'mirror images' instead of reflection for Q3. Students need to be aware of the technical vocabulary in a discipline course like Maths.
2) The need to draw diagrams and use of tables/visual representations, wherever necessary, for better visualisation and understanding of the question.
3) Revise thoroughly your O level pre-requisite knowledge, especially Trigonometry, which can be found in your notes and previous Revision Packages.
4) Learn to solve problems more effectively and efficiently. Solutions should also aim to be elegantly written.
5) Students will need to brush up on the mastery of topics such as Complex Numbers, APGP and Application of Differentiation.
6) In particular, students are advised to do more timed practice independently to speed up solving processes.
7) Practice and expose oneself to more Application Questions, applying reading and comprehension skills effectively.

Q	Solution	Comments
1(i)	$4(x+y)^2+(x-y)^2=20$ ---- (1) Differentiate (1) with respect to x $8(x+y)\left(1+\frac{dy}{dx}\right)+2(x-y)\left(1-\frac{dy}{dx}\right)=0$ $8(x+y)+8(x+y)\frac{dy}{dx}+2(x-y)-2(x-y)\frac{dy}{dx}=0$ $10x+6y+(8x+8y-2x+2y)\frac{dy}{dx}=0$ $\frac{dy}{dx}=-\frac{10x+6y}{6x+10y}=-\frac{2(5x+3y)}{2(3x+5y)}$ $\frac{dy}{dx}=-\frac{5x+3y}{3x+5y}$ (Shown)	Most students can differentiate implicitly correctly, with only a minority making y the subject first (which should not be the case). Some students did the expansion of the given equation before differentiating implicitly, which is also correct but time-consuming. It is important to use an efficient method and not do unnecessary work. **Common error:** Some students did not show the details of how the given answer was derived. **Learning point/Reminders:** For show/proof questions, details are required! The amount of work needed to arrive at the answer is more compared to questions that are not show/proofs in nature.
(ii)	Since the tangents are perpendicular to the line $y=x$, hence the gradient of tangents = - 1 $\frac{dy}{dx}=-\frac{5x+3y}{3x+5y}=-1$ $5x+3y=3x+5y$ $2x=2y$ $x=y---(*)$ Substituting into (1);	**Common Error:** 1. Students substituted the relation $y = x$ into the equation without any reasoning. (***Note:*** one needs to understand that it was a coincidence that the substitution was identical to the line provided) 2. Some students did not understand what was meant by 'perpendicular to the line $y = x$'. This resulted in incorrect gradients being deduced. **Learning points:** One should try to interpret and state the information provided in a mathematical form.

Q	Solution	Comments
	$4(x+x)^2+(x-x)^2=20$ $4(2x)^2=20$ $4x^2=5$ $x^2=\frac{5}{4}$ $x=\pm\frac{\sqrt{5}}{2}$ Given $y=x$ from (*) When $x=\frac{\sqrt{5}}{2}$, $y=\frac{\sqrt{5}}{2}$ When $x=-\frac{\sqrt{5}}{2}$, $y=-\frac{\sqrt{5}}{2}$ Hence, the points are $\left(\frac{\sqrt{5}}{2},\frac{\sqrt{5}}{2}\right)$ and $\left(-\frac{\sqrt{5}}{2},-\frac{\sqrt{5}}{2}\right)$ $y-\frac{\sqrt{5}}{2}=-\left(x-\frac{\sqrt{5}}{2}\right)$ $=-x+\frac{\sqrt{5}}{2}$ $y=-x+\sqrt{5}$	**Reminder:** Gradient of tangent $=\frac{dy}{dx}$ Students need to simplify the values of x and y. Students need to deduce that both positive and negative values of x and y are acceptable, and they need to find the equations of tangents for both values. Hence there will be 2 equations of tangents.

Q	Solution	Comments
	$y-\left(-\frac{\sqrt{5}}{2}\right)=-\left(x-\left(-\frac{\sqrt{5}}{2}\right)\right)$ $=-x-\frac{\sqrt{5}}{2}$ $y=-x-\sqrt{5}$ The equation of the tangents are $y=-x+\sqrt{5}$ and $y=-x-\sqrt{5}$.	**Checkpoint of answer?** Note that the tangents are perpendicular to the line $y=x$. Hence, gradient of tangent $=-1$ which could be observed from equation of tangent.

Q	Solution	Comments
2(i)	$y=\frac{6}{4-x^2}=\frac{6}{(2-x)(2+x)}$ Asymptotes are $x=2, x=-2, y=0$ Intersections with axes: When $x=0$, $y=\frac{6}{2(2)}=\frac{3}{2}$ (Also the stationary point) $y=6-x^2$ Intersections with axes: When $x=0$, $y=6 \Rightarrow (0, 6)$ When $y=0$, $6-x^2=0$ $x^2=6$ $x=\sqrt{6}$ or $-\sqrt{6}$ $(\sqrt{6}, 0)$ or $(-\sqrt{6}, 0)$	For all graphs, students should have the habit of calculating the features of graph, namely: Asymptotes Intersections with axes (x and y) Stationary points This is to help ensure that 1) Calculations are accurate, especially when exact coordinates are required, and 2) No features of graphs are missed out.

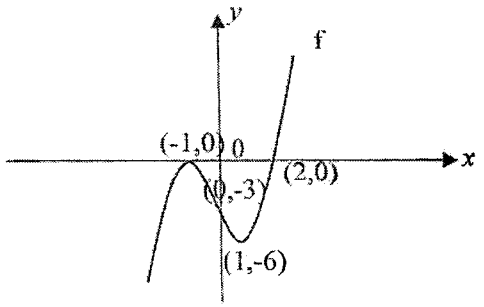
Q	Solution	Comments
2 (i)		For this question, most students were able to produce a graph which is of acceptable standard. However, students need to take note of the following: 1) Shape of graphs: One should note that both graphs are **symmetrical** about the y axes. This could be observed through the coordinates of the intersections with the x-axis as well as the vertical asymptotes for each of the graphs. 2) x and y axes are perpendicular to each other. So are horizontal and vertical lines! 3) For the graphs in part (i), **all** coordinates for turning/stationary points and intersections with axes as well as the equations of asymptotes are to be clearly labelled! 4) All equations of graphs should also be clearly labelled together with the origin being indicated. **Common errors:** 1) Calculating coordinates incorrectly (for both intersections with x and y axes) 2) No indicating of coordinates in exact form/equations of asymptotes as required. 3) Missing out the horizontal asymptote $y = 0$.

Q	Solution	Comments
		4) Incorrect placement of points. For example: $\left(0, \frac{3}{2}\right)$ lies in between the two vertical asymptotes! (**Note:** $\sqrt{6} > 2$) For a small group of students, it is alarming that one does not know a) the general equation of vertical/horizontal lines, and/or b) how to write a coordinate of a point (O-level work).
(ii)	The x-coordinates of the intersection points between the graphs $y = \frac{6}{4-x^2}$ and $y = 6 - x^2$ are -2.77, -1.53, 1.53 and 2.77 (to 3 sig. fig.) For $\frac{6}{4-x^2} < 6 - x^2$ From the graph above, $-2.77 < x < -2$ or $-1.53 < x < 1.53$ or $2 < x < 2.77$	**Common issues/errors:** 1. Some students were unable to identify the correct range of x to solve the inequality 2. Students writing the end answer with comma (,) instead of 'or' which represents a union of ranges 3. Incorrect rounding of values 4. Incomplete justification for range provided 5. Incorrect method of solving inequality by algebraic methodsp **Comments:** A large number of students are unable to see that the answer could be derived from (i) of the question. The inequality $\frac{6}{4-x^2} < 6 - x^2$ is basically comparing the y values of the two graphs drawn in (i). To solve graphically, the minimal requirement is to find out and indicate/state the x-coordinates of the intersection points required. Otherwise, the solution will not make sense. Solving this question by algebraic means is **not** recommended.

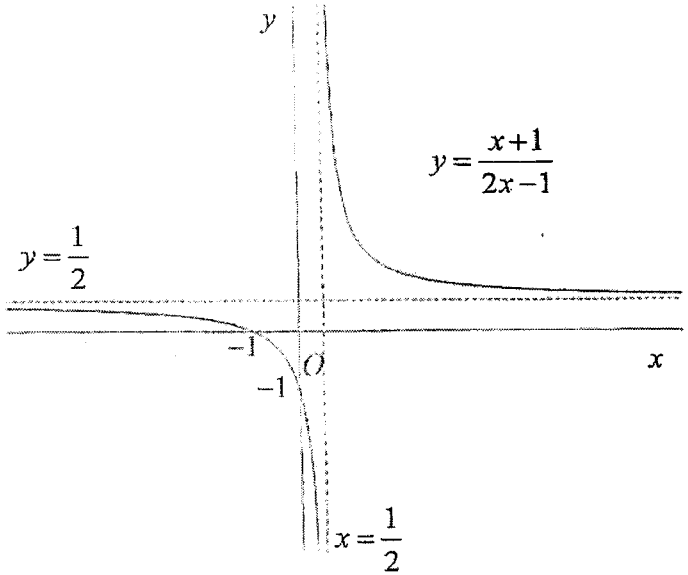
Q	Solution	Comments
		For students who tried to solve the inequality algebraically, it is worrying to know that students have forgotten the way to do so. Students need to be mindful that the word 'hence' may not always be there even though there might be a connection between the parts. Finally, all solutions which are **NON-EXACT** should be rounded off to 3 significant figures as required, unless otherwise stated.
(iii)	Replace x with $-x$, after the reflection about the y-axis, the solution is: $\Rightarrow 2 < x < 2.77$ or $-1.53 < x < 1.53$ or $-2.77 < x < -2$ Replace x with $x+4$, after the translation of 4 units in the negative x direction, $\Rightarrow 2 < x+4 < 2.77$ or $-1.53 < x+4 < 1.53$ or $-2.77 < x+4 < -2$ $\Rightarrow -2 < x < -1.23$ or $-5.53 < x < -2.47$ or $-6.77 < x < -6$ the solution set is therefore $\{x \in \mathbb{R} : -6.77 < x < -6$ or $-5.53 < x < -2.47$ or $-2 < x < -1.23\}$	Students need to understand the meaning of '**Deduce**'. There is a need to show **CLEARLY** all workings to derive the final solution, which also includes the need to write it as a set notation. Some students had the correct range deduced but failed to write it as a set form. There are students who are mixed up between the writing of a set through the inequality form or interval form! [REVISE] Also, there were two transformations provided, hence, both transformations will have to be accounted for before the final solution is provided. Again, there is no need to re-solve/re-draw the resulting graph (since the solution was to be deduced). For some students who tried, the wrong substitution was used especially for the translation. **Common mistake:** Replace x with $x-4$

Q	Solution	Comments
		Learning point: When the two graphs undergo a combination of linear transformations (i.e. scaling and translation), the corresponding range of x that satisfy the inequality will follow from (ii).

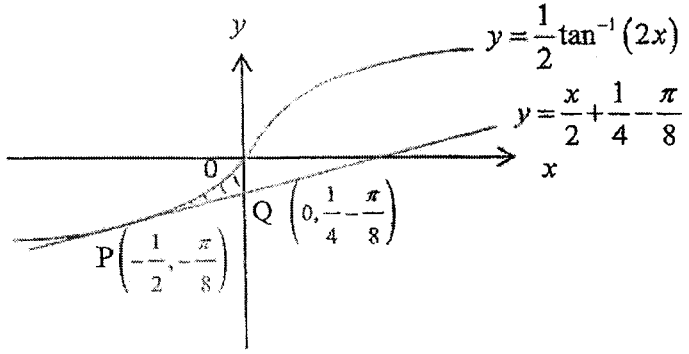
Q	Solution	Comments
3(i)	Given $f(-1)=f(2)=0$ but $-1\neq 2$ and $-1,2\in D_f$, f is not a one-to-one function. Hence, f does not have an inverse.	To explain the function is not 1 to 1 using the horizontal line test, students need to provide a specific line, e.g. $y=0$ instead of $y=k$ since it may not be true for all real values of k. Many students read the question wrongly and mentioned the line $y=-3$ cuts the graph of f twice at (-1, 0) and (2, 0). Students who used a specific line $y=k$ where $-3<k<0$ will need to elaborate more by showing a sketch as it is not obvious from the question. **Note:** Students should be familiar with both methods of justifying that the inverse of a function does not exist.
(ii)	Let $y=f(x)=ax^3+bx^2+cx+d$ Curve passes through $(0,-3)$ $d=-3$ $y=f(x)=ax^3+bx^2+cx-3$ Curve passes through $(-1,0)$ $-a+b-c=3$ ---------- (1) Curve passes through $(2,0)$ $8a+4b+2c=3$ ---------- (2) $\frac{dy}{dx}=3ax^2+2bx+c$ Tangent to the curve at $x=1$ is a horizontal line, $\frac{dy}{dx}=0$, $3a+2b+c=0$ ---------- (3)	Errors on this part mainly arise from incorrect simultaneous equations due to computation mistakes.

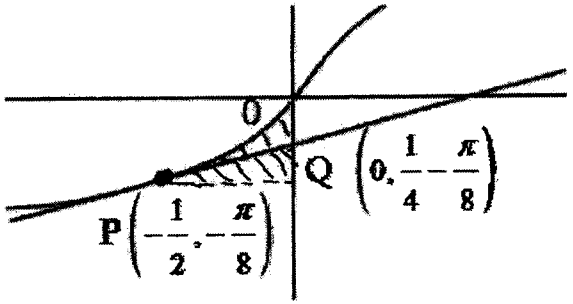
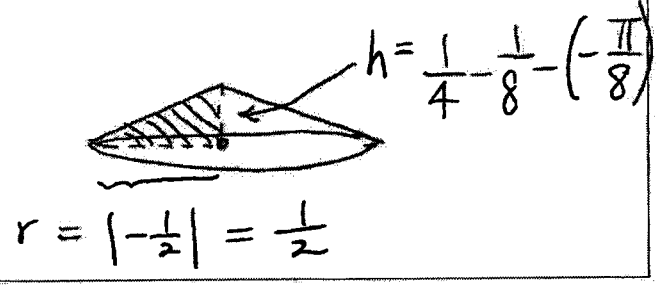
Q	Solution	Comments
	Solving (1), (2) and (3) using GC, $a=\frac{3}{2}$, $b=0$, $c=-\frac{9}{2}$, The equation of the curve is $y=\mathrm{f}(x)=\frac{3}{2}x^3-\frac{9}{2}x-3$	
(iii)		Many students did not label the axes and the origin properly. Others did not show a clear minimum point which is not at the y intercept. Some students did not indicate the required points in coordinate form as requested by the question. A number of students drew a quadratic graph when the expression is a cubic equation.
(iv)	Smallest $k=1$	Many students missed out on the word "smallest". **Note:** k can take on a range of values apart from 1. Requirement of k is stated in the question. Respond to the question!
(v)	The graphs of $y=\mathrm{f}(x)$ and $y=\mathrm{f}^{-1}(x)$ are reflections of each other about the line $y=x$.	Many students used inappropriate phrasing such as "along the line", "across the line", "symmetrical", "mirror images". The recommended words would be "reflection" and "about the line". Other students merely mentioned reflection without stating the line of reflection.

Q	Solution	Comments
(vi)	Since the graphs $y = f(x), y = f^{-1}(x)$ and $y = x$ intersect at the same point, the solution of $f^{-1}(x) = f(x)$ is the same as the solution of $f(x) = x$. $\Rightarrow \frac{3}{2}x^3 - \frac{9}{2}x - 3 = x$ $\Rightarrow 3x^3 - 11x - 6 = 0$ (shown) Solving the equation using GC, $x = 2.14$(3 s.f.) since $x \geq 1$	Students who got this part wrong did not realise that the graphs of $y = f(x), y = f^{-1}(x), y = x$ share a common intersection point. This could be observed by looking at the appropriate parts of the graph drawn in (iii). Inappropriate presentations include '$f(x) = f^{-1}(x) = x$' which implies that the rules , represented by $f(x)$ and $f^{-1}(x)$ is x as a whole, instead of solving for a common point of intersection. Many students tried to solve by algebraic method ended using quadratic formula when it is a **cubic** equation. Others who used GC **did not consider the domain of f** and failed to reject the values of x outside the domain of f.
(vii)	$R_f = [-6, \infty) \not\subseteq D_g = (-5, \infty)$ Hence gf does not exist	Many wrong comments were given. Students who were on the right track about checking whether R_f is not a subset of D_g did not verify it.

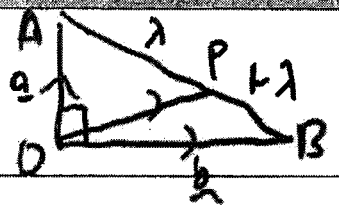
Q	Solution	Comments
4 (i)		**Common mistakes & suggestions for improvement:** • Horizontal asymptote not shown. Students are reminded that rational functions have either a horizontal or oblique asymptote. They will need to use long division to find it. • Did not label x- and y-intercepts /origin/ axes. Students are reminded to label all pertinent information/features of the curve
	From the graph above, $x \leq -1$ or $x > \frac{1}{2}$	**Common mistakes & suggestions for improvement:** • Many students used comma or and instead of "or". • Do not know when to have an equal sign in inequality. $x=-1$ satisfies the inequality, hence it has to be included in the solution and vertical asymptotes must be excluded. • Students are advised to write their final answers on a single line e.g $x \leq -1$ or $x > \frac{1}{2}$

Q	Solution	Comments
(ii)	$\int_{-2}^{0}\left\|\frac{x+1}{2x-1}\right\| dx$ $=\int_{-2}^{0}\left\|\frac{1}{2}+\frac{3}{2(2x-1)}\right\| dx$ $=\int_{-2}^{-1}\left(\frac{1}{2}+\frac{3}{2(2x-1)}\right) dx+\int_{-1}^{0}-\left(\frac{1}{2}+\frac{3}{2(2x-1)}\right) dx$ $=\left[\frac{1}{2}x+\frac{3}{4}\ln\lvert 2x-1\rvert\right]_{-2}^{-1}-\left[\frac{1}{2}x+\frac{3}{4}\ln\lvert 2x-1\rvert\right]_{-1}^{0}$ $=\left[-\frac{1}{2}+\frac{3}{4}\ln(3)-\left(-1+\frac{3}{4}\ln(5)\right)\right]-\left[0-\left(-\frac{1}{2}+\frac{3}{4}\ln 3\right)\right]$ $=-\frac{1}{2}+\frac{3}{4}\ln 3-\left(-1+\frac{3}{4}\ln(5)\right)-\frac{1}{2}+\frac{3}{4}\ln 3$ $=-1+\frac{3}{2}\ln 3+1-\frac{3}{4}\ln 5$ $=\frac{3}{2}\ln 3-\frac{3}{4}\ln 5$ (Shown)	**Common mistakes & suggestions for improvement:** • Many students did not realise that they have to split the definite integral limits using the graph in part (i) based on range of values of x for which the graph is above x-axis and below x-axis. [*Reference*: definition of modulus functions: $\lvert f(x)\rvert=\begin{cases} f(x) & , \text{ if } f(x)\geq 0 \\ -f(x) & , \text{ if } f(x)<0 \end{cases}$] • Several students forgot that they must do long division in order to integrate improper fractions • A small number of students forgot to include modulus for logarithmic function in the answer and at the end had $\ln(-1)$ in their working • Students are reminded to show all working, including calculation or use of properties of logarithm in their working for show questions to be duly credited.

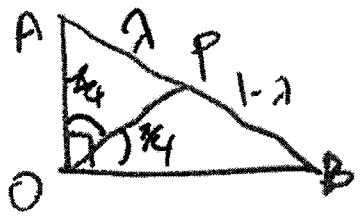
Q	Solution	Comments
5(i)	When $x=-\frac{1}{2}$, $y=\frac{1}{2}\tan^{-1}(-1)=\frac{1}{2}\left(-\frac{\pi}{4}\right)=-\frac{\pi}{8}$. When $x=0$, $y=\frac{0}{2}+\frac{1}{4}-\frac{\pi}{8}=\frac{1}{4}-\frac{\pi}{8}$ y-coordinate of $P=-\frac{\pi}{8}$; y-coordinate of $Q=\frac{1}{4}-\frac{\pi}{8}$	This part is generally well done. Reminders : • Leave answers in exact form in terms of π • Instead of leaving the answer as $\frac{1}{2}\tan^{-1}(-1)$, always evaluate your answer, especially for special angles • Answer the question by stating the y-coordinate
(ii)	 $y=\frac{1}{2}\tan^{-1}(2x)$ $2y=\tan^{-1}(2x)$ $\tan(2y)=2x$ $x=\frac{1}{2}\tan(2y)$	This part is very badly done. Students are advised to revise this topic and integration techniques thoroughly and understand the concepts well.

Q	Solution	Comments
	Required volume $= \pi \int_{-\frac{\pi}{8}}^{0} \left(\frac{1}{2}\tan(2y)\right)^2 dy - \frac{\pi}{3}\left(\frac{1}{2}\right)^2\left[\frac{1}{4} - \frac{\pi}{8} - \left(-\frac{\pi}{8}\right)\right]$ $= \frac{\pi}{4}\int_{-\frac{\pi}{8}}^{0} \tan^2(2y)\,dy - \frac{\pi}{48}$ $= \frac{\pi}{4}\int_{-\frac{\pi}{8}}^{0} \left(\sec^2(2y) - 1\right) dy - \frac{\pi}{48}$ $= \frac{\pi}{4}\left[\frac{1}{2}\tan(2y) - y\right]_{-\frac{\pi}{8}}^{0} - \frac{\pi}{48}$ $= \frac{\pi}{4}\left[0 - \left(-\frac{1}{2} + \frac{\pi}{8}\right)\right] - \frac{\pi}{48}$ $= \frac{\pi}{4}\left(\frac{1}{2} - \frac{\pi}{8}\right) - \frac{\pi}{48}$ $= \frac{\pi}{8}\left(\frac{5}{6} - \frac{\pi}{4}\right)$ units^3 Alternatively (more tedious mtd) : Required volume	**Common mistakes and reminders:** • As the area is rotated about the y-axis, it is incorrect to find the volume by integrating $\left(\frac{1}{2}\tan^{-1}(2x)\right)^2$ with respect to x. • Some students quoted the formula for volume of cone incorrectly! The correct formula is : Vol. of cone $= \frac{1}{3}\pi \times (\text{radius})^2 \times (\text{height})$ • Some identified the radius and/or height of the cone incorrectly. Correct radius and height :  

Q	Solution	Comments
	$=\pi\int_{-\frac{\pi}{8}}^{0}\left(\frac{1}{2}\tan(2y)\right)^2 dy-\pi\int_{-\frac{\pi}{8}}^{\frac{1}{4}-\frac{\pi}{8}}4\left(y-\left(\frac{1}{4}-\frac{\pi}{8}\right)\right)^2 dy$ $=\frac{\pi}{4}\int_{-\frac{\pi}{8}}^{0}\tan^2(2y)\,dy-4\pi\int_{-\frac{\pi}{8}}^{\frac{1}{4}-\frac{\pi}{8}}\left(y-\left(\frac{1}{4}-\frac{\pi}{8}\right)\right)^2 dy$ $=\frac{\pi}{4}\int_{-\frac{\pi}{8}}^{0}\left(\sec^2(2y)-1\right)\,dy-4\pi\int_{-\frac{\pi}{8}}^{\frac{1}{4}-\frac{\pi}{8}}\left(y-\left(\frac{1}{4}-\frac{\pi}{8}\right)\right)^2 dy$ $=\frac{\pi}{4}\left[\frac{1}{2}\tan(2y)-y\right]_{-\frac{\pi}{8}}^{0}-4\pi\left[\frac{\left(y-\left(\frac{1}{4}-\frac{\pi}{8}\right)\right)^3}{3}\right]_{-\frac{\pi}{8}}^{\frac{1}{4}-\frac{\pi}{8}}$ $=\frac{\pi}{4}\left[0-\left(-\frac{1}{2}+\frac{\pi}{8}\right)\right]-\frac{4}{3}\pi\left[\left(\frac{1}{4}-\frac{\pi}{8}-\left(\frac{1}{4}-\frac{\pi}{8}\right)\right)^3-\left(-\frac{\pi}{8}-\left(\frac{1}{4}-\frac{\pi}{8}\right)\right)^3\right]$ $=\frac{\pi}{4}\left(\frac{1}{2}-\frac{\pi}{8}\right)-\frac{4}{3}\pi\left[\left(\frac{1}{4}\right)^3\right]$ $=\frac{\pi}{4}\left(\frac{1}{2}-\frac{\pi}{8}\right)-\frac{\pi}{48}$ $=\frac{\pi}{8}\left(\frac{5}{6}-\frac{\pi}{4}\right)\text{ units}^3$	• Some students put a negative sign in front of $\pi\int_{-\frac{\pi}{8}}^{0}\left(\frac{1}{2}\tan(2y)\right)^2 dy$ without realising that $\pi\int_{-\frac{\pi}{8}}^{0}\left(\frac{1}{2}\tan(2y)\right)^2 dy$ is already positive as $\left(\frac{1}{2}\tan(2y)\right)^2\geq 0$ • Many do not know how to integrate $\tan^2(2y)$. For some of those who knew, they remembered the trigo identity wrongly • For those who used the alternative method, many expanded $\left(y-\left(\frac{1}{4}-\frac{\pi}{8}\right)\right)^2$ before integrating, which made their working very tedious

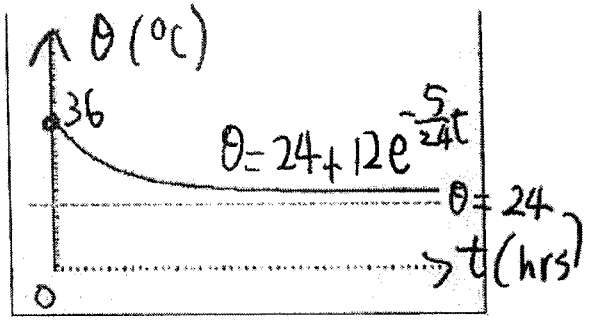
Q	Solution	Comments
6(i)	Using Ratio Theorem, $\overrightarrow{OP}=\dfrac{(1-\lambda)\mathbf{a}+\lambda\mathbf{b}}{1-\lambda+\lambda}$ $=(1-\lambda)\mathbf{a}+\lambda\mathbf{b}$  	**Common mistakes:** 1. Wrong formula for Ratio Theorem such as $\overrightarrow{OP}=\dfrac{(1-\lambda)\mathbf{a}+\lambda\mathbf{b}}{\lvert\mathbf{a}\rvert+\lvert\mathbf{b}\rvert}$ and $\overrightarrow{OP}=\dfrac{(1-\lambda)\mathbf{a}+\lambda\mathbf{b}}{\mathbf{a}+\mathbf{b}}$. 2. Writing of **aa** (note that this is meaningless) 3. Algebraic error: $\dfrac{(1-\lambda)\lvert\mathbf{a}\rvert^2+\lambda\mathbf{a}\bullet\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\rvert}\neq\dfrac{(1-\lambda)\lvert\mathbf{a}\rvert^2+\lambda\mathbf{a}\bullet\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\rvert}$ **Reminders:** 1. Dot in the dot product must be clearly seen 2. Put brackets for $(1-\lambda)\mathbf{a}+\lambda\mathbf{b}$ in the expansion of $\mathbf{a}\bullet[(1-\lambda)\mathbf{a}+\lambda\mathbf{b}]$ as $\mathbf{a}\bullet[(1-\lambda)\mathbf{a}+\lambda\mathbf{b}]\neq\mathbf{a}\bullet(1-\lambda)\mathbf{a}+\lambda\mathbf{b}$ 3. $\mathbf{a}\bullet\mathbf{a}\neq\mathbf{a}^2,[(1-\lambda)\mathbf{a}]\bullet[(1-\lambda)\mathbf{a}]\neq[(1-\lambda)\mathbf{a}]^2$ 4. $\mathbf{b}\bullet\mathbf{a}\neq-\mathbf{a}\bullet\mathbf{b}$ but $\mathbf{b}\times\mathbf{a}\neq-\mathbf{a}\times\mathbf{b}$
	$\cos(\angle AOP)=\dfrac{\overrightarrow{OA}\bullet\overrightarrow{OP}}{\lvert\overrightarrow{OA}\rvert\lvert\overrightarrow{OP}\rvert}$ $=\dfrac{\mathbf{a}\bullet[(1-\lambda)\mathbf{a}+\lambda\mathbf{b}]}{\lvert\mathbf{a}\rvert\lvert(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\rvert}$ $=\dfrac{(1-\lambda)\mathbf{a}\bullet\mathbf{a}+\lambda\mathbf{a}\bullet\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\rvert}$ $=\dfrac{(1-\lambda)\lvert\mathbf{a}\rvert^2+0}{\lvert\mathbf{a}\rvert\lvert(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\rvert}$ since $\mathbf{a}\bullet\mathbf{b}=0$ as **a** and **b** are perpendicular $=\dfrac{(1-\lambda)\lvert\mathbf{a}\rvert}{\lvert(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\rvert}$ (shown)	
(ii)	$[(1-\lambda)\mathbf{a}+\lambda\mathbf{b}]\bullet[(1-\lambda)\mathbf{a}+\lambda\mathbf{b}]$ $=(1-\lambda)\mathbf{a}\bullet(1-\lambda)\mathbf{a}+(1-\lambda)\mathbf{a}\bullet\lambda\mathbf{b}+\lambda\mathbf{b}\bullet(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\bullet\lambda\mathbf{b}$ $=(1-\lambda)^2\lvert\mathbf{a}\rvert^2+\lambda(1-\lambda)\mathbf{a}\bullet\mathbf{b}+\lambda(1-\lambda)\mathbf{b}\bullet\mathbf{a}+\lambda^2\lvert\mathbf{b}\rvert^2$ $=(1-\lambda)^2\lvert\mathbf{a}\rvert^2+\lambda^2\lvert\mathbf{b}\rvert^2$, since $\mathbf{a}\bullet\mathbf{b}=0$ given that **a** and **b** are perpendicular (Proven)	**Common mistakes and comments:** 1. As the question is a proving question, expansion of the expression needs to be shown. One cannot take the result provided for granted.

Q	Solution	Comments
	Given also that OP bisects $\angle AOB$, $\angle AOP = \frac{\pi}{4}$, $\cos\frac{\pi}{4} = \frac{(1-\lambda)\lvert\mathbf{a}\rvert}{\lvert(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\rvert}$ $\frac{1}{\sqrt{2}} = \frac{(1-\lambda)\lvert\mathbf{a}\rvert}{\lvert(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\rvert}$ $\frac{1}{2} = \frac{(1-\lambda)^2\lvert\mathbf{a}\rvert^2}{\lvert(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\rvert^2}$ $= \frac{(1-\lambda)^2\lvert\mathbf{a}\rvert^2}{\left[(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\right]\bullet\left[(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\right]}$ $= \frac{(1-\lambda)^2\lvert\mathbf{a}\rvert^2}{(1-\lambda)^2\lvert\mathbf{a}\rvert^2+\lambda^2\lvert\mathbf{b}\rvert^2}$ $(1-\lambda)^2\lvert\mathbf{a}\rvert^2+\lambda^2\lvert\mathbf{b}\rvert^2 = 2(1-\lambda)^2\lvert\mathbf{a}\rvert^2$ Hence $(1-\lambda)^2\lvert\mathbf{a}\rvert^2 = \lambda^2\lvert\mathbf{b}\rvert^2$ $\frac{\lvert\mathbf{a}\rvert^2}{\lvert\mathbf{b}\rvert^2} = \frac{\lambda^2}{(1-\lambda)^2}$ $\frac{\lvert\mathbf{a}\rvert}{\lvert\mathbf{b}\rvert} = \left\lvert\frac{\lambda}{(1-\lambda)}\right\rvert$	2. OP bisects $\angle AOB \not\Rightarrow P$ is the midpoint of AB (i.e. some mistook $\lambda = \frac{1}{2}$). Point P is a point on AB which the value of λ is still unknown! 3. The link from $\lvert(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\rvert^2$ to $(1-\lambda)^2\lvert\mathbf{a}\rvert^2+\lambda^2\lvert\mathbf{b}\rvert^2$ must be shown clearly. i.e., $\lvert(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\rvert^2 = \left[(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\right]\bullet\left[(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\right]$ $= (1-\lambda)^2\lvert\mathbf{a}\rvert^2+\lambda^2\lvert\mathbf{b}\rvert^2$ 4. Some students deduced $\frac{\lvert\mathbf{a}\rvert}{\lvert\mathbf{b}\rvert} = \frac{\lambda}{1-\lambda}$ by finding $\cos(\angle BOP)$ and setting $\cos(\angle AOP) = \cos(\angle BOP)$. However, as the question is a 'hence' question, the previous result needs to be utilised. 5. Reason must be stated to justify why $\frac{\lvert\mathbf{a}\rvert}{\lvert\mathbf{b}\rvert} = \left\lvert\frac{\lambda}{(1-\lambda)}\right\rvert \Rightarrow \frac{\lvert\mathbf{a}\rvert}{\lvert\mathbf{b}\rvert} = \frac{\lambda}{(1-\lambda)}$.



$\frac{\lambda}{1-\lambda} > 0$

Q	Solution	Comments
	$\frac{\lvert\mathbf{a}\rvert}{\lvert\mathbf{b}\rvert} = \pm\frac{\lambda}{1-\lambda} = \frac{\lambda}{1-\lambda}$, reject $-\frac{\lambda}{1-\lambda}$ since $0<\lambda<1$ and ratio of length is positive.	
7(i)	The rate of temperature change of a dead animal body is given by $\frac{d\theta}{dt} = -a(\theta-\theta_0)$, where $a>0$. $\frac{d\theta}{dt} = \frac{d\theta_{in}}{dt} - \frac{d\theta_{out}}{dt}$ $\frac{1}{(\theta-\theta_0)}\frac{d\theta}{dt} = -a$ $= 0 - a(\theta-\theta_0)$ Integrating both sides $\int \frac{1}{\theta-\theta_0}\,d\theta = -a\,dt$ $-a\int 1\,dt$ $\ln(\theta-\theta_0) = -at + C$, since $\theta-\theta_0>0$ where a and C are arbitrary constants. $\theta-\theta_0 = e^{-at+C} = Ae^{-kt}$, where $A=e^C$ and $k=a$ $\Rightarrow \theta = \theta_0 + Ae^{-kt}$ (Shown)	**Common mistakes & suggestions for improvement:** • Students are advised to avoid using the symbol for constants that is already found in the expression to be shown (in this case k). This is so that they don't get confused. For examples, a few students used $\frac{d\theta}{dt} = k(\theta-\theta_0)$, where $k<0$. In this case, they had a missing negative sign in the power $\theta = \theta_0 + Ae^{kt}$ and could not show the required result. • Students are also advised to make constants positive and indicate a negative sign in front if there is a rate of decrease e.g $\frac{d\theta}{dt} = -a(\theta-\theta_0)$, where a >0. • Many students did not use the information given $\theta>\theta_0$ before excluding the modulus sign in $\ln(\theta-\theta_0)$ • Many students did not use $\theta>\theta_0$ and end up using $A=\pm e^C$ which is incorrect in this case as sufficient information was given to for A to be positive
(ii)	$\theta_0 = 24$. When t=0 , $\theta = 36$ is $\frac{d\theta}{dt}$ = -2.5 ^{0}C, $\frac{d\theta}{dt} = -k(\theta-\theta_0)$, where $k>0$.	This part was generally well done. **Common mistakes & suggestions for improvement:** • Students need to take note that "initial value" and "initial rate" implied when $t=0$.

Q	Solution	Comments
	$-2.5 = -k(36\text{-}24) \therefore k = \frac{5}{24}$ Using $\theta = \theta_0 + Ae^{-kt}$ $36 = 24 + A$ $\therefore A = 12$	
iii	 $\theta = 24 + 12e^{-\frac{5}{24}t}$	**Common mistakes & suggestions for improvement:** • In context questions, students are reminded to label the axes appropriately with the relevant units. Several students used y and x without any units. • A number of students did not know that there was a horizontal asymptote. Students are reminded that exponential functions have a horizontal asymptote. In this case as $t \to \infty, e^{-\frac{5}{24}t} \to 0, \theta \to 24$, which is also the surrounding temperature.
iv	A constant rate of decrease is not possible as the temperature of the body of the dead animal will become 0°C or even negative at some point in time, which is lower than the surrounding temperature.	This part was poorly done. Several students quoted that the rate is proportional to the difference between its body temperature θ°C and the surrounding temperature θ_0°C and hence it cannot be constant. They need to answer the question using the new model that now the rate of decrease is a constant and explain what happens to the temperature of the dead animal in this case and what makes the model not possible.

Q	Solution	Comments
8(a)	Since the equation has all real coefficients, complex roots occur in complex conjugate pairs. (not negotiable) Since $z = \frac{5}{3} - \frac{\sqrt{11}}{3}i$ is a complex root $\Rightarrow$ its conjugate $z = \frac{5}{3} + \frac{\sqrt{11}}{3}i$ exists as a root of the equation. $(z-(-2))(z-\left(\frac{5}{3}-\frac{\sqrt{11}}{3}i\right))(z-\left(\frac{5}{3}+\frac{\sqrt{11}}{3}i\right))=0$ $(z+2)(z^2-\frac{10}{3}z+4)=0---(*)$ $z^3-\frac{4}{3}z^2-\frac{8}{3}z+8=0$ $\Rightarrow 3z^3-4z^2-8z+24=0---(\#)$ $\therefore a=-4, b=-8, c=24$	Many students were unable to articulate the reason for conjugate complex root to exist. Instead, quite a handful of students just cite "by conjugate root theorem" which is not advisable as there is **no such name registered officially**, unlike Pythagoras' theorem. **Common mistake:** Many students failed to realise that by writing $3z^3+az^2+bz+c=(z-(-2))(z-\left(\frac{5}{3}-\frac{\sqrt{11}}{3}i\right))(z-\left(\frac{5}{3}+\frac{\sqrt{11}}{3}i\right))$ both sides of the equation are actually **not** equal to each other. There are also a number of students who attempted to solve via the more tedious method, which involves substituting z = -2 and $z = \frac{5}{3} - \frac{\sqrt{11}}{3}i$ into the equation $3z^3+az^2+bz+c=0$. However most did not mange to solve for the unknowns successfully due to carelessness in algebraic manipulations.

Q	Solution	Comments
8b (i)	$w=\dfrac{e^{i\theta}+e^{i\phi}}{e^{i\theta}-e^{i\phi}}$ $=\dfrac{e^{i\left(\frac{\theta+\phi}{2}\right)}}{e^{i\left(\frac{\theta+\phi}{2}\right)}}\times\dfrac{e^{i\left(\frac{\theta-\phi}{2}\right)}+e^{-i\left(\frac{\theta-\phi}{2}\right)}}{e^{i\left(\frac{\theta-\phi}{2}\right)}-e^{-i\left(\frac{\theta-\phi}{2}\right)}}$ $=\dfrac{2\cos\left(\frac{\theta-\phi}{2}\right)}{2i\sin\left(\frac{\theta-\phi}{2}\right)}$ $=\dfrac{1}{i}\cot\left(\dfrac{\theta-\phi}{2}\right)$ $=-i\cot\left(\dfrac{\theta-\phi}{2}\right)$ $=e^{-i\frac{\pi}{2}}\cot\left(\dfrac{\theta-\phi}{2}\right)$	This part was poorly attempted by majority of the cohort. Advice: For this part, students have to analyse the end goal of the question which is to obtain $e^{-i\frac{\pi}{2}}\cot\left(\dfrac{\theta-\phi}{2}\right)$. Hence there is a need to modify the exponential expression in order to obtain $\left(\dfrac{\theta-\phi}{2}\right)$. The fastest way is to multiply by $e^{i\left(\frac{\theta+\phi}{2}\right)}$ to both numerator and denominator.

Q	Solution	Comments
	Alternative method:	The alternative method uses factor formulae.

$$w=\frac{e^{i\theta}+e^{i\phi}}{e^{i\theta}-e^{i\phi}}$$

$$=\frac{(\cos\theta+i\sin\theta)+(\cos\phi+i\sin\phi)}{(\cos\theta+i\sin\theta)-(\cos\phi+i\sin\phi)}$$

$$=\frac{(\cos\theta+\cos\phi)+i(\sin\theta+\sin\phi)}{(\cos\theta-\cos\phi)+i(\sin\theta-\sin\phi)}$$

$$=\frac{2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)+i\left(2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)\right)}{-2\left(\sin\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)\right)+i\left(2\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)\right)}$$

$$=\frac{\cos\left(\frac{\theta-\phi}{2}\right)\left(\cos\left(\frac{\theta+\phi}{2}\right)+i\sin\left(\frac{\theta+\phi}{2}\right)\right)}{\sin\left(\frac{\theta-\phi}{2}\right)\left(-\sin\left(\frac{\theta+\phi}{2}\right)+i\cos\left(\frac{\theta+\phi}{2}\right)\right)}$$

$$=\cot\left(\frac{\theta-\phi}{2}\right)\frac{\left(\cos\left(\frac{\theta+\phi}{2}\right)+i\sin\left(\frac{\theta+\phi}{2}\right)\right)}{i\left(\cos\left(\frac{\theta+\phi}{2}\right)+i\sin\left(\frac{\theta+\phi}{2}\right)\right)}$$

$$=\frac{1}{i}\cot\left(\frac{\theta-\phi}{2}\right)=e^{i\left(-\frac{\pi}{2}\right)}\cot\left(\frac{\theta-\phi}{2}\right)\text{ (Shown)}$$

Q	Solution	Comments
b (ii)	$\|w\| = \left\|e^{-i\frac{\pi}{2}} \cot\left(\frac{\theta-\phi}{2}\right)\right\| = \left\|e^{-i\frac{\pi}{2}}\right\| \left\|\cot\left(\frac{\theta-\phi}{2}\right)\right\| = \left\|\cot\left(\frac{\theta-\phi}{2}\right)\right\|$ $\arg(w) = \begin{cases} -\frac{\pi}{2}, & \text{if } \cot\left(\frac{\theta-\phi}{2}\right) > 0 \\ \frac{\pi}{2}, & \text{if } \cot\left(\frac{\theta-\phi}{2}\right) < 0 \end{cases}$	This part exposed students' lack of understanding of trigonometric properties. Note that $\cot\left(\frac{\theta-\phi}{2}\right)$ can be negative or positive. Hence there is a need to include the modulus sign. There are 2 cases to be considered for arg(w), depending on the sign of $\cot\left(\frac{\theta-\phi}{2}\right)$.

2022 JC2 Prelim Exam Paper 1 Application Questions (9) & (10)

Solving of Mathematics Application Questions

Step 1: Understanding the Problem	a) Read the problem thoroughly and you can help yourself by underlining the keywords, important conditions, assumptions and any relevant information. You may need to repeat this process once or twice to ensure that you understand the entire problem and that you do not miss out any key information. b) Make sure that you are clear about the requirements of the question such as which are the variables and which are the constants. If variables are not given, you are required to define them. c) At this stage, it might also be necessary for you to interpret various phrases and write it as mathematical statements or equations or expressions.
Step 2: Planning	a) Identify the topics, concepts and skills that the application questions are testing. There can be more than one topic /concept/ skill involved. b) Make the connection between parts of the question. Hence it is advisable that you scan through the entire question from the start to the end to identify possible links. c) Filter the information and select the most relevant ones for each part, before you start to solve.
Step 3: Solving	a) Implement the appropriate problem solving strategies which may include tabulating the information, guess and check, drawing a diagram etc. b) All the formulae and techniques (especially differentiation and integration) should be at your fingertips, so that you do not waste unnecessary time on searching through MF26. c) If the strategy you applied fails to work, revisit the problem to check whether you have misread the question or left out certain information. ***Tip: Every statement in the question will contain some form of information.***
Step 4: Check the Solutions/ Calculations	a) Check against the context of the Application Question on whether your answer makes any logical sense? Have you used GC to check the answers? b) Do not accept all answers blindly. You should always interpret the validity of your answers in the context of the Application Question.

Both Applications Questions were generally poorly done. Students are reminded that as application questions took up a heavy weightage of marks, it is important that you must have the stamina to carry on solving the paper and do well for both Application Questions!

Comments for Q(9)

Question 9 requires students to have the ability to list terms and recognize number patterns!

Students did not do well for this question for a number of reasons:

1) **Weak comprehension skills, failing to read the question carefully**– many students did not pick up the major key words , resulting in misinterpretation of the question. This is especially so in (i) and (iv).
2) **Poor number pattern recognition** – many students were unable to find the correct number pattern. For example, identifying incorrect number of terms, incorrect number of common ratio multiplied to a term
3) **Poor time management** – students spent too much time for the first 8 questions, resulting in having insufficient time for the last application question.
4) **Poor use of problem solving strategies** – it was observed that many students did not have the patience or skill to make of the table/listing of terms to correctly identify the number pattern required. This resulted in concluding an incorrect general form for terms/sum
5) **Disorganised** in workings without proper explanations
6) Many students also ignore the need to provide logical explanation when concluding a result. For eg, part (iii).
7) **Inability to make use of the right/efficient method in solving** – this is especially so in (iv) when r is **NOT** an integer

The coach of Besto running club designed a training programme such that runners begin with a 400 m run on the first training session. On each subsequent session, the distance covered is 250 m more than the distance covered on the previous session.

(i) Find the minimum number of sessions required for runners from Besto on the training programme to run at least 20 km in a training session. [3]

For another group of runners in Besto, a circuit training exercise was designed to build up their stamina.

In this exercise, this group of runners from Besto run from a starting point O to and from a series of points, $P_1, P_2, P_3, \cdots$, increasingly far away in a straight line. In the exercise, they start at O and run stage 1 from O to P_1, and back to O, then stage 2 from O to P_2, and back to O, and so on.

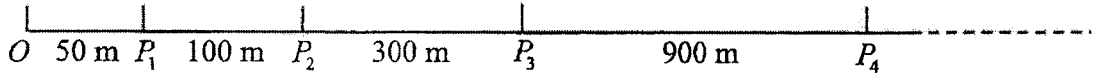


Fig. 1

The distances between the points are such that $OP_1 = 50$ m, $P_1P_2 = 100$ m, $P_2P_3 = 300$ m and $P_nP_{n+1} = 3P_{n-1}P_n$ (see Fig. 1).

(ii) Find an expression for the distance run by a runner from Besto who completes n stages of the circuit training exercise. [3]

(iii) Hence, find the distance from O and the direction of travel, of a runner from Besto undergoing the circuit training exercise after he has run exactly 42 km. [3]

Another running club, Choco, designed a different training programme. The runners in Choco began with running 400 m on the 1st session. On each subsequent session, the distance covered was increased by 10% of the distance covered on the previous session. From the 11th session onwards and for all subsequent sessions, the distance covered was increased by r% of the distance covered on the previous session.

(iv) Given that the runners from Choco club covered at least 20 km on the 70th training session, find the range of values of r. [4]

<table>
<tr>
<td>9(i)</td>
<td>Let T_n denote the distance covered by a runner from Besto on the nth training session.

Since T_n follows an arithmetic progression with common difference 250,
$T_n = 400 + 250(n-1)$
Given that $T_n \geq 20000$,
$400 + 250(n-1) \geq 20000$

<table><tr><td>n</td><td>T_n</td></tr><tr><td>79</td><td>$19900 < 20\,000$</td></tr><tr><td>80</td><td>$20150 > 20\,000$</td></tr><tr><td>81</td><td>$20400 > 20\,000$</td></tr></table>
The minimum value of n is 80.</td>
<td>A majority of students failed to see that the question is asking for the distance covered in a training session.

In step 1, the question to ask: 'is the question asking for total distance over n sessions OR distance covered in one session?'

Also, there was an instructional word 'at least'. This meant that an inequality should be formed with 20 km instead of an equation!

Common mistake:
1. Many students formulated the information as: $400 + 250(n-1) = 20000$
2. Many students thought that the question asked for the total sum of distance over n sessions.
3. Some students did not identify the minimum value for n (similar to Q3 on functions)
4. Some students were unable to formulate the general term.

Learning point:
Students should make it a habit to compare the values with the benchmark value so as to identify the correct minimum value.</td>
</tr>
<tr>
<td>(i) Alt</td>
<td>Let T_n denote the distance covered by a runner from Besto on the nth training session.

Since T_n follows an arithmetic progression with common difference 250,
$T_n = 400 + 250(n-1)$
Given that $T_n \geq 20000$,</td>
<td>Apart from those mentioned above, some students made careless algebraic mistakes in the calculation.

As the unknown to be solved is n, an integer, it is advised to make use of table of values as a more efficient method.</td>
</tr>
</table>

$400+250(n-1)\geq 20000$

$250(n-1)\geq 19600$

$n-1\geq 78.4$

$n\geq 79.4$

The minimum number of sessions for a runner to complete at least 20 km is 80.

(ii)

n	Total distance covered in the *nth* stage
1	$2(50)$
2	$2(50)+2(150)$ Do not simply oversimplify too early $=2(50)+2(3)(50)$ $=2(50)[1+3]$
3	$2(50)+2(150)+2(450)$ $=2(50)+2(3)(50)+2(3)^2(50)$ $=2(50)[1+3+3^2]$
...	...
n	$2(50)+2(3)(50)+2(3)^2(50)+\cdots+2(3)^{n-1}(50)$ $=2(50)[1+3+3^2+\cdots+3^{n-1}]$ $=100\left[\frac{1(3^n-1)}{3-1}\right]$ $=50(3^n-1)$

Many students were unable to deduce the correct expression. However, this was a question in your tutorial!

Analyse the question:
"distance run by a runner from Besto who completes n stages of the circuit training exercise."

Common errors:

1. Students are unable to formulate the terms in the table correctly, or used an inefficient form of terms, resulting in confusing oneself further.
2. Many do not understand the regime described.
3. Some students provided an incorrect final term.
4. Some students do not simplify the expression (This is rather serious).

Learning point:

1. Setting up the table of terms properly is helpful to identify patterns.
2. Proper headers in the table is also essential to help both the candidate and the examiner to understand the flow of solutions.

<table>
<tr>
<td>(iii)</td>
<td>To find the number of completed stages: $50(3^n-1) \le 42000$
<table>
<tr><td>n</td><td>$50(3^n-1)$</td></tr>
<tr><td>5</td><td>$12100 < 42\,000$</td></tr>
<tr><td>6</td><td>$36400 < 42\,000$</td></tr>
<tr><td>7</td><td>$109300 > 42\,000$</td></tr>
</table>
After completing 6 stages, the runner completed 36 400 m.

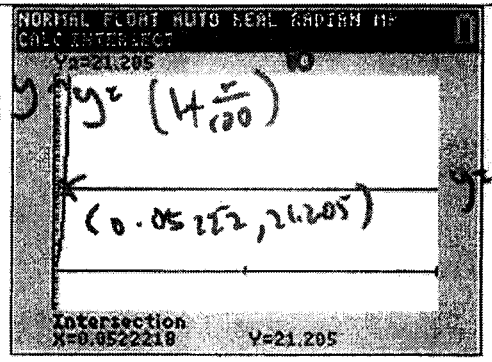
Distance remaining $= 42\,000 - 36\,400 = 5600$
Given that $OP_7 = 50 \times 3^6 = 36450 > 5600$, the runner from Besto is running away from O at a distance of 5600 m and has not reached P_7.</td>
<td>Students need to understand that
1. The non-integer value of n obtained cannot be fully used as a justification of the direction of travel.
2. A proper and logical explanation is required to justify the direction and distance.

This part was poorly attempted due to slipshod explanations and incorrect understanding of the question.</td>
</tr>
<tr>
<td>(iv)</td>
<td>On the 10^{th} session, a runner from Choco would have completed $400(1.1)^9$ m
From 11^{th} session onwards, using the new plan designed by Choco, a runner will complete $400(1.1)^9\left(1+\frac{r}{100}\right)^{60}$.

$400(1.1)^9\left(1+\frac{r}{100}\right)^{60} \ge 20000$

$(1+\frac{r}{100})^{60} \ge 21.205$

Let $x = \frac{r}{100}$</td>
<td>Common mistakes:
1. Misunderstanding of question! Many students misread the question, resulting in wrongly formulating the general terms!
2. One could actually make use of a table again to find out that the ratio $\left(1+\frac{r}{100}\right)$ should be raised to a power of 60.
3. Writing the ratio incorrectly as:
a. $(1.0r)^n$ (r is NOT a placeholder!)
b. r^n (forgetting that the distance is increased from previous sessions)
c. $(1+r)^n$ (r% was given, not r)
4. Not recognising that r is NOT an integer. This resulted in using an incorrect method (table of values) to determine the range of r.</td>
</tr>
</table>

	 From GC, $x \geq 0.05222$ $\frac{r}{100} \geq 0.05222$ $r \geq 5.222 = 5.22$ (to 3 sf)	Students are reminded to read the question carefully to pick out the key words to the question. For the graphical method, it would be good to replicate the screenshot on your solution. This replica does not need to be very detailed. The rationale is to allow the reader to understand how you deduced your answer. (see solution)

Comments for Q(10):

This question is generally poorly done.

Question (10) requires a sound grasp of trigonometry which is part of the Assumed Knowledge from O levels Additional Mathematics.

Students did not do well for this question for a few reasons:

1) **Weak foundation in Trigonometry** – many students were not able to solve trigonometric equations which is an O level skills;
2) **Poor algebraic manipulation skills**, which are essential for H2 Math
3) **Weak in differentiation skills** - some students could not differentiate correctly trigonometric functions
4) **Poor time management** – it was observed that students spent too much time for first 8-9 questions and did not have sufficient time for the last application question.
5) **Disorganised** in workings without proper explanations
6) Made **careless mistakes** unnecessarily
7) **Failed to read the question carefully**. In particular, for part (ii), the question requires students to show that the corresponding values of θ , given by θ_1 and θ_2 satisfy the equations $\tan\theta_1 = 1$ and $\sin(\theta_2 + \alpha) = k$ respectively. It is therefore non-negotiable that R formula must be used. However some students still went on to use GC to solve or other methods. By doing so, you will not be awarded any marks even if you obtained the same answers.

10 The diagram below shows the floorplan of a square lawn with a circular pond with its centre coinciding with the centre of the square lawn (see Fig. 2).

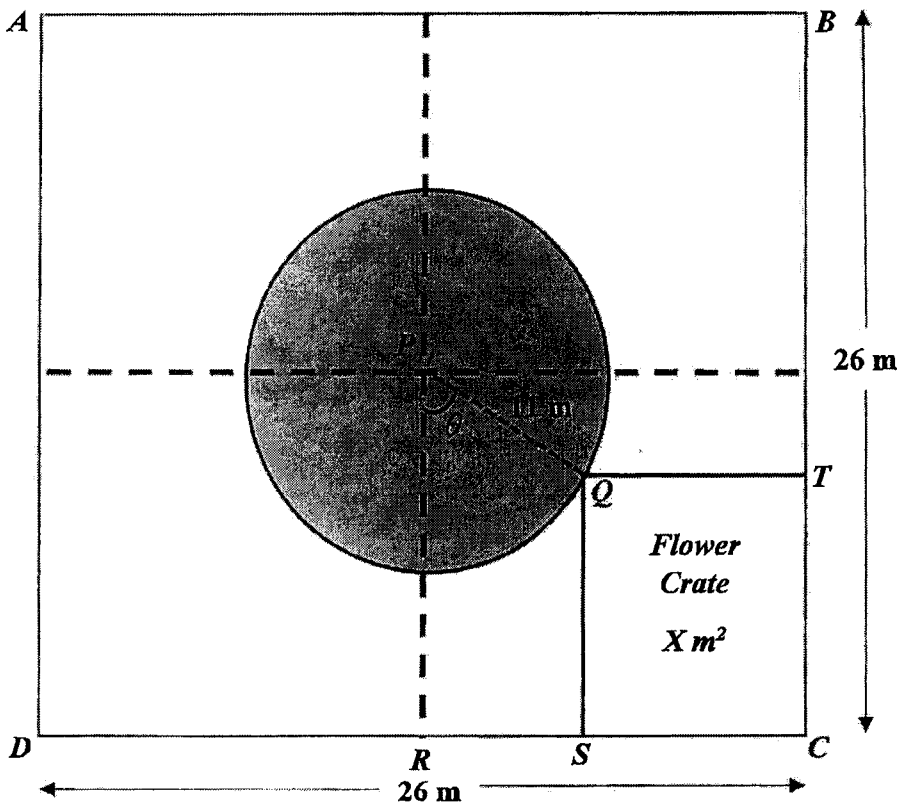


Fig. 2

The floorplan consists of a square lawn *ABCD* of side 26 m with a circular pond of radius 11 m built at the centre *P* of the square lawn. The owner intends to build a rectangular flower crate with base *QTCS*, with one corner of the base at *C*, and the opposite corner *Q* of the

base, touching the circular pond, where angle $RPQ = \theta$ radians, measured from RP and $0 \le \theta \le \frac{\pi}{4}$. The base of the flower crate has area $X\,\text{m}^2$.

(i) By finding an expression of X in terms of θ, show that $\frac{dX}{d\theta} = 11(\sin\theta - \cos\theta)(13 - 11\sin\theta - 11\cos\theta)$. [3]

(ii) For stationary values of X, show that the corresponding values of θ, given by θ_1 and θ_2 satisfy the equations $\tan\theta_1 = 1$ and $\sin(\theta_2 + \alpha) = k$ respectively, where k and α are constants in exact form. Hence, find the values of θ_1 and θ_2. [4]

(iii) Determine which of the values of θ found in part **(ii)** give a minimum value of X and which give a maximum value of X, and find these values. [3]

(iv) The owner wishes to build 4 identical flower crates with corners at A, B, C and D respectively, and he intends to cover the rest with grass. Find the smallest area of the square lawn to be covered by grass, giving your answers to 3 significant figures. [3]

10i	$X = (13 - 11\sin\theta)(13 - 11\cos\theta)$ $\frac{dX}{d\theta} = (-11\cos\theta)(13 - 11\cos\theta) + (13 - 11\sin\theta)(11\sin\theta)$ $= -143\cos\theta + 121\cos^2\theta + 143\sin\theta - 121\sin^2\theta$ $= 143\sin\theta - 143\cos\theta + 121(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$ $= 143\sin\theta - 143\cos\theta - 121(\sin\theta - \cos\theta)(\cos\theta + \sin\theta)$ $= 11(13(\sin\theta - \cos\theta)) - 121(\sin\theta - \cos\theta)(\cos\theta + \sin\theta)$ $= 11(\sin\theta - \cos\theta)(13 - 11\sin\theta - 11\cos\theta)$ (Shown)	Students are advised to use the most time efficient method to derive the expression for X. For the differentiation, many students did not manage to prove to the last part. Algebraic manipulation is a cohort wide weakness identified.

(ii)	$\frac{dX}{d\theta}=0$ $\Rightarrow 11(\sin\theta-\cos\theta)(13-11\sin\theta-11\cos\theta)=0$ $\Rightarrow \sin\theta-\cos\theta=0$ or $13-11\sin\theta-11\cos\theta=0$ $\Rightarrow \tan\theta=1$ *or* $11\sin\theta+11\cos\theta=13---(\#)$ Using *R*-formula to equation (1), $\sqrt{2}\sin(\theta+\alpha)=\frac{13}{11}$ where $\tan\alpha=1$ $\Rightarrow \alpha=\frac{\pi}{4}$ $\therefore \sqrt{2}\sin(\theta+\frac{\pi}{4})=\frac{13}{11}$ $\sin(\theta+\frac{\pi}{4})=\frac{13}{11\sqrt{2}}---(*)$ where $k=\frac{13}{11\sqrt{2}}$ and $\alpha=\frac{\pi}{4}$ Solving $\tan\theta=1$, $\Rightarrow \theta=\frac{\pi}{4}$	Surprisingly, many students did not know how to solve trigonometric equations and even more forgot about R-formula. Students are advised to practise solving trigonometric equations and to be familiar with the O level Trigonometry assumed knowledge required for H2 Math. Students are reminded that for applications of differentiation and integration, all angles must be expressed in radians, instead of degrees.

$\Rightarrow \sin(\theta+\frac{\pi}{4}) = 0.83567$

$\Rightarrow \theta+\frac{\pi}{4} = 0.98935$

$\Rightarrow \theta = 0.20396$ since $0 \le \theta \le \frac{\pi}{4}$

iii

Using first derivative test

θ	0.78	$\frac{\pi}{4}$	0.79
$\frac{dX}{d\theta}$	0.215 >0	0	-0.183 <0

Hence X is a maximum when $\theta_1 = \frac{\pi}{4}$

Using first derivative test

θ	0.203	0.20396	0.204
$\frac{dX}{d\theta}$	-0.06997 <0	0	0.00318 >0

Hence X is a minimum when $\theta_2 = 0.20396$

Students are reminded that whether you are using first derivative method or second derivative method, you are required to **evaluate the derivatives** and not by simply indicating '+' or '-' for 1st derivative test or '>' or '<0' for second derivative test.

Some students did not use the calculus method to justify the maximum or minimum value. Instead they evaluate X for θ_1 and θ_2 and compare the two values. However you are to be aware that by doing so, you can only deduce which is larger or smaller but you cannot show that the larger one is a maximum value, likewise for the smaller value!

Therefore students must remember that where Applications of differentiation are concerned, you must always use calculus methods to prove that X is a maximum or minimum.

	Alternatively, use second derivative test $\frac{dX}{d\theta} = -143\cos\theta + 121\cos^2\theta + 143\sin\theta - 121\sin^2\theta$ $\frac{d^2X}{d\theta^2} = 143\sin\theta - 242\cos\theta\sin\theta + 143\cos\theta - 242\sin\theta\cos\theta$ $= 143\sin\theta - 484\cos\theta\sin\theta + 143\cos\theta$ $\left.\frac{d^2X}{d\theta^2}\right\|_{\theta=\frac{\pi}{4}} = -39.8 < 0$ $\left.\frac{d^2X}{d\theta^2}\right\|_{\theta=0.20396} = 72.999 > 0$ Hence X is a maximum when $\theta = \frac{\pi}{4}$ and X is a minimum when $\theta = 0.20396$.	
iv	The greatest possible value of X is 27.3 m^2 when $\theta = \frac{\pi}{4}$. To find the minimum area covered by grass, we have to use the maximum area X. The area covered by grass $= 676 - 4(27.3) - \pi(11)^2$ $= 187$ m^2.	For part (iv), students are required to explain the choice of X corresponding to the question requirement, which is the minimum area of grass.

St Andrew's Junior College
2022 Preliminary Examination
H2 Mathematics Paper 2 (9758/02) Examiner's Comments

General Comments:

Paper 2 surfaced many conceptual gaps and lack of mastery in fundamental skills by students, despite some questions were questions that were like lecture examples and tutorial questions. This is true for both Pure Maths and Statistics questions.

The following issues were identified for Paper 2:

1. Students were unable to use the GC at the appropriate junctures to help them in solving problem efficiently.
2. Appropriate graphs required to solve questions were not included.
3. There were insufficient methods of solving, including weak fundamental skills such as Partial Fractions even from O-levels. This has resulted in time-management problem during the exam for a group of students.
4. Students were weak in explanation questions and often do not answer in the context of the question.
5. Many students also disregarded the nomenclature for both Pure Maths and Statistics topics, resulting in misunderstanding or inability to solve a question.
6. There was a lack of proper reading and application of comprehension skills, especially in the Statistics section.
7. For statistics questions, many did not state/describe/list the outcomes and/or outline how the calculations were done. One is reminded that clear and logical steps are expected in Mathematical solutions, especially for the A levels.

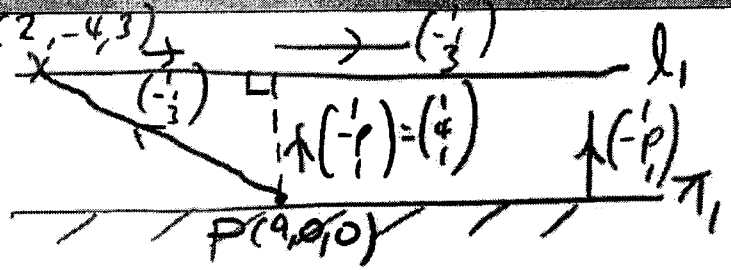
Students are therefore, advised on the following:

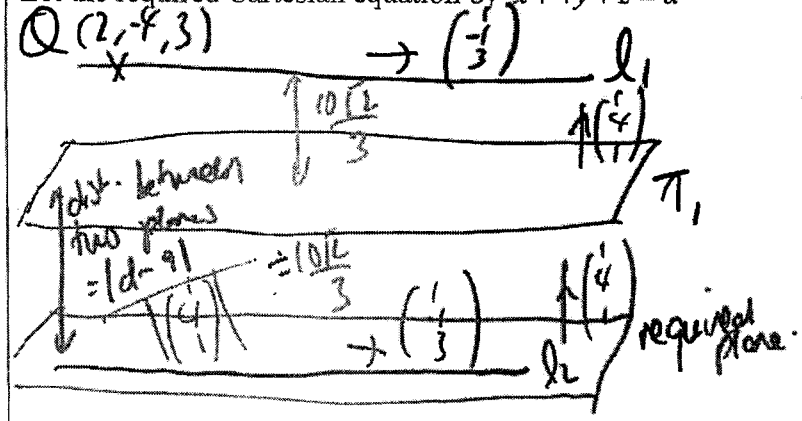
1. Review and revise lecture notes, tutorial and assignment questions thoroughly. This is to ensure that all concept gaps have been properly covered. Do also ensure that all fundamental skills required are covered.
2. Learn and identify junctures in solving that allows for use of your GC to aid in the solving of a problem. This would also apply to learning how to use your GC to help in checking your answers.
3. Students must review and learn how to define random variables properly and keep to the nomenclature taught.
4. Complete timed practices for both Paper 1 and 2, ensuring that your solutions are always written in a way that you would do for an examination. This is to help yourself sharpen on your problem solving and time management skills, while closing your conceptual gaps at the same time.

Section A: Pure Mathematics

Q	Solutions	Comments
1(i)	$\sum_{n=2}^{N}\frac{1}{(n+1)(n-1)}=\sum_{n=2}^{N}\frac{1}{n^2-1}$ $=\sum_{n=2}^{N}\left(u_n-u_{n-1}\right)$ $=\left[\begin{array}{l}u_2-u_1\\ +u_3-u_2\\ +u_4-u_3\\ \cdots\\ +u_{N-1}-u_{N-2}\\ +u_N-u_{N-1}\end{array}\right]$ $=u_N-u_1$ $=\frac{1}{N!}-\frac{1}{1!}$ $=\frac{1}{N!}-1$	There are 2 points to take note of: Firstly, the question requires students to understand the relationship that $u_n-u_{n-1}=\frac{1}{n^2-1}$ by rearranging the terms from $u_n=u_{n-1}+\frac{1}{n^2-1}$ Secondly, students need to show explicitly using method of difference to solve $\sum_{n=2}^{N}\left(u_n-u_{n-1}\right)$ and the relevant steps to obtain the solution. Some difficulties encountered were: 1. Carelessness which resulted in incorrect terms when substituting $n=N-2,\ n=N-1,\ n=N$ etc 2. Use 'n' or 'r' instead of 'N' when listing out the terms in the MOD. a. Students need to take care not to change the question on their own. b. Each unknown letter has a certain significance in an equation or expression. 3. Use of $\mathrm{f}(n)$ without defining what is $\mathrm{f}(n)$. (Moreover, it is not given or required in this question)

Q	Solutions	Comments
(ii)	As $N \to \infty$, $\frac{1}{N!} \to 0$ $\sum_{n=2}^{N} \frac{1}{(n+1)(n-1)} \to -1$ which is finite, hence $\sum_{n=2}^{N} \frac{1}{(n+1)(n-1)}$ converges and the sum to infinity is -1.	Generally, students were able to make the link between parts (i) and (ii) where the infinite sum (series) will tend to a constant. Students are reminded to state that there is a (unique) **finite** value in order for the infinite series to be convergent.
(iii)	$\sum_{n=8}^{N+5} \frac{1}{n(n-2)} = \sum_{n=7}^{N+4} \frac{1}{(n+1)(n-1)}$ (Replace n by $n+1$) $= \sum_{n=2}^{N+4} \frac{1}{(n+1)(n-1)} - \sum_{n=2}^{6} \frac{1}{(n+1)(n-1)}$ $= \frac{1}{(N+4)!} - 1 - \left(\frac{1}{6!} - 1\right)$ $= \frac{1}{(N+4)!} - \frac{1}{720}$	The correct replacement should be to replace n with $n+1$ to obtain the following: $\sum_{n=8}^{N+5} \frac{1}{n(n-2)} = \sum_{n=7}^{N+4} \frac{1}{(n+1)(n-1)}$. Some students did not know how evaluate the sum when the lower limit is not $n=2$, they cannot solve this by using method of difference as the question has stated "Hence", which required them to infer their answer from the previous part answer.

Q	Solutions	Comments
2(i)	 l_1 is parallel to Π_1, $\begin{pmatrix}1\\-1\\3\end{pmatrix}\bullet\begin{pmatrix}1\\-p\\1\end{pmatrix}=0\Rightarrow p=-4$ Let point P be the point (9, 0, 0) which lies on Π_1 and let point Q be (2, –4, 3). Distance between l_1 and Π_1 $=\dfrac{\left\lvert\overrightarrow{PQ}\bullet\begin{pmatrix}1\\4\\1\end{pmatrix}\right\rvert}{\left\lvert\begin{pmatrix}1\\4\\1\end{pmatrix}\right\rvert}=\dfrac{\left\lvert\begin{pmatrix}-7\\-4\\3\end{pmatrix}\bullet\begin{pmatrix}1\\4\\1\end{pmatrix}\right\rvert}{\sqrt{18}}=\dfrac{10\sqrt{2}}{3}$ units	• Students are not aware that one of the required conditions for a vector line not to intersect with a plane is that a direction vector of the line is perpendicular to a normal vector of the plane. (i.e. the line is parallel to the plane.) • A common method which is incorrect was to substitute $\mathbf{r}=\begin{pmatrix}2\\-4\\3\end{pmatrix}+\mu\begin{pmatrix}1\\-1\\3\end{pmatrix}$ into $\underset{\sim}{r}\bullet\begin{pmatrix}1\\-p\\1\end{pmatrix}\neq 9$ and ended up with $4p+(p+4)\mu\neq 4$ and derived that $p=-4$ by comparing coefficients of μ.iY • Students used wrong formulae to find the perpendicular distance such as $\dfrac{\left\lvert\begin{pmatrix}2\\-4\\3\end{pmatrix}\bullet\begin{pmatrix}1\\4\\1\end{pmatrix}\right\rvert}{\sqrt{18}}$ or $\dfrac{\left\lvert\begin{pmatrix}1\\-1\\3\end{pmatrix}\times\begin{pmatrix}1\\4\\1\end{pmatrix}\right\rvert}{\sqrt{18}}$, etc.

Q	Solutions	Comments
(ii)	Let the required Cartesian equation by $x+4y+z=d$  Distance between the two planes $= \dfrac{\lvert d-9 \rvert}{\left\lVert \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} \right\rVert}$ $\dfrac{\lvert d-9 \rvert}{3\sqrt{2}} = \dfrac{10\sqrt{2}}{3} \Rightarrow d = 29$ or -11 Since (2, –4, 3) lies on $x+4y+z=-11$, the required Cartesian equation of plane is $x+4y+z=29$	• Many students found the required cartesian equation through finding the foot of perpendicular of (2, –4, 3) onto Π_1, which will result in a longer and tedious solution. It is good to take note of this alternative method for a 2 marks question. • A number of students read the question wrongly and attempted to find the equation of l_2 instead. **Learning point:** It is important to read the requirement of the question carefully.

Q	Solutions	Comments
(iii)	Let $y=0$, $\left.\begin{matrix}3x-2z=10\\ x-z=5\end{matrix}\right\} x=0, z=-5$. So a common point between the planes is (0, 0, –5). A vector parallel to the $l_3 = \begin{pmatrix}3\\-1\\-2\end{pmatrix}\times\begin{pmatrix}1\\-a\\-1\end{pmatrix}=\begin{pmatrix}1-2a\\1\\1-3a\end{pmatrix}$ $l_3 : \mathbf{r}=\begin{pmatrix}0\\0\\-5\end{pmatrix}+\lambda\begin{pmatrix}1-2a\\1\\1-3a\end{pmatrix}, \lambda\in\mathbb{R}$ **Alternative Solution 1** Let $x=0$, $\left.\begin{matrix}-y-2z=10\\ -ay-z=5\end{matrix}\right\} y=0, z=-5$ $l_3 : \mathbf{r}=\begin{pmatrix}0\\0\\-5\end{pmatrix}+\lambda\begin{pmatrix}1-2a\\1\\1-3a\end{pmatrix}, \lambda\in\mathbb{R}$ **Alternative Solution 2** Let $z=0$, $\left.\begin{matrix}3x-y=10\\ x-ay=5\end{matrix}\right\} x=\frac{5-10a}{1-3a}, y=\frac{5}{1-3a}$	• Some students obtained a common point (0, 0, –5) by observation and did not verify that this point indeed lies on **both** planes. Other students simply picked a point on plane Π_2 and assumed that it is a point on l_3 without verifying whether that point is also on Π_3. • A significant number of students obtained a wrong normal vector because they have copied the normal vectors of Π_2 and Π_3 wrongly. • Some students were unable to find a common point on both planes by trying to solve the pair of simultaneous equations $3x-y-2z=10$ and $x-ay-z=5$ without letting one of the unknowns by 0. • As seen from the solutions, the most efficient method was to set $y=0$ as it would have eliminated the unknown a, allowing the use of GC to solve a pair of simultaneous equations.

Q	Solutions	Comments
	$l_3 : \mathbf{r} = \frac{5}{1-3a}\begin{pmatrix}1-2a\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}1-2a\\1\\1-3a\end{pmatrix}, \lambda \in \mathbb{R}$	

Alternative Solution 3

$3x - y - 2z = 10 \Rightarrow 3x - y = 10 + 2z$ --- (1)

$x - ay - z = 5 \Rightarrow x - ay = 5 + z$ --- (2)

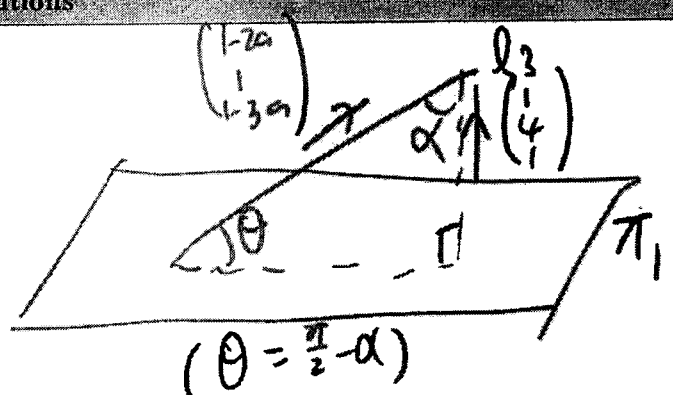
Solving (1) and (2),

$3x - y = 2x - 2ay \Rightarrow x = (1-2a)y$

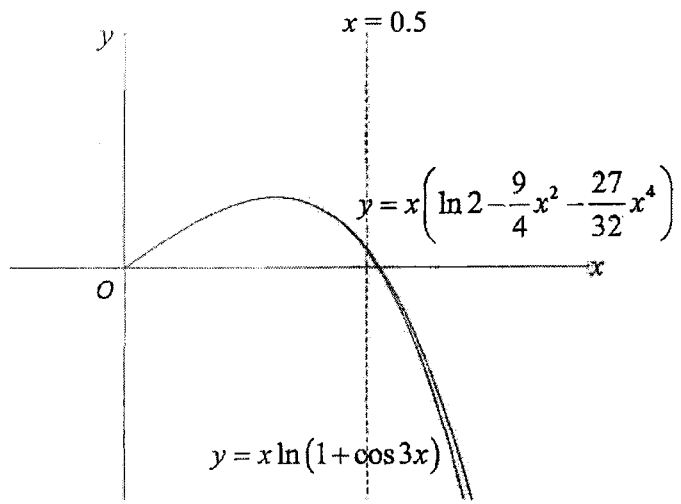
From (2): $y = \frac{1}{1-3a}(5+z), x = \frac{1-2a}{1-3a}(5+z)$

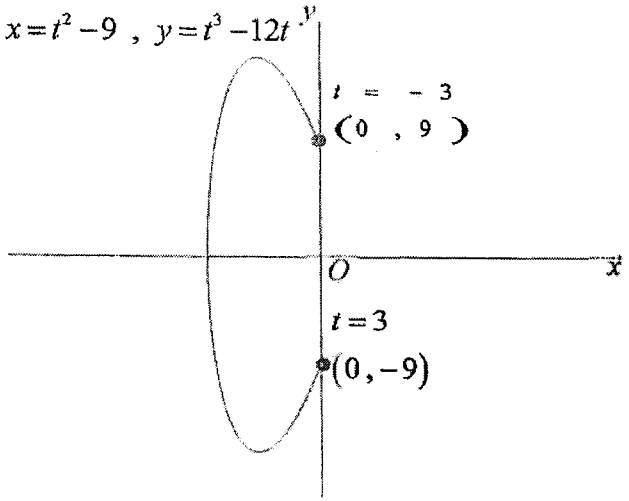
$$\begin{pmatrix}x\\y\\z\end{pmatrix} = \frac{1}{1-3a}\begin{pmatrix}5-10a+z\\5+z\\(1-3a)z\end{pmatrix}$$

$$l_3 : \mathbf{r} = \frac{5}{1-3a}\begin{pmatrix}1-2a\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}1-2a\\1\\1-3a\end{pmatrix}, \lambda \in \mathbb{R}$$

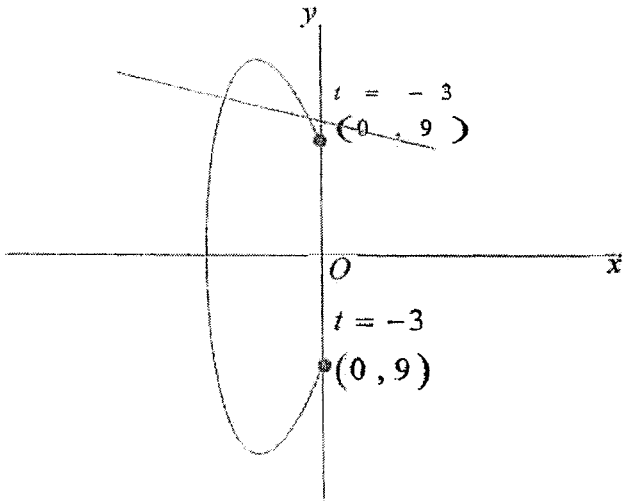
Q	Solutions	Comments
(iv)	 Given θ be the acute angle between l_3 and Π_1, $\dfrac{\left\lvert\begin{pmatrix}1-2a\\1\\1-3a\end{pmatrix}\bullet\begin{pmatrix}1\\4\\1\end{pmatrix}\right\rvert}{\sqrt{(1-2a)^2+1+(1-3a)^2}\sqrt{18}}=\sin\theta$ $\dfrac{\left\lvert\begin{pmatrix}1-2a\\1\\1-3a\end{pmatrix}\bullet\begin{pmatrix}1\\4\\1\end{pmatrix}\right\rvert}{\sqrt{(1-2a)^2+1+(1-3a)^2}\sqrt{18}}=\dfrac{\sqrt{3}}{18}$ (given) $\dfrac{\lvert(1-2a)+4+(1-3a)\rvert}{\sqrt{(1-2a)^2+1+(1-3a)^2}\sqrt{18}}=\dfrac{\sqrt{3}}{18}$	• Many students did not include the modulus sign in the numerator to ensure that the angle involved is an **acute** angle. • A handful of students used "$\cos\theta$" instead of "$\sin\theta$". • Some students did not continue to solve for a after they obtained $\dfrac{\lvert6-5a\rvert}{\sqrt{(1-2a)^2+1+(1-3a)^2}}=\dfrac{\sqrt{54}}{18}$. While others made mistakes in the algebraic manipulation. Since the question did not require exact solutions, it is more efficient to solve the question using G.C. A small group of students used the graphic calculator to solve for a from the equation marked '*' but did not provide any details how the answer is obtained. Students are advised to provide a simple sketch of the graph if they solved the above equation using a graphical method.

Q	Solutions	Comments
(ii)	$\int_0^{0.5} x\ln(1+\cos 3x)dx = \int_0^{0.5} x\left(\ln 2 - \frac{9}{4}x^2 - \frac{27}{32}x^4 + \ldots\right)dx$ $= \int_0^{0.5}\left((\ln 2)x - \frac{9}{4}x^3 - \frac{27}{32}x^5 + \ldots\right)dx$ $= \left[(\ln 2)\frac{x^2}{2} - \frac{9}{16}x^4 - \frac{9}{64}x^6 + \ldots\right]_0^{0.5}$ ≈ 0.04929 (5 d.p.)	Some students gave the answer in 5 **significant figures** instead of 5 **decimal places**. Students are reminded that answers should be rounded off instead of being truncated.
(iii)	Using GC, $\int_0^{0.5} x\ln(1+\cos 3x)dx = 0.04900$ (5 d.p.)	Some students gave the answer in 5 **significant figures** instead of 5 **decimal places**.

Q	Solutions	Comments
(iv)	 From the diagram, it can be seen that the graphs of $y = x\ln(1+\cos 3x)$ and $y = x\left(\ln 2 - \frac{9}{4}x^2 - \frac{27}{32}x^4\right)$ are close to each other mostly from $x = 0$ to $x = 0.5$. Hence, the approximated value of $\int_0^{0.5} x\left(\ln 2 - \frac{9}{4}x^2 - \frac{27}{32}x^4 + ...\right)dx$ from (ii) is approximately equal to the actual value of $\int_0^{0.5} x\ln(1+\cos 3x)dx = 0.04900$ (5 d.p.) in (iii).	Note: $\int_0^{0.5} x\ln(1+\cos 3x)\,dx =$ Area bounded by the curve, $y = x\ln(1+\cos 3x)$, the x – axis from $x = 0$ and $x = 0.5$. Likewise $\int_0^{0.5} x\left(\ln 2 - \frac{9}{4}x^2 - \frac{27}{32}x^4\right)dx =$ Area bounded by the curve, $y = x\left(\ln 2 - \frac{9}{4}x^2 - \frac{27}{32}x^4\right)$, the $x-$axis from $x = 0$ and $x = 0.5$. Accuracy of the integral depends on how much the curve of $y = x\left(\ln 2 - \frac{9}{4}x^2 - \frac{27}{32}x^4\right)$ deviates from $y = x\ln(1+\cos 3x)$. Therefore, the diagram should show the curves $y = x\ln(1+\cos 3x)$ and $y = x\left(\ln 2 - \frac{9}{4}x^2 - \frac{27}{32}x^4\right)$ from $x = 0$ and $x = 0.5$. The comment on the accuracy should include how much the 2 curves from deviate from each other from $x = 0$ and $x = 0.5$; not just how much the 2 curves from deviate from each other around $x = 0$. Students who calculated the percentage difference between the two values from part (ii) and part (iii) without reference from the diagram get no credits since this is part of the requirement stated.

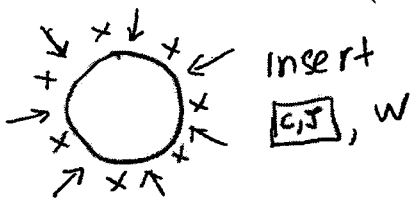
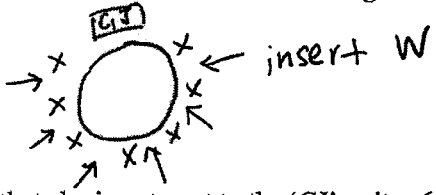
Q	Solutions	Comments
4(i)	$x=t^2-9$, $y=t^3-12t$  	Students need to be more detailed when sketching the diagram. They need to: 1) label the end-points in coordinates form as stated in the question 2) Draw full circle at end points since it is inclusive. 3) Axes, origin and equation of curve need to be indicated. **Learning point:** 1. In sketching the parametric equations, it is important to ensure that one adheres to the domain of the parameter stated.
(ii)	$x=t^2-9$, $y=t^3-12t$ $\frac{dx}{dt}=2t$, $\frac{dy}{dt}=3t^2-12$ $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{1}{\frac{dx}{dt}}=\frac{3t^2-12}{2t}$ ~~$l: 9y-2x+83=0$~~ $l: 9y+2x=83$	Students need to be well versed with differentiation. A few students made errors with differentiation of simple functions for this question. **Note:** Students also need to be clear that $\frac{dy}{dx}$ is the gradient of tangent and not the gradient of normal. When rejecting solutions, students are also reminded to state the reason(s) for rejection.

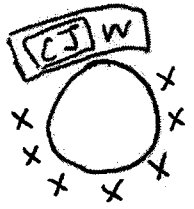
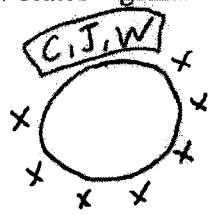
Q	Solutions	Comments
	$\frac{3t^2-12}{2t}=\frac{9}{2}$ $t^2-3t-4=0$ $(t-4)(t+1)=0$ $t=-1$ or $t=4$ Since $t\in[-3,3]$, $t=-1$.	
(iii)	Since the curve C intersects the line l at $Q\left(q^2-9, q^3-12q\right)$ $9(q^3-12q)+2(q^2-9)=83$ $9q^3-108q+2q^2-18-83=0$ $9q^3+2q^2-108q-101=0$ (Shown) Using G.C., $q=-2.9836$ or -1 or 3.7613 Since $-3\le q\le 3$ and $q\neq -1$, $q=-2.9836$. Hence, $x=(-2.9836)^2-9=-0.09813$, $y=(-2.9836)^3-12(-2.9836)=9.2436$ $Q(-0.0981, 9.24)$	Students need to state the reasons for rejecting the inappropriate values of q. (see (ii)) Student should be aware that point P correspond to $q=-1$. For accuracy, students are expected to use 5 significant figures or more when using results from the previous part. Similarly, students are also expected to leave their final answer to 3 significant figures. Students are reminded to leave their answers in coordinates form.

Q	Solutions	Comments
(iv)	 Required area	This question was not well done. Some students who find area by integrating with respect to y will find it challenging to proceed. Common mistakes made by the students include: • Identifying the region of the required area wrongly • Not changing the limits of the integral when the integrand is changed to be in terms of "t". • Curves sketched inaccurately thus affecting the workings/formulation.

Q	Solutions	Comments
	$= \int_{-8}^{-0.0981} y \, dx - \frac{1}{2}\left[-0.09813-(-8)\right][9.2436+11]$ $= \int_{-1}^{-2.9836} \left(t^3-12t\right)\left(\frac{dx}{dt}\right) dt - 79.981$ $= \int_{-1}^{-2.9836} \left(t^3-12t\right)(2t)\, dt - 79.981$ $= 30.324 \text{ units}^2$ $= 30.3 \text{ units}^2$ (to 3 sf)	

Section B: Probability and Statistics

Q	Solutions	Comments
5(i)	Let C be Caleb, J be James and W be the woman. Number of ways to arrange the 7 people (exclude C, J and W) = $(7-1)!$  Number of ways to insert the pair C & J together and the woman = $^7C_2 \times 2!$ Number of ways to arrange C and J within the pair = $2!$ Total number of ways = $(7-1)! \times {}^7C_2 \times 2! \times 2! = 60480$ Alternative mtd 1 : No. of ways to arrange the 'CJ' unit and 7 other men excluding the woman = $(8-1)! \times 2!$  No. of ways to insert W such that she is not next to the 'CJ' unit = 6 or $^6C_1$ Total number of ways = $(8-1)! \times 2! \times 6 = 60480$ Alternative mtd 2 (Complementary mtd 1): No. of arrangements where C and J are grouped together = $(9-1)! \times 2!$ No. of ways to arrange the 'CJ' unit and W = 2!	General comments for this question: It is essential to **state clearly** and **list down the various cases/steps** of consideration in the calculation of the number of ways **including a proper descriptor** for each of the cases/steps. Students who did poorly in this question should revise this topic thoroughly, and revise basic concepts such as addition and multiplication principles. They should also be familiar with the methods/approaches to solve P&C problems. Common mistakes for (i): Missing one or more steps (e.g. did not consider the arrangement of the 'CJ' unit and W in the slot in method) Students who used the Complementary method : $9! - 7! \times 3!$ is incorrect as this includes the cases where C and J are not together (due to 3! in the working)

	No. of arrangements where C and J group together and the 'CJ' unit is also next to W = $(8-1)!\times 2!\times 2!$  Note : 2! to arrange C and J and 2! to arrange the 'C,J' unit and W Total number of ways = No. of arrangements where C and J are grouped together - No. of arrangements where C and J group together and the 'CJ' unit is also next to W = $(9-1)!\times 2!-(8-1)!\times 2!\times 2!=60480$ Alternative mtd 3 (Complementary mtd 2): No. of arrangements where C and J are separated = $(8-1)!\times {}^{8}C_{2}\times 2!$ (using slot in mtd) Total number of ways = No. of arrangements with no restrictions - No. of arrangements where C and J are separated - No. of arrangements where C and J group together and the 'CJ' unit is also next to W = $(10-1)!-(8-1)!\times {}^{8}C_{2}\times 2!-(8-1)!\times 2!\times 2!=60480$	
(ii)	Number of ways (with identical seats) = number of ways without restriction – number of ways in which C, J and W seated together = $(10-1)!-(8-1)!3!=332640$  Number of ways (with different seats) =	Many students did not realize that the chairs are distinct (as the question states that "chairs of different colours"), so they found the number of required arrangements where chairs are identical instead.

Number of ways (with identical seats) × number of seats

$=\left[(10-1)!-(8-1)!3!\right] \times 10$

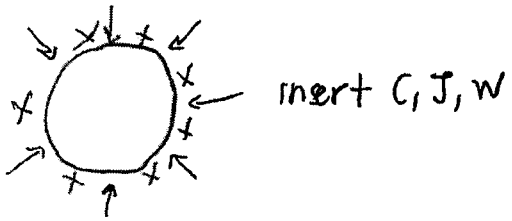
$= 332640 \times 10$

$= 3326400$

Alternative mtd :

Case 1 : C, J, W are all separated

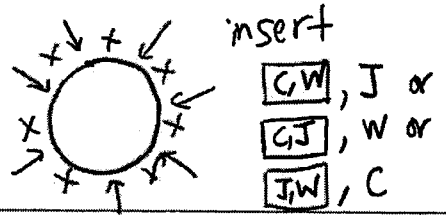
No. of arrangements (using slot in mtd) $= (7-1)! \times {}^{7}C_{3} \times 3! = 151200$



Case 2 : Any two of C, J, W are group together but separated from the 3rd person

No. of cases = ${}^{3}C_{2}$ (any two of C, J, W)

For each case, no. of arrangements (using slot in mtd) =
$(7-1)! \times {}^{7}C_{2} \times 2! \times 2!$

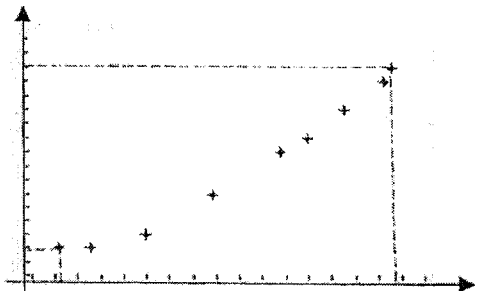


Students who used the alternative method often missed out Case 2. 'C, J and W not all together' does **NOT** only comprise of the cases where C, J, W are separated. It includes the cases where any two of C, J and W are together but separated with the third person.

	No. of arrangements for Case 2 $= {}^3C_2 \times (7-1)! \times {}^7C_2 \times 2! \times 2! = 181440$ Total no. of ways(with different seats) $=$ $(151200+181440)\times 10 = 3326400$	
6(i)	Let X be the random variable "no of students out of 30 students who could do the Differentiation question" $X \sim \text{B}(30, 0.3)$ $\text{P}(X \geq 6) = 1 - \text{P}(X \leq 5)$ $= 0.92341 \approx 0.923$ (3 sig. fig.)	Common mistakes: 1. Did not write down the **distribution**, although they defined the random variable. *Note:* The calculation of probability involving the random variable is meaningless without the distribution. 2. Interpreted $\text{P}(X \geq 6) = 1 - \text{P}(X < 6)$ which is incorrect! *Suggestion:* Make use of the number line to help identify the complement of the event $X \geq 6$. 3. Did not leave the final answer in 3 sig. fig.
(ii)	Let S be the random variable "no of students out of 8 who could do the Differentiation question " Let T be the random variable "no of students out of 22 who could do the Differentiation question " $S \sim \text{B}(8, 0.3)$ $T \sim \text{B}(22, 0.3)$	

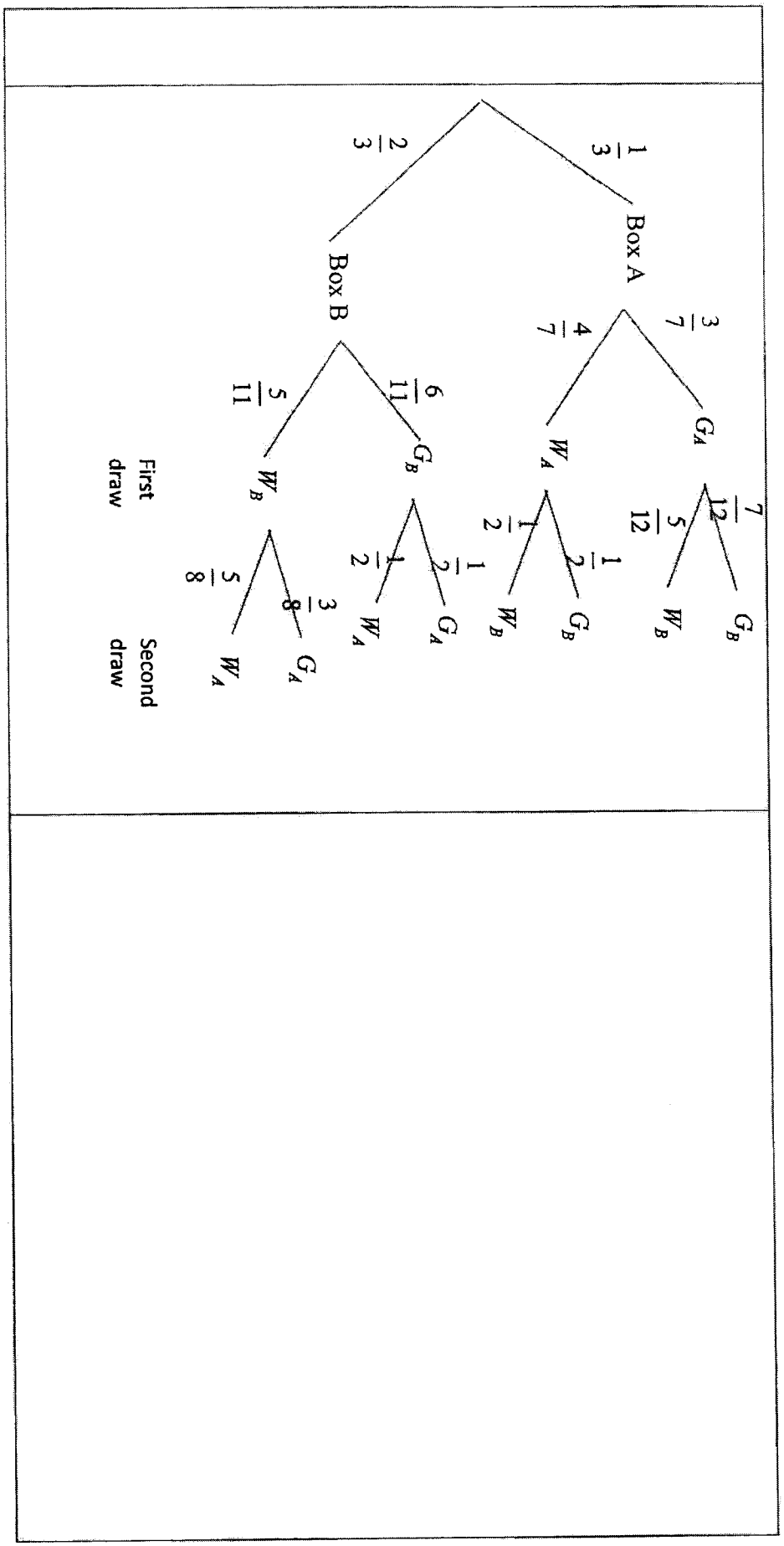
	P(only 2 among first 8 could do that question$\mid X \geq 6$) $= \frac{\text{P(only 2 among first 8 could do that question} \cap X \geq 6)}{\text{P}(X \geq 6)}$ $= \frac{\text{P}\left(\begin{array}{l}\text{only 2 among first 8 could do that question} \\ \cap\text{at least 4 among the next 22 could do the question}\end{array}\right)}{\text{P}(X \geq 6)}$ $= \frac{\text{P}(S=2 \cap T \geq 4)}{\text{P}(X \geq 6)}$ $= \frac{\text{P}(S=2)\text{P}(T \geq 4)}{\text{P}(X \geq 6)}$ $= \frac{\text{P}(S=2)[1-\text{P}(T \leq 3)]}{\text{P}(X \geq 6)}$ $= \frac{0.296475 \times 0.931937}{0.92341} = \frac{0.276297}{0.92341}$ $= 0.29921 \approx 0.299$ (to 3 sig. fig.)	This part was badly done as students did not consider/recognise the conditional probability, resulting them not being able to obtain $\text{P}(T \geq 4)$. This was the important part to illustrate that the students have understood the question. It is a good practice to have 5 sig. fig. when students were solving it and leave as 3 sig. fig as their final answer (as stated on the cover page for non-exact answers)
(iii)	Let Y be the random variable "no of students out of n who could do the Differentiation question " $Y \sim \text{B}(n, 0.3)$ $\text{P}(Y \leq 5) > 0.9$ From G.C, n | $\text{P}(Y \leq 5)$ 10 | $0.95265 < 0.9$ 11 | $0.92178 < 0.9$ 12 | $0.88215 > 0.9$	There was a lack of evidence on how they used their GC table to solve for largest possible value of n. A table with correct header and 3 pair of values as shown in the solution was expected.

	Therefore, the largest possible value of n is 11.	Students need to write "largest possible value of n" as it was stated in the question. *Caution!*: The GC commands should **NOT** appear in your solutions!
7(i)	$P = 0.92588M + 69.804$ $\frac{763+k}{9} = 0.92588(27.333) + 69.804$ $763 + k = 9(95.111)$ $k = 855.999 - 763$ $k = 92.999 = 93.0$ (3sf) (Shown)	General comments for this question: Students are good with the calculation parts but rather weak in answering questions that require them to given explanations. They need to learn to be precise and clear in their explanation by using correct and specific terms. Common mistake for (i): Substituting the observed point (25.9, k) into the equation of the regression line - this is incorrect as the **observed points may not lie on the regression line.** **Note/Learning Point:** The only point we are certain that will lie on the regression line is $(\overline{M}, \overline{P})$.

(ii)	P/ mm Hg 102 89  	Students should indicate the units for the variables in the axes' labels and indicate the range of values of M and P respectively on the scatter diagram. The relative distance between points must be accurate and the points should be drawn by following the general trend shown in the GC.
(iii)	$\ln P = aM + b$ $\Rightarrow P = e^{aM+b}$ From the scatter diagram, as M increases, P increases at an increasing rate, hence a is positive. [From the scatter diagram, as M increases, P increases. Hence, ln P will also increase. Therefore a is positive] Product moment correlation coefficient between M and ln P = 0.988128=0.988 (to 3 sf)	Many students mentioned that the scatter diagram or graph has a positive gradient. This is incorrect as scatter diagram comprises of points, so it is not appropriate to mention 'gradient'. Instead students should comment on the **trend (how does P behaves as M increases)** of the points shown by the scatter diagram. While most students were able to get the answer correct for the second part, some students either forgot to write out "product moment correlation coefficient" or left their answers to 5 significant figures when they should give the end answer to 3 significant figures. ***Reminder:*** You will need to define product moment correlation coefficient the first time you use it in the question.
(iv)	From (ii), as M increases, P increases at an increasing rate instead of constant rate.	Students have to be careful when writing their explanations. It is quite clear from the question that they need to compare the two

	From (iii), the PMCC between M and $\ln P$ is closer to 1 as compared to the PMCC between M and P. Hence $\ln P = aM + b$ is the better model.	product moment correlation coefficients(r values) to justify which model is a better fit. Incorrect phrasing : 'r value for $\ln P = aM + b$' Correct phrasing : 'r value between M and $\ln P$' When comparing the two r values, the one which $\lvert r \rvert$ closer to 1(not the 'higher' one as r value can be negative) will give a model which better fits the data. Also, the question does require the candidate to look at the scatter plot/diagram in (ii) (and the scatter diagram is NOT a continuous graph!), so mention of the relationship between M and P is essential to justify which model is a better fit.
(v)	It may not be reliable to estimate John's DBP using the model as John's BMI is outside the given BMI data range of 19.2 to 33.6 and extrapolation is needed.	When explaining whether the estimate is reliable, students cannot just rely on "key words" without ensuring that the entire prose makes sense. Students need to mention that $M = 35$ is outside the given data range AND thus extrapolation is required which makes the estimation unreliable. Explanations should also be written in the context of the question.

8(i)	Let G_A and G_B represent the event that the ball drawn is green from Box A and Box B respectively. Let W_A and W_B represent the event that the ball drawn is white from Box A and Box B respectively.	Students should define the events for the probability properly in order to make use of the respective symbols/representations. Most students were able to draw the tree diagram required, though some had forgotten to label the events or the probabilities along branches of the tree. The labels should make a distinction from which box were the balls drawn from. There were a handful who had written the event along the branches and probabilities at the end of the branches, however the convention should be the reverse. Students are reminded to use properties of tree diagram to check their drawing, e.g. branches from same point should sum to 1, sub-branches represent conditional probability etc. ***Reminder:*** Probability tree is a tool which one should and must know how to construct with precision and accuracy.



(ii)

P(wins the game) = P(second ball is white)

$$= P(G_A, W_B) + P(W_A, W_B) + P(G_B, W_A) + P(W_B, W_A)$$

$$= \frac{1}{3}\left[\left(\frac{3}{7}\right)\left(\frac{5}{12}\right) + \left(\frac{4}{7}\right)\left(\frac{1}{2}\right)\right] + \frac{2}{3}\left[\left(\frac{6}{11}\right)\left(\frac{1}{2}\right) + \left(\frac{5}{11}\right)\left(\frac{5}{8}\right)\right]$$

$$= \frac{81}{154}$$

P(first ball from Box A| Player wins the game)

$$= \frac{\text{P(first ball from } A \cap \text{player wins the game)}}{\text{P(player wins the game)}}$$

$$= \frac{\text{P}\left(\begin{array}{l}\text{score of die is less than 3 followed by}\\ \text{drawing a ball from A}\\ \text{and the second ball drawn is white}\end{array}\right)}{\text{P(player wins the game)}}$$

$$= \frac{\frac{1}{3}\left[\left(\frac{3}{7}\right)\left(\frac{5}{12}\right) + \left(\frac{4}{7}\right)\left(\frac{1}{2}\right)\right]}{\frac{81}{154}}$$

$$= \frac{143}{486}$$

Common mistakes/ comments

1. It is insufficient to show only the calculations without listing the outcomes.

2. Notation Mistake:
   P(first ball from A $\cap$ player wins the game)
   $\neq$ P(first ball from A) $\cap$ P(player wins the game)

3. $P(G_A, W_B) \neq P(G_A) \times P(W_B)$ but
   $P(G_A, W_B) \neq P(G_A) \times P(W_B \mid G_A)$

4. Conceptual Mistakes:
   P(first ball from A $\cap$ player wins the game)
   $\neq$ P(first ball from A) $\times$ P(player wins the game)
   The reason is the event that first ball is from A and the event that player wins the game are not independent.

9(i)	Given that X is a discrete random variable, $\sum_{\text{all } x} \mathrm{P}(X=x)=1$. $2p+2q=1$ ---(1) Also $\mathrm{E}(X^2)=\sum_{\text{all } x} x^2\mathrm{P}(X=x)=13.3$ $4p+9q+16q+25p=13.3$ $\Rightarrow 29p+25q=13.3$ ---(2) Solving (1) and (2), by GC, $p=\frac{1}{5}, q=\frac{3}{10}$ By symmetry, $\mathrm{E}(X)=3.5$ $\mathrm{Var}(X)=\mathrm{E}(X^2)-[\mathrm{E}(X)]^2$ $=13.3-3.5^2$ $=1.05$	Students who came out with the probability distribution table tend to do better. Common mistakes made by the students include: 1) Did not state "Given that X is a **discrete random variable**, $\sum_{\text{all } x}\mathrm{P}(X=x)=1$" when formulating $2p+2q=1$. Please note that this is a property unique to discrete random variables. 2) Confuse with Binomial Distribution concept by stating $p+q=1$ 3) Students wrote $\mathrm{Var}(X)=\mathrm{E}(X^2)-[\mathrm{E}(X)]^2$ correctly but did not square $\mathrm{E}(X)$ when solving.
(ii)	Since $n=30$ is sufficiently large, by Central Limit Theorem	Some students wrongly assumed that the distribution of X was normally distributed as it is nowhere stated in the question. While some students remembered that the Central Limit Theorem was to

	$\overline{X} \sim N\left(3.5, \frac{1.05}{30}\right)$ approximately $P\left(\overline{X} > 3.8\right) = 0.054405 = 0.0544$ (3 s.f.)	be applied, they concluded that X was normally distributed. The Central Limit Theorem is only applied when X is NOT a normal distribution and it ONLY tells us the distribution of $\overline{X}$ NOT X.

2022 JC2 Paper 2 Prelims Application Questions (10) & (11)

Solving of Mathematics Application Questions

Step 1: Understanding the Problem	a) Read the problem thoroughly and you can help yourself by underlining the keywords, important conditions, assumptions and any relevant information. You may need to repeat this process once or twice to ensure that you understand the entire problem and that you do not miss out any key information. b) Make sure that you are clear about the requirements of the question such as which are the variables and which are the constants. If variables are not given, you are required to define them. c) At this stage, it might also be necessary for you to interpret various phrases and write it as mathematical statements or equations or expressions.
Step 2: Planning	a) Identify the topics, concepts and skills that the application questions are testing. There can be more than one topic /concept/ skill involved. b) Make the connection between parts of the question. Hence it is advisable that you scan through the entire question from the start to the end to identify possible links. c) Filter the information and select the most relevant ones for each part, before you start to solve.
Step 3: Solving	a) Implement the appropriate problem solving strategies which may include tabulating the information, guess and check, drawing a diagram etc. b) All the formulae and techniques (especially differentiation and integration) should be at your fingertips, so that you do not waste unnecessary time on searching through MF26. c) If the strategy you applied fails to work, revisit the problem to check whether you have misread the question or left out certain information. ***Tip: Every statement in the question will contain some form of information.***
Step 4: Check the Solutions/ Calculations	a) Check against the context of the Application Question on whether your answer makes any logical sense? Have you used GC to check the answers? b) Do not accept all answers blindly. You should always interpret the validity of your answers in the context of the Application Question.

10 In this question you should state clearly the parameters of any distributions that you use.

A supermarket sells honeydews and watermelons. The masses, in kilograms, of the honeydews and the watermelons each follow a normal distribution. The means and standard deviations of these distributions are shown in the following table:

	Mean (kg)	**Standard deviation (kg)**
Honeydew	1.5	0.2
Watermelon	8.5	0.3

You may assume that the masses of the fruits (watermelon and honeydew) are independent of one another.

(i) Find the probability that for 3 randomly chosen honeydews, two of the honeydews each has mass less than 1.8 kg and one of the honeydews has mass more than 1.8 kg. [2]

(ii) Find the probability that the total mass of 5 randomly chosen honeydews is less than the mass of one randomly chosen watermelon. [3]

The supermarket wants to pack fruits into gift packs to be donated to needy families. Each gift pack consists of one randomly chosen honeydew and one randomly chosen watermelon.

(iii) 90% of the gift packs have masses differ from the mean mass of gift packs by less than m kg, find the value of m. You may assume that the packing material has negligible mass. [3]

The selling price of honeydews is \$3.50 per kilogram and the selling price of watermelons is \$0.70 per kilogram. Lam has a budget of \$10.

(iv) Lam intends to buy one honeydew and one watermelon. Find the probability that Lam is able to pay for his purchase. [3]

Let the probability that a honeydew and watermelon cost at most \$5 **each** be k.

(v) Explain, without any further calculation, why the probability in **(iv)** is at least k. [1]

General Comments for Q10:

One must take proper care to define random variables and state the distributions along with the random variables whenever possible.
Random variables are represented by uppercase letters and its corresponding values are represented by its corresponding lowercase letters (please do not mix this up!).
While most students were able to do well, there were also some who scored poorly for this question. Those who did not score for this question are strongly advised to revise and practice more context questions on Normal Distribution. Reasons for not doing well for this question are largely due to poor concepts, incorrect use of properties of Expectation & Variance and skills in interpreting questions and inadequate explanations of working steps & lack of precision in working steps and mathematical notations.

Students are advised to be clear on whether Standard deviation or Variance is given in a question. Many students did not square the standard deviation before applying the properties of Variance.
There was evidence that students did not know how to use the Graphing Calculator correctly to find the value of probability. We had several scripts with everything correct, except for the final answer. It seems like either they did not take the square root of the variance or they swapped their values of μ and σ.

Qn	Solutions	Marker's Comment
(i)	Let H be the mass of a honeydew in kg. $H \sim \mathrm{N}(1.5,\ 0.2^2)$ Let W be the mass of a watermelon in kg. $W \sim \mathrm{N}(8.5,\ 0.3^2)$ Required probability $=3\times \mathrm{P}(H<1.8)^2\,\mathrm{P}(H>1.8)$ $=0.17454$ $=0.175$	**Common mistakes & suggestions for improvement:** • Most of the students did not consider the permutation of the weight of the 3 honeydews. Students are encouraged to understand the problem and list down the possibilities.

(ii)	Let $T = H_1 + H_2 + H_3 + H_4 + H_5 - W$ $\mathrm{E}(T) = \mathrm{E}(H_1 + H_2 + H_3 + H_4 + H_5 - W)$ $= 5\mathrm{E}(H) - \mathrm{E}(W)$ $= 5(1.5) - 8.5 = -1$ $\mathrm{Var}(T) = \mathrm{Var}(H_1 + H_2 + H_3 + H_4 + H_5 - W)$ $= 5\mathrm{Var}(H) + \mathrm{Var}(W)$ $= 5(0.2)^2 + 0.3^2 = 0.29$ $T \sim \mathrm{N}(-1, 0.29)$ $\mathrm{P}(T < 0)$ $= 0.96834$ $= 0.968$	**Common mistakes & suggestions for improvement:** • Use $5H$ instead of $H_1 + H_2 + H_3 + H_4 + H_5$. Students are advised to know the notations very well and furthermore it affects the way the Variance is calculated. Students are also advised to underline the key word, in this case "total mass" and "less" so as to formulate it correctly. **[Interpretation:** $5H$: five times the mass of a randomly chosen honeydew $H_1 + H_2 + H_3 + H_4 + H_5$: total mass of five randomly chosen honeydews] • Applying properties of variance incorrectly. Students need to revise and remember that the **Variance** of linear combinations of independent random variables is only **added.** • Students are reminded to write down the distributions with the simplified value of the correct parameters (expectation and variance) • Many students used did not square the standard deviation to obtain the correct variance
(iii)	Let $F = H + W$ $\mathrm{E}(F) = \mathrm{E}(H + W)$ $\quad$ $\mathrm{Var}(F) = \mathrm{Var}(H + W)$ $= \mathrm{E}(H) + \mathrm{E}(W)$ $\quad$ $= \mathrm{Var}(H) + \mathrm{Var}(W)$ $= 1.5 + 8.5 = 10$ $\quad$ $= 0.2^2 + 0.3^2 = 0.13$	**Common mistakes & suggestions for improvement:** • A few students did not relate that mass of the gift pack to be the sum of the mass of each fruit. • Used $\mathrm{Var}(F) = 0.2 + 0.3 = 0.5$ instead of $\mathrm{Var}(F) = 0.2^2 + 0.3^2 = 0.13$.

	$F \sim \mathrm{N}(10, 0.13)$ $\mathrm{P}(\lvert F-10\rvert < m) = 0.9$ $\mathrm{P}(10-m<F<10+m) = 0.9$ $10+m = 10.593$ $\Rightarrow \quad m = 0.593$	• Many students did not interpret the question correctly. The keyword was "differs", which most of time involves modulus. For example, "A differs from B by less than m" can be formulated as $\lvert A-B\rvert < m$ • Students are advised to rename a linear combinations of random variables e.g $F = H+W$
(iv)	Let $C = 3.5H+0.7W$ $\mathrm{E}(F) = \mathrm{E}(3.5H+0.7W)$ $= 3.5\mathrm{E}(H)+0.7\mathrm{E}(W)$ $= 3.5(1.5)+0.7(8.5)$ $= 11.2$ $\mathrm{Var}(F) = \mathrm{Var}(3.5H+0.7W)$ $= 3.5^2\mathrm{Var}(H)+0.7^2\mathrm{Var}(W)$ $= (3.5)^2(0.2^2)+(0.7)^2(0.3^2)$ $= 0.5341$ $C \sim \mathrm{N}(11.2, 0.5341)$ $\mathrm{P}(C \le 10) = 0.050296$=0.0503 (to 3 sf)	This part was well done. **Common mistakes:** • Missed out the equal sign in $\mathrm{P}(C \le 10)$. Students need to be aware that Lam can afford to buy the gift pack even if the total cost is \$10. • Many students did not write down the distribution. *Note:* The calculation of probability involving the random variable is meaningless without the distribution. • A number of students did not square 3.5 and 0.7. There is a need to learn the properties of variance properly.
(v)	The event that a honeydew and watermelon cost at most \$5 **each** is a subset of the event that the total cost of one honeydew and one watermelon is \$10.	This part was very poorly done.

11 The branch manager of a bank would like to find out about the satisfactory level of the customer services provided by the branch. He would like to survey 80 customers of the branch to find out about their opinion on the wait time.

(i) Describe how the branch manager could obtain a random sample of 80 customers to conduct his survey. [2]

The mean amount of time a customer needs to wait in the queue until they were served was known to be at least 15 minutes. After the survey was conducted, he realised that the waiting time for each customer before they were served at the counter was of a major concern. A change in processes at the branch was implemented. After a month, the branch manager of the bank decided to record the waiting time, t minutes, for a customer at the branch for 50 different customers to evaluate if the changes were effective. The results are summarised by

$$\sum(t-15)=-60\ ,\ \sum(t-15)^2=1168\,.$$

(ii) Find the unbiased estimates of the population mean and variance. [2]

(iii) Test, at the 5% level of significance, whether the change in processes have been effective. [4]

(iv) Explain what is meant by the p-value obtained in **(iii)** in the context of the question. [1]

(v) The quality service manager claimed that that the mean waiting time before there were any changes in processes was in fact k minutes. With the same data collected from the same 50 customers, find the range of values of k if there is insufficient evidence to conclude that there was a change in the mean waiting time at 2% level of significance. [3]

General Comments for Q11:

This question was generally not well done. There is a major misreading of the question and lack of conceptual understanding in both the topics of Sampling and Hypothesis Testing. At this stage, many students were still unable to provide proper nomenclatures that represent both the random variables and its corresponding values (see Q10).

Students must recognise that calculation of unbiased estimates is a basic skill to be acquired and mastered. Hypothesis testing is a structured test whereby the flow of test, including the way the test and conclusions are worded, cannot be modified at one's preference. Students are reminded and strongly advised to memorise the structure of the test.

Explanations are also poorly attempted, either without a proper understanding of terminologies or are inaccurately written, often leaving out the context of the question. This is a rather serious problem especially for Statistics questions which typically involves a context.

(i)	The branch manager obtains a sampling frame consisting of all the customers of the branch, **numbering all the customers with a distinct number from 1 to *N*.** **Randomly select 80** of these customers by **generating 80 distinct random numbers (using a random number generator)** and **select the corresponding** customers.	Many students misread the question! (**Serious error**!) Many students provided the reason as to why the sample is a random sample instead of 'how' (referring to the method) a random sample could be obtained! Before any numbers could be generated, the numbering should be properly stated/defined. Also, the random numbers generated should be both distinct and properly quantified as 80, as required by the sample size. As it is a descriptor of method, it is also advised to suggest a way the numbers could be obtained. **Common mistakes (other than misreading):** 1. Fail to recognise the sampling units required. Some students said that the sample should be obtained from a sampling frame of employees. 2. Did not state how the customers should be numbered. 3. Some students wrote statements such as 'numbering all customers from 1 to 200'. However, no one actually knows how many customers there are in the branch. 4. Identifying the sampling frame incorrectly. 5. Many only wrote 'generating 80 random numbers'. However, one must recognise that the random numbers generated could possibly be repeated. Hence, the word 'distinct' is needed. 6. The final step of choosing corresponding customers was omitted. That would mean that the selection process has not been completed. 7. Many students described methods which **DO NOT** produce a 'random sample'. *Reminder:* Students should apply the knowledge that has been acquired in H2 Maths first. Do not bring in content that has been acquired from other subjects if you have not clarified it with your

		tutors! You might unknowingly have applied it incorrectly or have misconceptions.
(ii)	Let T be the random variable denoting the waiting time of a customer at the branch in minutes and μ be the population mean waiting time. Unbiased estimate of population mean, $\bar{t}$ $=\frac{\sum(t-15)}{50}+15$ $=\frac{-60}{50}+15=13.8$ (Exact) Unbiased estimate of population variance, s^2 $=\frac{1}{n-1}\left[\sum(t-15)^2-\frac{\left(\sum(t-15)\right)^2}{n}\right]$ $=\frac{1}{50-1}\left(1168-\frac{(-60)^2}{50}\right)$ $=22.367$ $=22.4$ (to 3 sf)	One should define the random variable and population mean prior to calculating unbiased estimates of population mean/variance as far as possible. Otherwise, the definition must occur before hypothesis test is conducted! Some students do not even know how to define a random variable properly/accurately. **Common mistakes**: 1. In this question, data is represented by t, not x! Hence, the unbiased estimate of population mean is $\bar{t}$ not $\bar{x}$. One needs to be careful with notations. 2. Not defining the symbols $\bar{t}$ and s^2. 3. Exact value for $\bar{t}$ not given and answer for s^2 was not rounded off to 3 significant figures (as required by the cover page). 4. Many students still do not know how to calculate the unbiased estimate of population mean/variance. (Unfamiliarity of formulae) 5. There was a handful of students who did not read the sample size off carefully for (ii) onwards **Note:** Short forms should not be used. Also, unbiased estimates of population mean/variance are **NOT** the population mean and variance. • Var $(t) = 0$ since t is a data value observed. • Var (T) is the population variance, not the unbiased estimate.
(iii)	Test $H_0: \mu = 15$ against $H_1: \mu < 15$	**Common problems:** Poor hypothesis testing structure was observed

	at 5% significance level. Under H_0, since $n = 50 > 30$ is large, $\overline{T} \sim N\left(15, \frac{22.367}{50}\right)$ approximately by Central Limit Theorem. Using a 1-tailed z-test, The test statistic value $\bar{t} = 13.8$ gives z_{calc}= -1.7942 and p-value = 0.036393=0.0364 ≤ 0.05 p-value = P(T ≤ 13.8 | μ=15) Since p-value = 0.0364 ≤ 0.05, we reject H_0 and conclude that at the 5% level of significance, there is sufficient evidence to conclude that the mean waiting time is less than 15 minutes.	Misreading of the claim and sample size used Improper terminologies **Common errors for:** Null and Alternative Hypothesis 1. Some students wrote: $H_0 = 15$ against $H_1 < 15$ or similar forms 2. Some wrote: $H_0 : \mu \geq 15$ **Reminder:** The null hypothesis states that the population mean is **equal** to a specific value. 3. Incorrect alternative hypothesis. There was a good proportion of students who wrote $H_1 : \mu > 15$ or $H_1 : \mu \neq 15$ 4. No mention of level of significance throughout the test, other than the conclusion. This is part of the set up! Distribution of Sample Mean Waiting Time 1. 'Under H_0' was missing. **Note:** This would then not allow the mean of $\overline{T}$ to take the value of 15 2. Incorrect mean and/or variance for $\overline{T}$ used 3. Did not know that Central Limit Theorem was to be used 4. Distribution of $\overline{T}$ is not approximated 5. Small group of students stated that T follows a normal distribution 6. Some students did not know that Central Limit Theorem was used to approximate $\overline{T}$ and not T. The test method 1. Did not state the test statistic value 2. $\bar{t}$ is the test statistic value, **NOT** the test statistic. **Note:** Test statistics are random variables, in this question, $\overline{T}$ for the p-value test. 3. Students who tried using the critical value method did not have a proper structure.

		Conclusion Students are to pay close attention to how conclusions should be written. **Prompts:** 1. What is the relationship between the null and alternative hypothesis? If the claim was 'at least 15 min' , what should the alternative hypothesis be? 2. Do we know the distribution of T? 3. Do we need to apply Central Limit Theorem? 4. What is the sample size used? 5. What is the p-value obtained? 6. What is the rejection criteria for hypothesis testing? (both p-value method and critical value method) 7. In this question, what does it mean by 'change in processes have been effective'? [**Note:** have you focused on the wrong words?]
(iv)	The p-value of **0.0364** is the **probability that sample mean waiting time of a customer** in the branch is **at most 13.8 minutes** when the **(population) mean waiting time of a customer in the branch is actually 15 minutes.** *Note:*	This part is poorly done. **Common errors** are as follow: 1. Many students defined level of significance instead 2. Students who tried did not a. State the p-values (context) b. Define in the context of the question c. Does not know what the p-value actually means (students on this point does not even define anything close to level of significance) d. Does not distinguish between sample mean waiting time of a customer and population mean waiting time of a customer. **Question:** What definition did the question ask for? Level of significance or p-value?

(v)

Test $H_0 : \mu = k$

against $H_1 : \mu \neq k$

at 2% significance level.

Under H_0, since $n = 50 > 30$ is large,

$\overline{T} \sim N\left(k, \frac{22.367}{50}\right)$ approximately by Central Limit Theorem.

Using a 2-tailed z-test, the test statistic $Z = \frac{\overline{T} - k}{\sqrt{\frac{22.367}{50}}} \sim N(0,1)$

Given that there is insufficient evidence to reject H_0,

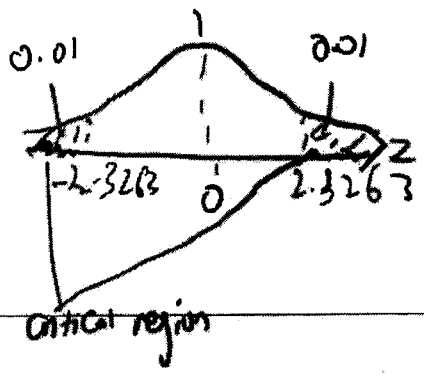
$$-2.3263 < \frac{13.8 - k}{\sqrt{\frac{22.367}{50}}} < 2.3263$$

$$-1.5559 < 13.8 - k < 1.5559$$

$$-15.3559 < -k < -12.2441$$

$$12.2441 < k < 15.3559$$

$$12.2 < k < 15.4$$



Common errors:

1. Students could not identify the correct alternative hypothesis.
2. Students could not recognise that the claim mean has been changed to k.
3. There was an unnecessary change of random variable when the data collected was still on the waiting time of a customer to be served.
4. Students did not recognise that k was actually the claimed population mean and thought that it was the test statistic value instead.
5. No distribution of Z was provided before conducting the test using the critical value method
6. Students who identified the correct alternative hypothesis, i.e. 2-tailed test, did not know how to identify the correct critical region and subsequently, the range to reject H_0.
7. There was a sizeable number of students who do not know how to conduct standardisation of the random variable.

Suggestions:

1. Read the question carefully, labelling all the values given with the correct terminologies.
2. Make use of the distribution curve to help in identifying the correct region.