# 2025 ACJC H2 Math Preliminary Examination Paper 1 Markers' Report

| Qn | Solution | Remarks |
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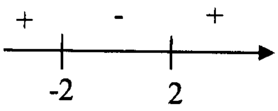
| 1(a) | $\frac{x}{x+2} \geq \frac{2}{2-x}$<br>$\frac{x}{x+2} - \frac{2}{2-x} \geq 0$<br>$\frac{x(2-x)-2(x+2)}{(x+2)(2-x)} \geq 0$<br>$\frac{-x^2-4}{(x+2)(2-x)} \geq 0$<br>$\frac{x^2+4}{(x+2)(x-2)} \geq 0$<br>As $x^2+4>0$ for all real $x$,<br>consider:<br><br>$(x-2)(x+2) \geq 0$<br>$x<-2$ or $x>2$ | This is a standard inequality question and generally it was quite well done.<br><br>Method:<br>Move expressions to one side, common denominator, sign test<br><br>Some common issues:<br>1. Multiplying -1 to both sides of inequality should result in an inequality sign flip.<br>2. When considering $x^2+4$, it is not sufficient to just look at whether it has roots (it matters whether it is always positive/negative).<br>3. $(x+2)(2-x)$ is a n-shaped quadratic curve.<br>4. In the final solution, $x \neq \pm 2$ is often forgotten. |
| 1(b) | $\frac{\lvert x\rvert}{\lvert x\rvert+2} \geq \frac{2}{2-\lvert x\rvert}$<br>Replace $x$ with $\lvert x\rvert$,<br>$\lvert x\rvert<-2$ (no solutions) or $\lvert x\rvert>2$<br>$x<-2$ or $x>2$ | The modulus function is still quite poorly understood by a significant minority.<br>$\lvert x\rvert<-2$ has NO real solutions. |
| 2(a) | Since vertical asymptote is $x=-1$, $d=1$.<br>Since the oblique asymptote is $y=2x+2$, consider<br>$y=2x+2+\frac{k}{x+1}$<br>$=\frac{(2x+2)(x+1)+k}{x+1}$<br>$=\frac{2x^2+4x+2+k}{x+1}$<br>Comparing with $y=\frac{ax^2+bx+c}{x+d}$, $a=2$ and $b=4$.<br><br>**Alternatively,**<br>Since vertical asymptote is $x=-1$, $d=1$.<br>$y=\frac{ax^2+bx+c}{x+1}=ax+(b-a)+\frac{c-(b-a)}{x+1}$<br>(by long division).<br>Comparing coefficients of $y=ax+(b-a)$ with $y=2x+2$,<br>$a=2$ and $b=4$. | Graphing question, generally well-done.<br><br>For students doing long division, there were still a minority who made careless slips in the procedure, resulting (thankfully for this part) in wrong remainders. Those who got the quotient ($ax+(b-a)$) would not have been able to show the given result.<br><br>Also, as this is a 'show' question, the presentation of their method should be clear and substantial. |

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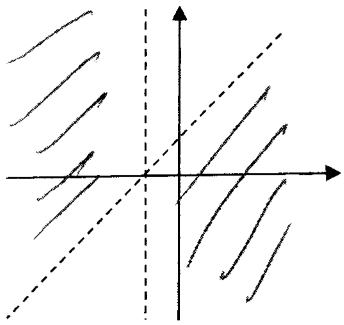
| **2(b)** | $y=\dfrac{2x^2+4x+c}{x+1}=2x+2+\dfrac{c-2}{x+1}$, $c\neq 2$.<br><br>For the graph to have no stationary points, the graphs will have to be in these 2 segments.<br><br>As $x\to\infty$,<br>$y\to(2x+2)^-$<br>Hence<br>$c-2<0 \;\Rightarrow\; c<2$.<br><br>**Alternatively,**<br>$\dfrac{dy}{dx}=2-\dfrac{c-2}{(x+1)^2}$<br>Considering $\dfrac{dy}{dx}=0$,<br>$\dfrac{c-2}{(x+1)^2}=2$<br>$c-2=2(x+1)^2$<br>$2x^2+4x+4-c=0$<br>For the graph to have no stationary points, the $\dfrac{dy}{dx}=0$ equation should have no solutions,<br>$16-4(2)(4-c)<0$<br>$-16+8c<0$<br>$c<2$ | This part proved more challenging than **(a)**. Those who got the remainder ($c-2$) wrong would not have been able to get the correct answer.<br><br>Most students considered the discriminant < 0, but a significant percentage of these solutions considered the discriminant of $2x^2+4x+c$ (which is the numerator of $y$). This is conceptually wrong. Students should be considering the discriminant of $\dfrac{dy}{dx}=0$.<br><br>There were also many careless mistakes differentiating $\dfrac{c-2}{x+1}$, with some even arriving at a ln function. |
| **3(a)** | $y=x^{xy}$<br>$\ln y=xy\ln x$<br>$\dfrac{\ln y}{y}=x\ln x$<br>$\left(\dfrac{1-\ln y}{y^2}\right)\dfrac{dy}{dx}=x\left(\dfrac{1}{x}\right)+(1)\ln x$<br>$\left(\dfrac{1-\ln y}{y^2}\right)\dfrac{dy}{dx}=1+\ln x$<br>$\dfrac{dy}{dx}=\dfrac{y^2(1+\ln x)}{1-\ln y}$ | Well done. Most realised the need to take ln on both sides before differentiating implicitly.<br><br>Those who rearranged to $\dfrac{\ln y}{y}=x\ln x$ were most successful, while some who differentiated $\ln y=xy\ln x$ missed out the third term in applying the product rule on the RHS.<br><br>Some mistakes observed:<br>1. $x^{xy}\neq x^x x^y$<br>2. $\dfrac{\ln x}{\ln y}\neq \ln(x-y)$<br>3. Assuming the base is a constant, e.g. $\dfrac{d}{dx}\left(x^{xy}\right)\neq \ln x\times x^{xy}\times\dfrac{d}{dx}(xy)$<br>4. Assuming the index is a constant, e.g. $\dfrac{d}{dx}\left(x^{xy}\right)\neq\dfrac{d}{dx}(xy)\times x^{xy-1}$ |

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| | **Alternatively,**<br>$y = x^{xy}$<br>$\ln y = xy\ln x$<br>$\frac{1}{y}\frac{dy}{dx} = x\ln x\frac{dy}{dx} + y(1+\ln x)$<br>$\frac{dy}{dx} = \frac{y(1+\ln x)}{\frac{1}{y} - x\ln x}$<br>$= \frac{y^2(1+\ln x)}{1 - xy\ln x}$<br>$= \frac{y^2(1+\ln x)}{1 - \ln y}$ | |
| **3(b)** | Tangent // to $y$-axis, $\frac{dy}{dx}$ is undefined.<br>$1 - \ln y = 0$<br>$\ln y = 1$<br>$\therefore y = e$<br>$\ln e = xe\ln x \Rightarrow x\ln x = \frac{1}{e}$<br>$\therefore x = 1.32$ (by GC)<br>The coordinates of the point is $(1.32, e)$. | Most equated the denominator of $\frac{dy}{dx}$ to zero correctly. Many got stuck solving $x\ln x = \frac{1}{e}$, and they should be reminded to use the GC. A handful did not give the final answer in coordinates, swopped $x$- and $y$-coordinates, or gave $\left(1.32, \frac{1}{e}\right)$ as the final answer. |
| **4** | Volume $= \pi\int_e^{e^2} y^2\,dx = \pi\int_e^{e^2} x^2(\ln x)^2\,dx$<br>$\int x^2(\ln x)^2\,dx = \frac{x^3}{3}(\ln x)^2 - \int \frac{x^3}{3}(2\ln x)\left(\frac{1}{x}\right)dx$<br>$= \frac{1}{3}x^3(\ln x)^2 - \frac{2}{3}\int x^2\ln x\,dx$<br>$= \frac{1}{3}x^3(\ln x)^2 - \frac{2}{3}\left(\frac{x^3}{3}\ln x - \int \frac{x^3}{3}\left(\frac{1}{x}\right)dx\right)$<br>$= \frac{1}{3}x^3(\ln x)^2 - \frac{2}{3}\left(\frac{x^3}{3}\ln x - \frac{1}{3}\int x^2\,dx\right)$<br>$= \frac{1}{3}x^3(\ln x)^2 - \frac{2}{3}\left(\frac{x^3}{3}\ln x - \frac{x^3}{9}\right) + c$<br>$= \frac{1}{27}x^3\left(9(\ln x)^2 - 6\ln x + 2\right) + c$<br>$\pi\int_e^{e^2} x^2(\ln x)^2\,dx = \frac{\pi}{27}\left[x^3\left(9(\ln x)^2 - 6\ln x + 2\right)\right]_e^{e^2}$<br>$= \frac{\pi}{27}\left(e^6(36-12+2) - e^3(9-6+2)\right)$<br>$= \frac{\pi}{27}\left(26e^6 - 5e^3\right) = \frac{\pi e^3}{27}\left(26e^3 - 5\right)$<br>$a = 26, b = -5$ | Most students knew the procedure to be carried out, i.e. obtain the correct limits, apply the formula, apply integration by parts, and evaluate the limits.<br><br>Almost all cited the correct volume formula, with common mistakes (i) using $\pi\int y\,dx$ and $2\pi\int y^2\,dx$; and (ii) finding the wrong region or limits of integration.<br><br>Most could apply integration by parts correctly once, but many made sign errors applying it twice.<br><br>Some made algebraic slips at the outset that made them think that only one integration by parts was necessary. These mistakes include:<br>1. $(\ln x)^2 \neq \ln(x^2) = 2\ln x$<br>2. $\int (x\ln x)^2\,dx \neq \left(\int x\ln x\,dx\right)^2$ |

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| **5(a)** | $\mathbf{a}\cdot\mathbf{n}=\mathbf{b}\cdot\mathbf{n}$<br>$(\mathbf{b}-\mathbf{a})\cdot\mathbf{n}=0$<br>$\overrightarrow{AB}\cdot\mathbf{n}=0$<br>$\overrightarrow{AB}\perp\mathbf{n}$<br>$\therefore \overrightarrow{AB}$ is <u>parallel</u> to the plane $p$. | Not well done.<br><br>It is insufficient to say that **a** and **b** are coplanar with normal vector **n**, as the property that $\mathbf{r}\cdot\mathbf{n}$ is a constant for planes is derived from this proof.<br><br>In writing proofs, students should ensure logical consistency, i.e.<br>1. It should not insinuate that $\mathbf{a}\cdot\mathbf{n}=0$ or $\mathbf{b}\cdot\mathbf{n}=0$ (both are given to be non-zero quantities)<br>2. $\mathbf{a}\cdot\mathbf{n}=\mathbf{b}\cdot\mathbf{n}$ does <u>not</u> imply that $\mathbf{a}=\mathbf{b}$<br><br>State *geometrical relationships* precisely (e.g. "parallel" or "perpendicular"), as opposed to:<br>1. More colloquial expressions like "lies in" and "contains", which is also vague here as neither $A$ nor $B$ are on the plane $p$;<br>2. "length of projection" when neither vector is unit and is a geometrical *interpretation*; and<br>3. "direction vector of the plane" which applies to *lines*. |
| **5(b)** | $\mathbf{r}=\lambda(\mathbf{b}-\mathbf{a})$<br>(Since origin lies on $p$.) | Not well done. Common errors are to think that $A$ or $B$ lie on $P$ (and hence on the line), or that the line is parallel to **n**. Some did not realise that $\mathbf{r}\cdot\mathbf{n}=0$ meant that the origin is a known point on $p$.<br><br>Abuse of notation, e.g. $l=\mathbf{r}+\lambda(\mathbf{a}-\mathbf{b})$, was observed. |
| **5(c)** | Substituting $\mathbf{r}=\mathbf{c}+\mu\mathbf{n}$ into $\mathbf{r}.\mathbf{n}=0$,<br>$(\mathbf{c}+\mu\mathbf{n}).\mathbf{n}=0$<br>$\mathbf{c}.\mathbf{n}+\mu\|\mathbf{n}\|^2=0$<br>$\mu=-\frac{\mathbf{c}.\mathbf{n}}{\|\mathbf{n}\|^2}$<br>$\therefore \overrightarrow{OF}=\mathbf{c}-\left(\frac{\mathbf{c}.\mathbf{n}}{\|\mathbf{n}\|^2}\right)\mathbf{n}$<br><br>**Alternatively,**<br>$\overrightarrow{FC}=(\overrightarrow{OC}.\hat{\mathbf{n}})\hat{\mathbf{n}}=\frac{(\mathbf{c}.\mathbf{n})}{\|\mathbf{n}\|^2}\mathbf{n}$<br>$\therefore \overrightarrow{OF}=\overrightarrow{OC}+\overrightarrow{CF}=\mathbf{c}-\frac{(\mathbf{c}.\mathbf{n})}{\|\mathbf{n}\|^2}\mathbf{n}$ | Not well done. Many could not use the line equation or vector projection formula, probably due to the abstraction of this question.<br><br>Many arrived at $\mathbf{c}-\left(\frac{\mathbf{c}.\mathbf{n}}{\|\mathbf{n}\|^2}\right)\mathbf{n}$, but went ahead to "cancel" by **n** to get $\mathbf{c}-\frac{\mathbf{c}}{\mathbf{n}}\mathbf{n}=\mathbf{0}$. The final answer mark was withheld for this conceptual error, and students are reminded that this is not a valid vector operation. In a similar vein, $\mathbf{n}\cdot\mathbf{n}=\mathbf{n}^2$ was also seen.<br><br>Some who attempted the line equation approach only considered $\overrightarrow{CF}\cdot\mathbf{n}$ instead of the equation of line $CF$.<br><br>More mistakes were seen by students using the vector projection approach, including: |

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| | | 1. Stating $\left(\overrightarrow{OC}.\hat{\mathbf{n}}\right)\hat{\mathbf{n}}$ is $\overrightarrow{CF}$ or $\overrightarrow{OF}$<br>2. Using $\overrightarrow{FC}=\left\|\overrightarrow{OC}.\hat{\mathbf{n}}\right\|\hat{\mathbf{n}}$<br>3. Using $\overrightarrow{OF}=\left\|\overrightarrow{OC}\times\hat{\mathbf{n}}\right\|\hat{\mathbf{n}}$ |
| **6(a)** | $\pi(r-a)^2(h-a)=1000\pi$<br>$h-a=\frac{1000}{(r-a)^2}\Rightarrow h=\frac{1000}{(r-a)^2}+a$<br><br>$V=\pi r^2h-\pi(r-a)^2(h-a)$<br>$=\pi r^2\left(\frac{1000}{(r-a)^2}+a\right)-1000\pi$<br>$=\frac{1000\pi r^2}{(r-a)^2}+\pi r^2a-1000\pi$<br>$=1000\pi\left[\frac{r^2}{(r-a)^2}-1\right]+\pi r^2a$<br>$k=1000$ | Most students were able to get at least 2/3 marks.<br><br>Common mistakes were<br>1. unnecessary expansion of $(r-a)^2(h-a)$. Students should look at the form to be shown as a guide<br>2. a lot of unnecessary working from not seeing the internal cylinder volume as ,  resulting in loss of time<br>3. misread $1000\pi$ to be $100\pi$<br><br>Note also that since *h* is defined in the question as the external height, some students still use *h* for the height of the internal cylinder, which resulted in presentation error even though they manage to show the expression. |
| **6(b)** | $\frac{\mathrm{d}V}{\mathrm{d}r}=1000\pi\left[\frac{2r(r-a)^2-2r^2(r-a)}{(r-a)^4}\right]+\pi r^2a$<br>$=1000\pi\left[\frac{2r(r-a)[(r-a)-r]}{(r-a)^4}\right]+2\pi ar$<br>$=-\frac{2000\pi ar(r-a)}{(r-a)^4}+2\pi ar$<br>$=-\frac{2000\pi ar}{(r-a)^3}+2\pi ar$<br>$\frac{\mathrm{d}V}{\mathrm{d}r}=-\frac{2000\pi ar}{(r-a)^3}+2\pi ar=0$<br>$\frac{2000\pi ar}{(r-a)^3}=2\pi ar$<br>$(r-a)^3=1000\Rightarrow r=10+a$ | Most can differentiate using quotient/product rule correctly, although some reversed the order of the numerator when using quotient rule.<br><br>However many were not able to solve $\frac{\mathrm{d}V}{\mathrm{d}r}=0$ due to inadequacies in algebraic manipulation, such as<br>1. unnecessary expansion, instead of factorisation and cancellation of common terms<br>2. not simplifying the expression, e.g. $r-a$ can be cancelled in $\frac{2000\pi ar(r-a)}{(r-a)^4}$ |

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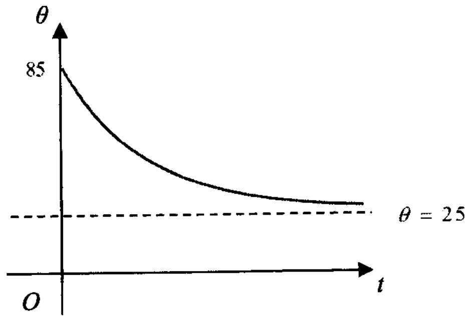
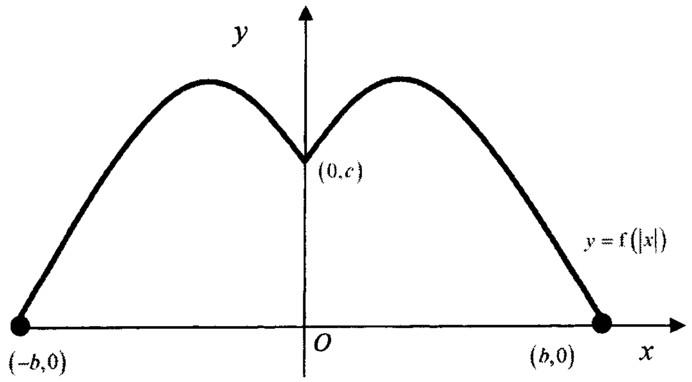
| 7(a) | $\frac{d\theta}{dt}=k(\theta-25)$<br>$\int \frac{1}{\theta-25}\,d\theta=kt+c$<br>$\Rightarrow \ln\lvert\theta-25\rvert=kt+c$<br>$\Rightarrow \theta-25=Ae^{kt}$<br>When $t=0, \theta=85 \Rightarrow A=60 \Rightarrow \theta-25=60e^{kt}$<br>When<br>$t=20, \theta=55 \Rightarrow 30=60e^{20k} \Rightarrow k=\frac{1}{20}\ln\frac{1}{2}(=)-0.0347$<br>$\therefore \theta=25+60e^{\left(\frac{1}{20}\ln\frac{1}{2}\right)t}=25+60\left(\frac{1}{2}\right)^{\frac{t}{20}}=25+60e^{-0.0347t}$ | There are many who got full marks for this question even though it is in some sense unguided, showing that most students know the process of solving differential equations and find particular solutions clearly.<br><br>There were a few who used the conditions right at the start. students need to know that if the conditions involve time, the conditions can only be used after obtaining the general solution. |
| 7(b) |  | Most students were able to get the graph using the form given.<br><br>Comon errors were<br>1. sketching for $t<0$<br>2. the curve did not cut the $y$-axis<br>3. not indicating the horizontal asymptote or $y$-intercept |
| 8(a) (i) |  | Most got this correct. Some who didn't study of course got it wrong. They reflected $x<0$ about the $y$-axis. |

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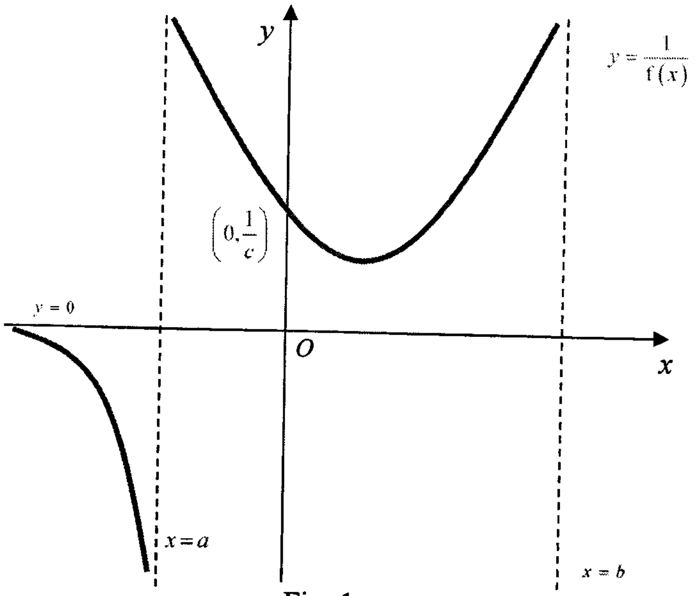
| **8(a) (ii)** | <br>Fig. 1 | Quite well done too.<br><br>Comon errors<br>1. no $y$-intercept indicated<br>2. extra curve on $x>b$ |
| **8(a) (iii)** | Scale the graph of $y=f(2x+1)$ by factor 2 parallel to the $x$-axis. (To obtain $y=f\left(2\left(\frac{x}{2}\right)+1\right)=f(x+1)$.)<br>Translate the graph of $y=f(x+1)$ 1 unit in the positive $x$-direction. (To obtain $y=f((x-1)+1)=f(x)$.)<br><br>**Alternatively,**<br>Translate the graph of $y=f(2x+1)$ 0.5 units in the positive $x$-direction. (To obtain $y=f(2(x-0.5)+1)=f(2x)$.)<br>Scale the graph of $y=f(2x)$ by factor 2 parallel to the $x$-axis. (To obtain $y=f\left(2\left(\frac{x}{2}\right)\right)=f(x)$.) | There were a lot of reading errors in this question. Many read as obtaining $y=f(2x+1)$ from $y=f(x)$.<br><br>Those who did translation first often translated by 1 unit in the positive direction. This will result in a replacement of $x$ by $x-1$, hence will give $y=f(2(x-1)+1)=f(2x-1)$ and not $y=f(2x)$.<br><br>There were many who did scaling parallel to the $y$-axis.<br><br>Marks were also deducted for students who use the terms "shift" or "scale by 2 units" |
| **8(b) (i)** | Since $y=f(x)$ is a quadratic curve, the maximum point occurs at $x=\frac{a+b}{2}$.<br>Hence the maximum value of $k$ is $\frac{a+b}{2}$ for $y=g^{-1}(x)$ to exist. | About one third got this right. Many put $x=\frac{b-a}{2}$. |

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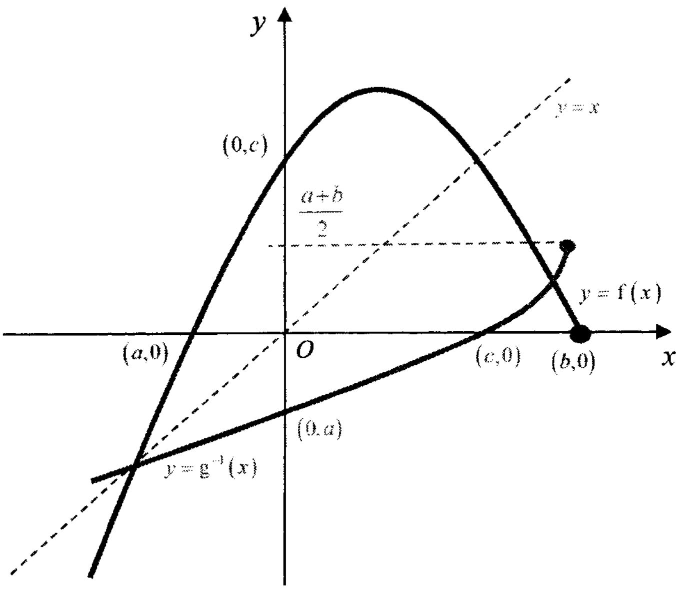
| **8(b) (ii)** | <br>Fig. 1<br>$R_{g^{-1}}=D_g=\left(-\infty,\dfrac{a+b}{2}\right]$ | This was poorly done.<br><br>There were many answers that gave the reflection of the whole curve, instead of the part $x\le\dfrac{a+b}{2}$.<br><br>There were many who gave me curves that are not one-one. (The curve "turn" too much and became concave upwards)<br><br>There were also many answers that gave the y-intercept as $(0,-a)$ because it is below the $x$-axis. Marks were not deducted but students should realise that $a$ need not always be a positive number. |
| **8(b) (iii)** | The solution to $\mathrm{g}^{-1}(x)=x$ is the $x$-coordinate of the intersection point between the graph of $y=\mathrm{g}^{-1}(x)$ and the line $y=x$.<br>As the graphs of $y=\mathrm{g}^{-1}(x)$ and $y=\mathrm{g}(x)$ are the reflections of each other in the line $y=x$, if the graph of $y=\mathrm{g}^{-1}(x)$ intersects the line $y=x$, then the graph of $y=\mathrm{g}(x)$ will also intersect the line $y=x$ at the same point.<br>Hence, the solution to $\mathrm{g}^{-1}(x)=x$ will also satisfy the equation $\mathrm{g}^{-1}(x)=\mathrm{g}(x)$. | The explanation "$y=\mathrm{g}^{-1}(x)$ and $y=\mathrm{g}(x)$ are the reflections of each other in the line $y=x$" is not sufficient, they have to mention that the two curves intersect at $y=x$ or suggest that is the case.<br><br>The language used is also particularly bad as students do not how to describe curves. They will say "$\mathrm{g}(x)$ and $\mathrm{g}^{-1}(x)$ intersect at $x$", instead of "the graphs of $\mathrm{g}(x)$ and $\mathrm{g}^{-1}(x)$ intersect at the line $y=x$ ". |
| **9(a)** | Substitute $(1,3,-2)$ into $3x+c(y+z)-2=0$,<br>$3(1)+c(3-2)-2=0\Rightarrow c=-1$ | |

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| | **Alternatively:**<br>$\pi_1 : \mathbf{r}.\begin{pmatrix}3\\c\\c\end{pmatrix}=2$<br>$\begin{pmatrix}1\\3\\-2\end{pmatrix}.\begin{pmatrix}3\\c\\c\end{pmatrix}=3+3c-2c=2 \Rightarrow c=-1$ | |
| **9(b)** | Sub $\mathbf{r}=\begin{pmatrix}1+2s+t\\3-s\\-2+3s-t\end{pmatrix}$ into $\mathbf{r}.\begin{pmatrix}3\\-1\\-1\end{pmatrix}=2$<br>$\begin{pmatrix}1+2s+t\\3-s\\-2+3s-t\end{pmatrix}.\begin{pmatrix}3\\-1\\-1\end{pmatrix}=2$<br>$3+6s+3t-3+s+2-3s+t=2$<br>$4s+4t=0$<br>$s=-t$<br>$\therefore$ line of intersection is<br>$\mathbf{r}=\begin{pmatrix}1-2t+t\\3+t\\-2-3t-t\end{pmatrix}=\begin{pmatrix}1-t\\3+t\\-2-4t\end{pmatrix}=\begin{pmatrix}1\\3\\-2\end{pmatrix}-t\begin{pmatrix}1\\-1\\4\end{pmatrix}=\begin{pmatrix}1\\3\\-2\end{pmatrix}+\alpha\begin{pmatrix}1\\-1\\4\end{pmatrix}$<br>**Alternatively:**<br>$\mathbf{n}_2=\begin{pmatrix}2\\-1\\3\end{pmatrix}\times\begin{pmatrix}1\\0\\-1\end{pmatrix}=\begin{pmatrix}1\\5\\1\end{pmatrix}$<br>Since $A(1,3,-2)$ lies in both planes and<br>$\begin{pmatrix}3\\-1\\-1\end{pmatrix}\times\begin{pmatrix}1\\5\\1\end{pmatrix}=\begin{pmatrix}4\\-4\\16\end{pmatrix} // \begin{pmatrix}1\\-1\\4\end{pmatrix}$<br>$\therefore$ line of intersection is $\mathbf{r}=\begin{pmatrix}1\\3\\-2\end{pmatrix}+\alpha\begin{pmatrix}1\\-1\\4\end{pmatrix}=\begin{pmatrix}1+\alpha\\3-\alpha\\-2+4\alpha\end{pmatrix}$<br>Cartesian form:<br>$\pi_1 : 3x-y-z=2$<br>$\pi_2 : x+5y+z=14$<br>From GC, line of intersection is **not** accepted. | To show the line of intersection between 2 planes, GC method is not accepted.<br>Line of intersection is given by $\mathbf{r}=\mathbf{a}+\lambda(\mathbf{n}_1\times\mathbf{n}_2)$ where $\mathbf{a}$ is the p.v. of the point $(1,3,-2)$ that lies in both planes. |
| **9(c)** | Since $P$ lies on the line, $\overrightarrow{OP}=\begin{pmatrix}1+\alpha\\3-\alpha\\-2+4\alpha\end{pmatrix}$<br>$\overrightarrow{BP}=\begin{pmatrix}1+\alpha\\3-\alpha\\-2+4\alpha\end{pmatrix}-\begin{pmatrix}2\\-3\\7\end{pmatrix}=\begin{pmatrix}-1+\alpha\\6-\alpha\\-9+4\alpha\end{pmatrix}$<br>$\left\|\overrightarrow{BP}\right\|=\left\|\begin{pmatrix}-1+\alpha\\6-\alpha\\-9+4\alpha\end{pmatrix}\right\|=3\sqrt{2}$ | Students who had difficulty did not use "any point that lies on the line $l$" is given by $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1+\alpha\\3-\alpha\\-2+4\alpha\end{pmatrix}$ for some α. |

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| | $\sqrt{(-1+\alpha)^2+(6-\alpha)^2+(-9+4\alpha)^2}=3\sqrt{2}$<br>$1-2\alpha+\alpha^2+36-12\alpha+\alpha^2+81-72\alpha+16\alpha^2=18$<br>$18\alpha^2-86\alpha+118=18$<br>$9\alpha^2-43\alpha+50=0$<br>$\alpha=2$ or $\frac{25}{9}$<br>The points are<br>$\begin{pmatrix}1+2\\3-2\\-2+4(2)\end{pmatrix}=\begin{pmatrix}3\\1\\6\end{pmatrix}$ $\begin{pmatrix}1+\frac{25}{9}\\3-\frac{25}{9}\\-2+4\left(\frac{25}{9}\right)\end{pmatrix}=\begin{pmatrix}\frac{34}{9}\\\frac{2}{9}\\\frac{82}{9}\end{pmatrix}=\frac{1}{9}\begin{pmatrix}34\\2\\82\end{pmatrix}$ | |
| **9(d)** | $\pi_2:\mathbf{r}.\begin{pmatrix}1\\5\\1\end{pmatrix}=\begin{pmatrix}1\\3\\-2\end{pmatrix}.\begin{pmatrix}1\\5\\1\end{pmatrix}=1+15-2=14$<br>$\pi_3:\mathbf{r}.\begin{pmatrix}1\\5\\1\end{pmatrix}=\begin{pmatrix}2\\-3\\7\end{pmatrix}.\begin{pmatrix}1\\5\\1\end{pmatrix}=2-15+7=-6$<br>Distance $=\frac{\lvert 14-(-6)\rvert}{\sqrt{1^2+5^2+1^2}}=\frac{20}{\sqrt{27}}=\frac{20}{3\sqrt{3}}$<br>**Alternatively (for distance),**<br>$\overrightarrow{AB}=\begin{pmatrix}2\\-3\\7\end{pmatrix}-\begin{pmatrix}1\\3\\-2\end{pmatrix}=\begin{pmatrix}1\\-6\\9\end{pmatrix}$<br>$\lvert\overrightarrow{AB}.\hat{\mathbf{n}}\rvert=\left\lvert\begin{pmatrix}1\\-6\\9\end{pmatrix}.\frac{1}{\sqrt{27}}\begin{pmatrix}1\\5\\1\end{pmatrix}\right\rvert=\frac{1}{3\sqrt{3}}\left\lvert\begin{pmatrix}1\\-6\\9\end{pmatrix}.\begin{pmatrix}1\\5\\1\end{pmatrix}\right\rvert=\frac{20}{3\sqrt{3}}$<br>Hence the perpendicular from $\pi_3$ to $\pi_2$ is $\frac{20}{3\sqrt{3}}$. | Using the equations of planes:<br>$\pi_1:\mathbf{r}.\mathbf{n}=p_1$<br>$\pi_2:\mathbf{r}.\mathbf{n}=p_2$<br>to find the distance between two planes $=\frac{\lvert p_1-p_2\rvert}{\lvert\mathbf{n}\rvert}$.<br>If $p_1>0,\ p_2<0$ then the planes lies on opposite sides of the origin. Adding distances from orign is the distance between two planes. Otherwise students should avoid using $\frac{\lvert p_1\rvert+\lvert p_2\rvert}{\lvert\mathbf{n}\rvert}$ as a formula, it <u>will not</u> work if both $p_1>0,\ p_2>0$ or both $p_1<0,\ p_2<0$. |

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| **10(a)** | $\omega^4-2\omega^3+10\omega^2+p\omega+q=0$<br>Since the coefficients are all real, complex roots occur in conjugate pairs. Therefore $2-3\text{i}$ is also a root.<br>$\therefore \omega^4-2\omega^3+10\omega^2+p\omega+q$<br>$=\left(\omega-(2+3\text{i})\right)\left(\omega-(2-3\text{i})\right)\left(\omega^2+a\omega+b\right)$<br>$=\left(\omega^2-4\omega+13\right)\left(\omega^2+a\omega+b\right)$<br>Comparing coefficients:<br>$\omega^3:-2=-4+a\Rightarrow a=2$<br>$\omega^2:10=-4a+13+b\Rightarrow b=5$<br>$\therefore \omega^4-2\omega^3+10\omega^2+p\omega+q\left(\omega^2-4\omega+13\right)\left(\omega^2+2\omega+5\right)$<br>$\Rightarrow p=-20+26=6$<br>$\Rightarrow q=13\times5=65$<br>The other roots are<br>$\omega=\dfrac{-2\pm\sqrt{4-4(5)}}{2}=\dfrac{-2\pm\sqrt{-16}}{2}=-1\pm2\text{i}$<br><br>**Alternatively,**<br>$(2+3\text{i})^2=-5+12\text{i}$<br>$(2+3\text{i})^3=(-5+12\text{i})(2+3\text{i})=-46+9\text{i}$<br>$(2+3\text{i})^4=(-5+12\text{i})^2=-119-120\text{i}$<br>Substitute $\omega=2+3\text{i}$ into $\omega^4-2\omega^3+10\omega^2+p\omega+q=0$<br>$(-119-120\text{i})-2(-46+9\text{i})$<br>$+10(-5+12\text{i})+p(2+3\text{i})+q=0$<br>Compare real and imaginary parts,<br>$\text{Re}:-119+92-50+2p+q=0\Rightarrow 2p+q=77$<br>$\text{Im}:-120-18+120+3p=0\Rightarrow p=6\Rightarrow q=65$<br>Since the coefficients are all real, complex roots occur in conjugate pairs. Therefore $2-3\text{i}$ is also a root.<br>$\therefore \omega^4-2\omega^3+10\omega^2+6\omega+65$<br>$=\left(\omega-(2+3\text{i})\right)\left(\omega-(2-3\text{i})\right)\left(\omega^2+a\omega+b\right)$<br>$=\left(\omega^2-4\omega+13\right)\left(\omega^2+a\omega+b\right)$<br>Comparing coefficients:<br>$\omega^0:65=13b\Rightarrow b=5$<br>$\omega^3:-2=-4+a\Rightarrow a=2$<br>Therefore the other factor is $\omega^2+2\omega+5$.<br>Hence the other roots are<br>$\omega=\dfrac{-2\pm\sqrt{4-4(5)}}{2}=\dfrac{-2\pm\sqrt{-16}}{2}=-1\pm2\text{i}$ | Lots of errors when expanding $2+3\text{i}$ up to power $(2+3\text{i})^4$.<br>Lots of errors when expanding factors of conjugate roots.<br><br>Regards to comparing coefficients, some students needed more effort than others. And for other students, long division works well.<br><br>Some students expressed the quadratic factor as<br>$\left(\omega-(a+b\text{i})\right)\left(\omega-(a-b\text{i})\right)$<br>or<br>$(\omega-a)(\omega-b)$<br>where the latter clearly does not suffice. |

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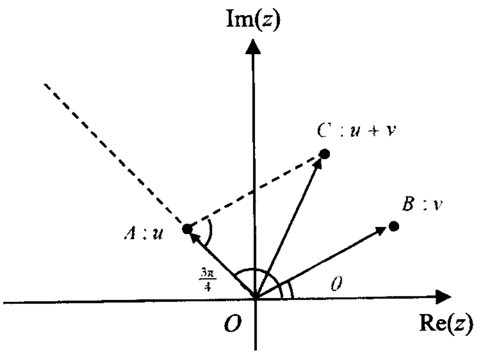
| **10(b)<br>(i)** | $\lvert u\rvert=\sqrt{\left(-\sqrt{2}\right)^2+\sqrt{2}^2}=2$<br>Basic angle $=\tan^{-1}\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{\pi}{4}$<br>$\arg u=\pi-\dfrac{\pi}{4}=\dfrac{3\pi}{4}$ | This is wrong:<br>$\lvert u\rvert=\sqrt{\left(-\sqrt{2}\right)^2+\left(\sqrt{2}\mathbf{i}\right)^2}=0$<br>The <u>modulus</u> of **u** is the length of OA.<br>The <u>argument</u> of **u** is the angle (in radians) that OA made with the positive Re($z$).<br>Students can use GC to get angle/arg. |
| **10(b)<br>(ii)** |  | Argand diagram mostly lack details.<br>Given $0<\arg\mathbf{v}<\dfrac{\pi}{4}$ but many students drew OB at 45°, resulting in a rectangle or square OACB, instead of a parallelogram.<br>Since $\lvert\mathbf{v}\rvert=3>2$, the length of OB should be drawn longer in comparison to OA. |
| **10(b)<br>(iii)** | $\angle AOB=\dfrac{3\pi}{4}-\theta$<br>$\angle OAC=\pi-\left(\dfrac{3\pi}{4}-\theta\right)=\theta+\dfrac{\pi}{4}$<br>Using cosine rule on $\Delta OAC$<br>$OC^2=AO^2+AC^2-2(AO)(AC)\cos\angle OAC$<br>$\lvert u+v\rvert^2=2^2+3^2-2(2)(3)\cos\left(\theta+\dfrac{\pi}{4}\right)$<br>$=13-12\cos\left(\theta+\dfrac{\pi}{4}\right)$<br>$a=13, b=-12, K=\dfrac{\pi}{4}$<br><br>**Alternative Method 1**<br>$\lvert u+v\rvert^2=(u+v)\cdot(u+v)$<br>$=\lvert u\rvert^2+2u.v+\lvert v\rvert^2$ $\qquad u.v=\lvert u\rvert\lvert v\rvert\cos\angle AOB$<br>$=2^2+3^2+2(2)(3)\cos\left(\dfrac{3\pi}{4}-\theta\right)$<br>$=13+12\cos\left(\dfrac{3\pi}{4}-\theta\right)$<br>$=13+12\cos\left(\theta-\dfrac{3\pi}{4}\right)$ $\qquad a=13, b=12, K=-\dfrac{3\pi}{4}$ | O level geometry & cosine rule.<br><br>Alternative method 1 uses $\lvert z\rvert^2=z.z$.<br><br>Alternative method 2 is for students who applied trigo to rewrite complex no. $v$.<br>$x=\mathrm{Re}(v)=3\cos\theta$<br>$y=\mathrm{Im}(v)=3\sin\theta$<br>$v=x+\mathrm{i}y=3(\cos\theta+\mathrm{i}\sin\theta)$ |

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| | **Alternative Method 2**<br>$\lvert u+v\rvert^2 = \lvert -\sqrt{2}+\mathrm{i}\sqrt{2}+3(\cos\theta+\mathrm{i}\sin\theta)\rvert^2$<br>$=(-\sqrt{2}+3\cos\theta)^2+(\sqrt{2}+3\sin\theta)^2$<br>$=(2-6\sqrt{2}\cos\theta+9\cos^2\theta)+(2+6\sqrt{2}\sin\theta+9\sin^2\theta)$<br>$=4+9(\sin^2\theta+\cos^2\theta)-6\sqrt{2}(\cos\theta-\sin\theta)$<br>$=13-6\sqrt{2}\left(\sqrt{2}\cos\left(\theta+\frac{\pi}{4}\right)\right)$<br>$=13-12\cos\left(\theta+\frac{\pi}{4}\right)$ | |
| **11(a)** | $\frac{\mathrm{d}x}{\mathrm{d}t}=2at \qquad \frac{\mathrm{d}y}{\mathrm{d}t}=-\frac{a}{t^2}$<br>$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{a}{t^2}\div 2at=-\frac{1}{2t^3}$<br>At $P\left(ap^2,\frac{a}{p}\right)$, $t=p \quad \therefore \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2p^3}$<br>$y-\frac{a}{p}=-\frac{1}{2p^3}(x-ap^2)$<br>$y=-\frac{1}{2p^3}x+\frac{a}{2p}+\frac{a}{p}$<br>$y=-\frac{1}{2p^3}x+\frac{3a}{2p}$<br>$2p^3y+x=3ap^2$ | Most students were able to apply chain rule correctly to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $t$ and use the coordinates of the point to arrive at the correct equation.<br><br>The value of $t=p$ should be substituted into the expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ first to obtain the value of the gradient <u>before</u> forming the equation of the tangent. An equation with 3 variables $(x, y, t)$ is not an equation for a line.<br><br>As this is a "show" question and the final answer is provided, students must be reminded to show <u>sufficient</u> algebraic working to demonstrate how their formula for equation of tangent leads to the required solution. The examiner cannot fill in the blanks for the steps that are not written down explicitly. |
| **11(b)** | $x=a \Rightarrow p^2=1 \therefore p=\pm 1$<br>Substitute $p=\pm 1$ into the equations of tangents in **(a)**:<br>$2(1)^3y+x=3a(1)^2 \Rightarrow 2y+x=3a$ or $y=-\frac{1}{2}x+\frac{3a}{2}$<br>$2(-1)^3y+x=3a(-1)^2 \Rightarrow -2y+x=3a$ or $y=\frac{1}{2}x-\frac{3a}{2}$<br><br>Angle between the tangents is<br>$\tan^{-1}\frac{1}{2}-\tan^{-1}\left(-\frac{1}{2}\right)=53.1^\circ$ or $0.927$ radians | Most students were able to find $t=\pm 1$, but some did not realise that these were the values of $p$ to substitute into the equation of the tangent from part **(a)**. Many of them end up with algebraic errors as they tried to find the equations of tangents from scratch, for example in finding the values of the gradients and the coordinates of the points. Another common error was carelessness in matching the gradient to the correct coordinates.<br><br>To find the angle between the tangents, the concept to use is that the gradient of a line that makes an angle of $\theta$ with the positive $x$-axis is $\tan\theta$. Students who didn't know this often tried to find the |

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| | | angle by geometry, which wasn't always successful as students used incorrect values from the equations of the tangent for the calculation. A common misconception was assuming the tangents were at right angles as their gradients differ by a factor of −1 – the correct concept is that two lines are perpendicular if the *product* of their gradients is −1. |
| **11(c) (i)** | $(q-p)^2(q+2p)=(q^2-2qp+p^2)(q+2p)$<br>$=q^3 \cancel{-2q^2p}+p^2q\cancel{+2pq^2}-4p^2q+2p^3$<br>$=q^3-3p^2q+2p^3$ | This is a straightforward question that is meant to help students for **(c)(ii)**. Again, since this is a "show" question, students must write down sufficient algebraic working to reach the final answer. |
| **11(c) (ii)** | Sub $Q\left(aq^2,\frac{a}{q}\right)$ into $2p^3y+x=3ap^2$,<br>$2p^3\left(\frac{a}{q}\right)+aq^2=3ap^2$<br>$2ap^3+aq^3=3ap^2q$<br>$q^3-3p^2q+2p^3=0$<br>$(q-p)^2(q+2p)=0$<br>Since $q\neq p \ \therefore q=-2p$ | A significant number of students were stuck after substituting the coordinates of $Q$ into the equation of the tangent. Some tried to solve for $q$ using some form of the "zero product rule", but end up with an answer that also contains $q$. These students usually did not realise that the result in **(c)(i)** was meant to be a hint to help students factorise the cubic expression for $q$.<br><br>For those who arrived at the solutions $q=p$ or $-2p$, many of them did not go on to reject the first answer, not realising that $Q$ must be a different point from $P$ and therefore $q\neq p$.<br><br>Some students did not understand the task required and attempted to find the equation of the tangent at point $Q$ instead. |
| **11(d)** | **Method 1: Area between 2 graphs**<br>Area $=\int_{p^2}^{4p^2}\frac{1}{2p^3}(-x+3p^2)-\left(-\frac{1}{\sqrt{x}}\right)\mathrm{d}x$<br>$=\frac{1}{2p^3}\left[-\frac{x^2}{2}+3p^2x\right]_{p^2}^{4p^2}+\left[2x^{\frac{1}{2}}\right]_{p^2}^{4p^2}$<br>$=\frac{1}{2p^3}\left(\left(-\frac{16p^4}{2}+12p^4\right)-\left(-\frac{p^4}{2}+3p^4\right)\right)+(4p-2p)$<br>$=\frac{1}{2p^3}\left(4p^4-\frac{5p^4}{2}\right)+2p$<br>$=\frac{3p^4}{4p^3}+2p$<br>$=\frac{11p}{4}$ | There was a roughly equal proportion of students who tried to find the area with respect to the $x$-axis and the $y$-axis. The easiest approach was to observe that the region is simply bounded by the tangent $y=\frac{3p^2-x}{2p^3}$ from above and the curve $y=-\frac{1}{\sqrt{x}}$ from below, so the formula for area between 2 curves would suffice. Students were more likely to make algebraic errors if they tried to divide the region into more parts.<br><br>Some of the common algebraic errors include: |

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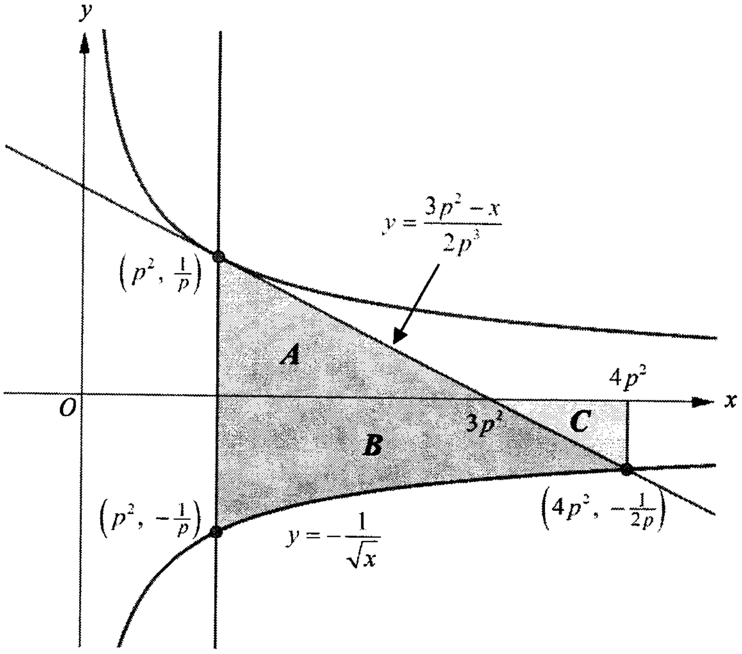
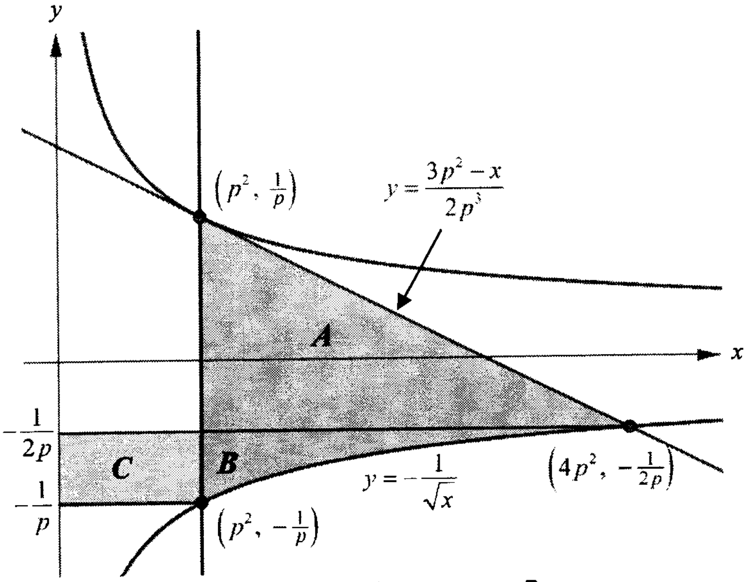
| | **Method 2: Area with respect to $x$-axis**<br><br>Area of $A$ $=\frac{1}{2}\times(3p^2-p^2)\times\frac{1}{p}$<br>$=p$<br>Area of $B$ and $C=-\int_{p^2}^{4p^2}-\frac{1}{\sqrt{x}}\,dx$<br>$=\int_{p^2}^{4p^2}x^{-\frac{1}{2}}\,dx$<br>$=\left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_{p^2}^{4p^2}$<br>$=2\sqrt{4p^2}-2\sqrt{p^2}$<br>$=2(2p)-2p$<br>$=2p$<br>Area of $C$ $=\frac{1}{2}\times(4p^2-3p^2)\times\frac{1}{2p}$<br>$=\frac{1}{4p}$<br>$\therefore$ Required area $=p+2p-\frac{1}{4p}$<br>$=\frac{11p}{4}$ | • Writing the equation of the curve as $y=-\frac{1}{x}$ or $y=\sqrt{1-x}$<br>• Using the wrong curve, or not accounting for the negative area bounded by the curve and $x$-axis<br>• Using the wrong limits, or mixing up the $x$- and $y$-coordinates<br>• Forgetting to subtract the areas of certain triangles and rectangles<br>• Carelessness in the actual integration<br><br>Students should **sketch a diagram in the solution space** in the Printed Answer Booklet to complement their workings. Any sketches or indications done on the question booklet will not be seen by the marker, so their descriptions of "rectangles" and "triangles" would not make sense to the marker. |

**Method 3: Area with respect to $y$-axis**




Area of $A$ $=\frac{1}{2}\times\left(4p^2-p^2\right)\times\left[\frac{1}{p}-\left(-\frac{1}{2p}\right)\right]$

$=\frac{9p}{4}$

Area of $B$ and $C$ $=\int_{-\frac{1}{p}}^{-\frac{1}{2p}}\frac{1}{y^2}\,\mathrm{d}y$

$=\left[-\frac{1}{y}\right]_{-\frac{1}{p}}^{-\frac{1}{2p}}$

$=2p-p$

$=p$

Area of $C$ $=p^2\times\left[-\frac{1}{2p}-\left(-\frac{1}{p}\right)\right]$

$=\frac{p}{2}$

$\therefore$ Required area $=\frac{9p}{4}+p-\frac{p}{2}$

$=\frac{11p}{4}$

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| **12(a)** | Amount Sarah plans to repay (before graduation):<br>$\frac{1}{5}\times\frac{90}{100}\times 8250\times 4=\$5940$<br><br>Number of repayments made from August 2021 to July 2025 is 48 (i.e. $n=48$).<br>The repayments follow an arithmetic progression (AP) with first term $a=x$ and common difference $d=2$.<br><br>The sum of an arithmetic progression is<br>$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$<br>$5940=\frac{48}{2}\left[2x+(48-1)2\right]$<br>$x=76.75$ | Many students were unable to handle the many details involved in the calculation of the loan amount and repayment, often missing out at least one piece of information. The most common error in calculating the amount repaid in the first phase was omitting to multiply by 4, as \$8250 was the **annual** tuition fee for the **4-year course**.<br><br>Students often applied the wrong formula, using $a+(n-1)d$ instead of the formula for the sum of an AP. The number of repayments was also a common error – 47 was commonly seen, perhaps due to careless counting, but other numbers were also seen, which indicated inadequacy in understanding the problem.<br><br>Students should also not round off exact answers unnecessarily. |
| **12(b)** | Amount of outstanding loan after Sarah's graduation:<br>$\frac{4}{5}\times\frac{90}{100}\times 8250\times 4=\$23760$<br><br>On 31st August 2025, apply interest of 0.4% to outstanding loan to get $\$\left(23760(1.004)\right)$.<br>On 1st September 2025, make 1st repayment of \$$y$ to the bank.<br><br>$\therefore$ Outstanding loan after 1st repayment<br>$=\$\left(23760(1.004)-y\right)$ or $\$(23855.04-y)$ | To answer parts **(b)** and **(c)** correctly, students needed to understand the interest and repayment scheme and be very meticulous with the timeline of both elements when tracking how the outstanding loan changes.<br><br>The first important point to note is that **interest** is the **first** element to be applied to the post-graduation loan amount. This is then followed by **repayment** on the next day. Students who misinterpreted and did repayment before interest would have been unsuccessful in both parts.<br><br>Next, students must multiply the correct number reflecting the interest rate of 0.4%, which is 100.4% or 1.004. Common errors involving the wrong number of zeroes in this common ratio were often seen.<br><br>Part **(c)** was poorly done because students did not "show" sufficiently all the information needed to lead to the given result. Most students realised the formula for the sum of a geometric progression (GP) was needed, but the application of this formula was slipshod. Students must write down the full series before the formula can be applied, and this full series must be sufficiently |
| **12(c)** | On 30th September 2025, apply interest of 0.4%.<br>On 1st October 2025, make 2nd repayment of \$$y$.<br>Outstanding loan after 2nd repayment<br>$=\left(23760(1.004)-y\right)1.004-y$<br>$=23760(1.004)^2-1.004y-y$<br><br>Outstanding loan after 3rd repayment (on 1st Nov 2025)<br>$=\left(23760(1.004)^2-1.004y-y\right)1.004-y$<br>$=23760(1.004)^3-1.004^2y-1.004y-y$<br><br>Outstanding loan after $n^{\text{th}}$ repayment<br>$=23760(1.004)^n-1.004^{n-1}y-...-1.004^2y-1.004y-y$<br>$=23760(1.004)^n-y\left(1+1.004+1.004^2+...+1.004^{n-1}\right)$<br>$=23760(1.004)^n-y\left(\frac{1.004^n-1}{1.004-1}\right)$<br>$=23760(1.004)^n-250y\left(1.004^n-1\right)$ (shown) | |

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| | | written to demonstrate the pattern – simply writing down "$1.004^{n-1}+...+1$" does not mean that this series is a GP with common ratio 1.004.<br><br>Another technique students must grasp is to identify and track the **correct amount** for the sequence. The required sequence was the outstanding loan after repayment, but there were students who tried to arrive at the pattern by tracking the amount after interest was applied. It would have been impossible to get the required result in this way.<br><br>Also, students must understand that patterns can only be established with at least 3 terms in the sequence. Since the amount after first repayment has already been done in part **(b)**, students must write down the outstanding loan after at least 2 more repayments in order to generate a pattern.<br><br>In short, when a question requires students to "show" a result, they must be careful to show all necessary steps that demonstrate application of concept and not simply summarise workings and take shortcuts that occur in their minds. |
| **12(d)** | To repay remaining loan by the end of 3 years (i.e. 36 monthly repayments of \$$y$), let $n=36$.<br><br>To fully repay the loan, we want outstanding loan amount after 36 repayments to be zero (or less).<br><br>$23760(1.004)^{36}-250y(1.004^{36}-1)\le 0$<br>$y\ge 709.9769$<br>$\therefore\ y=709.98$ (nearest cent) | This was generally well done by students who correctly substituted $n$ to be 36 in part **(c)**'s result. Some students seemed to overthink the problem and used 37 instead. Some forgot to convert years to months, while a small portion of students did not know how many months there were in a year.<br><br>Another common problem was equating the expression for the outstanding loan from part **(c)** to a number other than zero (e.g. 5940). This perhaps comes from not understanding the significance of the expression in part **(c)** – to fully repay the loan is to bring the outstanding loan amount down to zero. |

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| **12(e)** | After 24 monthly repayments of \$500 (i.e. $n=24$, $y=500$), outstanding loan amount<br>$=23760(1.004)^{24}-250(500)\left((1.004)^{24}-1\right)$<br>$=13580.49$<br><br>Outstanding loan amount as of 1st August 2027<br>$=13580.49$<br>In a similar method to **(c)**, the outstanding loan amount after $m$ monthly repayments of \$1000 (i.e. $y=1000$) is<br>$13580.49(1.004)^{m}-250(1000)\left((1.004)^{m}-1\right)$<br><br>To fully repay the loan, we need<br>$13580.49(1.004)^{m}-250(1000)\left((1.004)^{m}-1\right)\le 0$<br>$m\ge 13.991185$<br>Least $m=14$<br><br>Sarah would require another 14 monthly repayments to fully repay her loan, with her first repayment of \$1000 on 1st September 2027.<br><br>Hence, she would clear her loan on (1st) October 2028.<br><br>The outstanding loan amount after her 13th monthly repayment of \$1000 (on 1st Sep 2028) is<br>$13580.49(1.004)^{13}-250(1000)\left((1.004)^{13}-1\right)$<br>$=987.2529$<br>Interest is applied on 30th Sep 2028. Hence her final repayment amount is<br>$987.2529\times 1.004=\underline{\$991.20}$ (nearest cent)<br><br>(Alternatively, outstanding loan amount after 14th monthly repayment is<br>$13580.49(1.004)^{14}-250(1000)\left((1.004)^{14}-1\right)$<br>$=-8.7981$<br>Hence the final repayment is $1000-8.7981=\$991.20$ (nearest cent). | This question was poorly done. Many students disregarded the interest rate completely and simply considered the total outstanding loan amount after graduation minus 24 repayments of \$500. Some students repeated the working from part **(c)** all over again without realising that the required outstanding loan amount after 24 monthly repayments of \$500 can be calculated using the same result from part **(c)**.<br><br>For the repayments of \$100, students need to realise that the first term in the result in part **(c)** should change to reflect the new outstanding loan amount at the start of this new repayment phase.<br><br>For those who made it to this part, most of them obtained $n=14$, but then went on to incorrectly calculate the month and year as November 2028. Also, only a handful correctly spotted that to obtain the amount of the final repayment, if they had substituted $n=13$ into the expression for the outstanding loan amount, they need to multiply 1.004 to apply interest on the loan amount before the final repayment takes place. (On a related note, students also misread or misunderstood "final repayment" to mean "total repayment".) |

# 2025 ACJC H2 Math Preliminary Examination Paper 2 Markers' Report

| Qn | Solution | Remarks |
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| 1(a) | $l_{OB}: \mathbf{r} = \lambda\overrightarrow{OB} = \lambda\begin{pmatrix}5\\-4\\3\end{pmatrix}$<br><br>Since $R$ lies on the line $OB$, $\overrightarrow{OR} = \lambda\begin{pmatrix}5\\-4\\3\end{pmatrix}$.<br><br>$\overrightarrow{AR} = \overrightarrow{OR} - \overrightarrow{OA} = \begin{pmatrix}5\lambda-3\\-4\lambda-1\\3\lambda-3\end{pmatrix}$<br><br>$\overrightarrow{AR}\cdot\begin{pmatrix}5\\-4\\3\end{pmatrix} = 0 \Rightarrow \begin{pmatrix}5\lambda-3\\-4\lambda-1\\3\lambda-3\end{pmatrix}\cdot\begin{pmatrix}5\\-4\\3\end{pmatrix} = 0$<br><br>$25\lambda - 15 + 16\lambda + 4 + 9\lambda - 9 = 0$<br><br>$\therefore \lambda = \frac{2}{5}$<br><br>$\overrightarrow{OR} = \frac{2}{5}\begin{pmatrix}5\\-4\\3\end{pmatrix}$<br><br>$\overrightarrow{OR} = \frac{\overrightarrow{OA} + \overrightarrow{OA'}}{2} = \frac{2}{5}\begin{pmatrix}5\\-4\\3\end{pmatrix}$<br><br>$\overrightarrow{OA'} = 2\overrightarrow{OR} - \overrightarrow{OA} = \frac{4}{5}\begin{pmatrix}5\\-4\\3\end{pmatrix} - \begin{pmatrix}3\\1\\3\end{pmatrix} = \begin{pmatrix}1\\-\frac{21}{5}\\-\frac{3}{5}\end{pmatrix} = \frac{1}{5}\begin{pmatrix}5\\-21\\-3\end{pmatrix}$<br><br>**Alternatively (for finding foot of perpendicular from point to line),**<br>$\overrightarrow{OR} = (\mathbf{a}.\mathbf{b})\mathbf{b}$<br><br>$= \left(\begin{pmatrix}3\\1\\3\end{pmatrix}.\frac{1}{\sqrt{50}}\begin{pmatrix}5\\-4\\3\end{pmatrix}\right)\frac{1}{\sqrt{50}}\begin{pmatrix}5\\-4\\3\end{pmatrix}$<br><br>$= \frac{1}{50}\left(\begin{pmatrix}3\\1\\3\end{pmatrix}.\begin{pmatrix}5\\-4\\3\end{pmatrix}\right)\begin{pmatrix}5\\-4\\3\end{pmatrix} = \frac{15-4+9}{50}\begin{pmatrix}5\\-4\\3\end{pmatrix} = \frac{2}{5}\begin{pmatrix}5\\-4\\3\end{pmatrix}$ | Generally well done. Students understood that the procedure involves $\overrightarrow{AR}\cdot\overrightarrow{OB} = 0$ or applying the projection vector formula to find $\overrightarrow{OR}$ and using the midpoint theorem to find $\overrightarrow{OA'}$. However, some students failed to recognize that points $O$, $R$, and $B$ are collinear, hence, $\overrightarrow{OR} = \lambda\overrightarrow{OB}$. This oversight led to unnecessarily complicated expressions and careless mistakes in the calculations. eg $\overrightarrow{AR} = \begin{pmatrix}x-3\\y-1\\z-3\end{pmatrix}$ or $\overrightarrow{OR} = \begin{pmatrix}5\\-4\\3\end{pmatrix} + \lambda\begin{pmatrix}5\\-4\\3\end{pmatrix}$. |

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| **2(a)** | $f(x)=\frac{1}{\sqrt{1-4x^2}}=\left(1-4x^2\right)^{-\frac{1}{2}}$<br>$=1-\frac{1}{2}(-4x^2)+\frac{-\frac{1}{2}(-\frac{3}{2})}{2!}(-4x^2)^2+\frac{-\frac{1}{2}(-\frac{3}{2})(-\frac{5}{2})}{3!}(-4x^2)^3+\ldots$<br>$=1+2x^2+6x^4+20x^6+\ldots$ | Most students knew the procedure to be carried out, i.e. express f(x) in power form and use the maclaurin expansion formula $(1+x)^n$ to obtain the first 4 non-zero terms. A handful of students used the repeated differentiation method; however, no marks were awarded as this did not follow the instructions of the question.<br><br>Common mistakes:<br>1. Expressing f($x$) in an incorrect power form due to carelessness, eg. $\left(1-4x^2\right)^{-1}$ or $\left(1-4x^2\right)^{\frac{1}{2}}$.<br>2. Omitting the negative sign in the expansion, eg $1-\frac{1}{2}(4x^2)+\frac{-\frac{1}{2}(-\frac{3}{2})}{2!}(4x^2)^2+\ldots$<br>3. Taking out 4 and then applying the Maclaurin expansion formula directly, which is a misapplication of the formula. |
| **2(b)** | $\int\frac{1}{\sqrt{1-4x^2}}\,\mathrm{d}x=\int 1+2x^2+6x^4+20x^6\,\mathrm{d}x$<br>$\frac{1}{2}\sin^{-1}2x=x+\frac{2}{3}x^3+\frac{6}{5}x^5+\frac{20}{7}x^7+c$<br>$\sin^{-1}2x=2x+\frac{4}{3}x^3+\frac{12}{5}x^5+\frac{40}{7}x^7+d$<br>$\sin^{-1}0=0\quad\therefore d=0$<br>$\sin^{-1}2x=2x+\frac{4}{3}x^3+\frac{12}{5}x^5+\frac{40}{7}x^7$ | Most students recognized that $\frac{\mathrm{d}}{\mathrm{d}x}\sin^{-1}2x=\frac{2}{\sqrt{1-4x^2}}$; however, many either forgot to include the factor of 2 or were unsure how to use the results from 2(a) to proceed. Several students attempted repeated differentiation, which is a lengthy and tedious process. While some successfully obtained the result by integrating the previous answer, they forgot to include the constant of integration $C$, and consequently were unable to explain why $C$ disappears in the final solution. |

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| **3(a) (i)** | $\sum_{r=1}^{n} r(r+1)^2 = \sum_{r=1}^{n} r(r^2+2r+1) = \sum_{r=1}^{n}(r^3+2r^2+r)$<br>$= \frac{n^2(n+1)^2}{4} + 2\left[\frac{n(n+1)(2n+1)}{6}\right] + 1+2+...+n$<br>$= \frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{3} + \frac{n}{2}(n+1)$<br>$= \frac{3n^2(n+1)^2}{12} + \frac{4n(n+1)(2n+1)}{12} + \frac{6n}{12}(n+1)$<br>$= \frac{n(n+1)}{12}\left[3n(n+1)+4(2n+1)+6\right]$<br>$= \frac{n(n+1)}{12}(3n^2+3n+8n+4+6)$<br>$= \frac{n(n+1)}{12}(3n^2+11n+10)$<br>$= \frac{n(n+1)(n+2)(3n+5)}{12}$ | Mostly well done, except that some students were unfamiliar with the AP formula for $\sum_{r=1}^{n} r$. Students who combined the three fractions by expansion had difficulty arriving at the final result; a better approach is to factorize instead. |
| **3(a) (ii)** | $\sum_{r=5}^{n-1}(r+2)(r+3)^2$<br>Replace $r$ with $r-2$<br>$\sum_{r-2=5}^{r-2=n-1}(r-2+2)(r-1+3)^2$<br>$= \sum_{r=7}^{n+1}(r)(r+1)^2$<br>$= \sum_{r=1}^{n+1} r(r+1)^2 - \sum_{r=1}^{6} r(r+1)^2$<br>$= \frac{(n+1)(n+2)(n+3)(3(n+1)+5)}{12} - \frac{6(6+1)(6+2)(3(6)+5)}{12}$<br>$= \frac{(n+1)(n+2)(n+3)(3n+8)}{12} - 644$ | Many students tried replacing $r$ with $r+2$ in the expression from part (i) but failed to recognize the connection to part (ii). A more straightforward approach is to replace $r$ with $r-2$ in the expression given in part (ii). Additionally, some students made careless mistakes when applying the change of limits formula or substituting values into the result from part (i). |
| **3(b)** | $u_2 = \frac{1}{1-2} = -1$<br>$u_3 = \frac{1}{1-(-1)} = \frac{1}{2}$<br>$u_4 = \frac{1}{1-\frac{1}{2}} = 2$<br>$u_{2025} = u_3 = \frac{1}{2}$ | Very well done—almost all students were able to find the three terms. However, some students failed to recognize the pattern and were therefore unable to find $u_{2025}$. |

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| **4(a)** | Let $y=\dfrac{1}{x^2+6x+5}$.<br>$x^2+6x+5=\dfrac{1}{y}$<br>$(x+3)^2-4=\dfrac{1}{y}$<br>$(x+3)^2=\dfrac{1}{y}+4$<br>$x=-3+\sqrt{\dfrac{1}{y}+4}$ since $x\ge -3$<br>$f^{-1}(x)=-3+\sqrt{\dfrac{1}{x}+4}$ | Most students were able to recognise the need to make *x* the subject in order to find the rule of the inverse function.<br><br>Students need to be reminded that "±" must be considered on taking the square root, together with the reason for rejecting the negative root. The expression inside the square root must be simplified.<br><br>The following mistakes should **NOT** be made:<br>$(x+1)(x+5)=\dfrac{1}{y}\Rightarrow x+1=\dfrac{1}{y}$ or $x+5=\dfrac{1}{y}$<br>$(x+3)^2=\dfrac{1}{y}+4\Rightarrow x+3=\sqrt{\dfrac{1}{y}}+2$ |
| **4(b)** | Let $y=\dfrac{1}{x^2+6x+5}$.<br>$yx^2+6yx+5y=1$<br>$yx^2+6yx+5y-1=0$<br>This quadratic equation must have real roots for $y\in R_f$.<br><br>Discriminant $\ge 0$<br><br>$36y^2-4y(5y-1)\ge 0$<br>$16y^2+4y\ge 0$<br>$y(16y+4)\ge 0$<br>$y\le -\dfrac{1}{4}$ or $y>0$ $(\text{as } y\ne 0)$<br>$R_f=\left(-\infty,-\dfrac{1}{4}\right]\cup(0,\infty)$ | Many students had no idea how to approach this question **algebraically**.<br><br>Some students did partial fractions just to be able to do differentiation to obtain the stationary point. There were no tests to determine that this is a maximum point. There were also missing working for the interval $(0,\infty)$, though some students tried to explain graphically.<br><br>Some students were not familiar with the set notation, especially the "$\cup$" sign and the interval notation. It is **WRONG** to write $(\infty,0)$.<br><br>Some students did not handle inequalities with care, as "$4y$" should be included as a factor in $16y^2+4y\ge 0$. Thus, the result from $4y+1\ge 0$ was wrong and incomplete. |
| **4(c)** | $R_f=\left(-\infty,-\dfrac{1}{4}\right]\cup(0,\infty)\xrightarrow{g}\left(0,e^{-0.25}\right]\cup(1,\infty)$<br>$R_{gf}=\left(0,e^{-0.25}\right]\cup(1,\infty)$ | Some students gave the wrong rule for the composite function gf, which should be $gf(x)=e^{\frac{1}{x^2+6x+5}}$.<br><br>As exact range of gf is required, $gf(-3)=e^{\frac{1}{-4}}$ or $g\left(-\dfrac{1}{4}\right)=e^{-\frac{1}{4}}$ or $g(0)=1$ in **exact** form is expected. |

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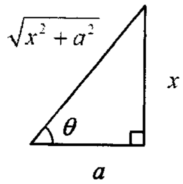
| **5(a)** | $\displaystyle\int \frac{1}{x\sqrt{x^2+a^2}}\,\mathrm{d}x$ $\qquad x = a\tan\theta$<br>$\displaystyle= \int \frac{1}{a\tan\theta\sqrt{a^2\tan^2\theta+a^2}}\left(a\sec^2\theta\right)\mathrm{d}\theta$ $\qquad \displaystyle\frac{\mathrm{d}x}{\mathrm{d}\theta} = a\sec^2\theta$<br>$\displaystyle= \int \frac{1}{a\tan\theta\left(a\sec\theta\right)}\left(a\sec^2\theta\right)\mathrm{d}\theta$ $\qquad \displaystyle\tan\theta = \frac{x}{a}$<br>$\displaystyle= \int \frac{\sec\theta}{a\tan\theta}\,\mathrm{d}\theta$<br><br>$\displaystyle= \frac{1}{a}\int \operatorname{cosec}\theta\,\mathrm{d}\theta$<br>$\displaystyle= -\frac{1}{a}\ln\left\|\operatorname{cosec}\theta+\cot\theta\right\|+c$ $\qquad \displaystyle\sin\theta = \frac{x}{\sqrt{x^2+a^2}}$<br>$\displaystyle= -\frac{1}{a}\ln\left\|\frac{\sqrt{x^2+a^2}+a}{x}\right\|+c$<br>$\displaystyle= \frac{1}{a}\ln\left(\frac{x}{\sqrt{x^2+a^2}+a}\right)+c\ (\because x, a > 0)$ | It's very strange to see solutions expanding $\sec^2\theta$ instead of simplifying $a^2\tan^2\theta + a^2$ to $a^2\sec^2\theta$.<br><br>Some students were not able to simplify $\frac{\sec\theta}{\tan\theta}$ to $\operatorname{cosec}\theta$, while some others were not able to use MF27 for $\int \operatorname{cosec}\theta\,\mathrm{d}\theta = -\ln\left\|\operatorname{cosec}\theta+\cot\theta\right\|+c$ and the modulus sign was often missing.<br><br>Many steps were wasted to convert $\operatorname{cosec}\theta+\cot\theta$ in terms of $\tan\theta$ for $\frac{x}{a}$ instead of using the right-angled triangle to do so.<br><br>As this is a "show" question, it would be good to include the explanation that $x, a > 0$ for converting the modulus sign to round brackets at the end. |
| **5(b) (i)** | $\displaystyle\frac{\mathrm{d}x}{\mathrm{d}t} = xy$ and $\displaystyle\frac{\mathrm{d}y}{\mathrm{d}t} = x^2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^2}{xy} = \frac{x}{y}$<br>$\displaystyle\Rightarrow y\frac{\mathrm{d}y}{\mathrm{d}x} = x \Rightarrow \int y\,\mathrm{d}y = \int x\,\mathrm{d}x$<br>$\displaystyle\Rightarrow \frac{y^2}{2} = \frac{x^2}{2}+c \Rightarrow y^2 = x^2+d$<br>When $t=0, x=4, y=5,\ d = 5^2-4^2 = 9$<br>$\therefore y^2 = x^2+9$ or $y = \sqrt{x^2+9}\ (\because y > 0)$ | Integral signs were missing when separating the variables.<br><br>After multiplying throughout by 2 for $\frac{y^2}{2} = \frac{x^2}{2}+c$, the arbitrary constant should be changed to $2c$ or $d$ where $d = 2c$.<br><br>As this is a "show" question again, it would be required to include the reason that $y > 0$ as "±" must be considered on taking the square root at the end. |
| **5(b) (ii)** | $\displaystyle\frac{\mathrm{d}x}{\mathrm{d}t} = xy = x\sqrt{x^2+9}$<br>$\displaystyle\Rightarrow \int \frac{1}{x\sqrt{x^2+9}}\,\mathrm{d}x = \int 1\,\mathrm{d}t$<br>Using **(a)** and $a = 3$,<br>$\displaystyle\Rightarrow t = \frac{1}{3}\ln\left(\frac{x}{\sqrt{x^2+9}+3}\right)+c$<br>When $\displaystyle t=0, x=4,\ c = 0-\frac{1}{3}\ln\left(\frac{4}{\sqrt{4^2+9}+3}\right) = \frac{1}{3}\ln 2$<br>$\displaystyle\Rightarrow t = \frac{1}{3}\ln\frac{x}{\sqrt{x^2+9}+3}+\frac{1}{3}\ln 2$ | Some students could not understand the instructions to obtain a differential equation in terms of $x$ and $t$ only, with a few of them simply substitute the initial value of $y$ to carry on wrongly.<br><br>Some students could not see the need to apply part (a) after separating the variables, while some others took the value of $a$ to be 9 instead of 3.<br><br>The question asked for an expression of $t$ in terms of $x$, so there was no need to waste extra steps to do the usual expressing $x$ in terms of $t$, to find the arbitrary constant. |

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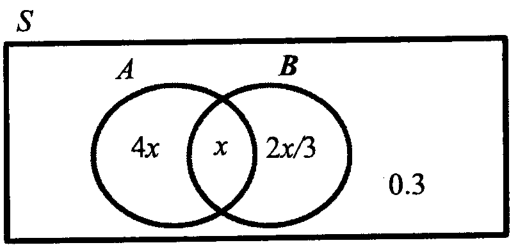
| 6(a) | Given $\text{P}(B\mid A)=0.2$, $\text{P}(B\mid A)=0.2$ and $\text{P}(A'\cap B')=0.3$.<br><br>$\text{P}(B\mid A)=0.2 \Leftrightarrow \dfrac{\text{P}(A\cap B)}{\text{P}(A)}=0.2 \Leftrightarrow \text{P}(A)=5\,\text{P}(A\cap B)$ (1)<br>$\text{P}(A\mid B)=0.6 \Leftrightarrow \dfrac{\text{P}(A\cap B)}{\text{P}(B)}=0.6 \Leftrightarrow \text{P}(B)=\dfrac{5}{3}\text{P}(A\cap B)$ (2)<br><br>Given $\text{P}(A'\cap B')=0.3$,<br>$\therefore \text{P}(A\cup B)=1-\text{P}(A'\cap B')=0.7$<br><br>Now $\text{P}(A\cup B)=\text{P}(A)+\text{P}(B)-\text{P}(A\cap B)$ ...(3)<br>Sub. (1) and (2) into (3)<br>$0.7=5\,\text{P}(A\cap B)+\dfrac{5}{3}\text{P}(A\cap B)-\text{P}(A\cap B)$<br>$\text{P}(A\cap B)=\dfrac{21}{170}$ or 0.124 (3 sig fig)<br><br>**Alternatively,**<br>$\text{P}(B\mid A)=0.2 \Leftrightarrow \dfrac{\text{P}(A\cap B)}{\text{P}(A)}=0.2 \Leftrightarrow \text{P}(A)=\dfrac{\text{P}(A\cap B)}{0.2}$<br>$\text{P}(A\mid B)=0.6 \Leftrightarrow \dfrac{\text{P}(A\cap B)}{\text{P}(B)}=0.6 \Leftrightarrow \text{P}(B)=\dfrac{\text{P}(A\cap B)}{0.6}$<br>Let $x=\text{P}(A\cap B)$.<br><br>$\text{P}(A)=\dfrac{x}{0.2}=\dfrac{10x}{2}=5x$ ...(1)<br>$\text{P}(B)=\dfrac{x}{0.6}=\dfrac{10x}{6}=\dfrac{5x}{3}$ ...(2)<br><br>Given $\text{P}(A'\cap B')=0.3$<br>From the Venn Diagram,<br>$4x+x+\dfrac{2x}{3}+0.3=1$<br>$\dfrac{17x}{3}=\dfrac{7}{10}$<br>$x=\dfrac{21}{170}$ or 0.124 (3 s.f.) | Generally, well-done.<br>The definition for conditional probability was well-known, and the usage of appropriate probability results led to many correct solutions.<br><br>The only issue that recurred a few times were students using $\text{P}(A\cap B)=\text{P}(A)\text{P}(B)$. This is <u>not true</u> unless the events $A$ and $B$ are independent. This was also an issue with (b). |

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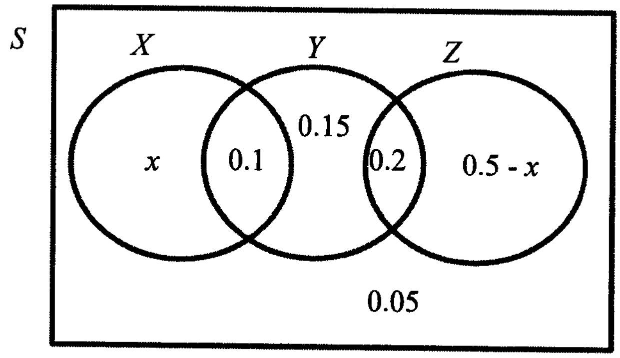
| 6(b) | $P[(X\cup Y\cup Z)']=1-P(X\cup Y\cup Z)$<br>$=1-0.95=0.05$<br>Since $X$ and $Z$ are mutually exclusive,<br>$P(Y\cap X'\cap Z')=P(Y\cap X')-P(Y\cap Z)$<br>$=0.35-0.2=0.15$<br>$y=P(Z\cap Y')$<br>Let $x=P(X\cap Y')$.<br>$P(Z\cap Y')=1-0.5-x=0.5-x$<br>We have the following Venn diagram:<br><br><br>$\therefore 0\le x\le 0.5$<br>If $P(X\cap Y')=0$ then $P(X)=0.1$(minimum).<br>If $P(X\cap Y')=0.5$ then $P(X)=0.6$(maximum). | Most students utilised a Venn diagram to visualise the situation, and the idea of mutual exclusivity was well-understood.<br><br>Those who got this correct usually have one of the following 2 approaches:<br>(1) assigning an unknown (method shown), or<br>(2) considering visually how to maximise $X$'s area (probability) by "moving $Z$ into or out of $Y$".<br><br>A more common careless mistake was the mistreatment of the 0.1 probability (forgetting that it contributes to the probability of $X$). |
| 7(a) | Step 1: Select 4 people out of 12 people to play bridge: ${}^{12}C_4$<br>Step 2: Select 4 people out of the remaining 8 people to play carrom: ${}^{8}C_4$<br>Step 3: Select the remaining 4 people to play mahjong: ${}^{4}C_4$<br>By Principle of Multiplication, we have<br>${}^{12}C_4\times{}^{8}C_4\times{}^{4}C_4=34650$<br>Number of ways to break 12 people into 3 groups of 4 to play 3 different games = 34650 | Students kept trying to permutate the people within each group (leading to 4!), or permutate the groups (leading to 3!). |
| 7(b) | **Method 1: By complementation**<br><br>Step 1: Put Aaron and Sandy into one of the 3 groups (Bridge, Carrom or UNO): 3<br>Step 2: Select 2 people from the other 10 people to complete the group Aaron and Sandy are in: ${}^{10}C_2$<br>Step 3: Select 4 out of the remaining 8 people to join one of the group that Aaron and Sandy are not in: ${}^{8}C_4$<br>Step 4: Select the remaining 4 people to form the last group: ${}^{4}C_4$<br><br>By complementation, we have<br>$34650-3\times{}^{10}C_2\times{}^{8}C_4\times{}^{4}C_4=25200$<br>Number of ways where Aaron and Sandy are not in the same group<br>= 25200 | Same issue as above occurred, but students generally could try to apply complementation, or by considering cases. |

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| | **Method 2: Direct Method**<br><br>Step 1: Assign Aaron to one of the 3 groups: ${}^{3}C_{1}$<br>Step 2: Assign Sandy to one of the 2 groups where Aaron is not in: ${}^{2}C_{1}$<br>Step 3: Select 3 people to complete the group Aaron is in: ${}^{10}C_{3}$<br>Step 4: Select 3 people to complete the group Sandy is in: ${}^{7}C_{3}$<br>Step 5: Select 4 people to form the last group: ${}^{4}C_{4}$<br><br>By Principle of Multiplication, we have<br>${}^{3}C_{1}\times{}^{2}C_{1}\times{}^{10}C_{3}\times{}^{7}C_{3}\times{}^{4}C_{4}=3\times2\times120\times35\times1=25200$<br><br>Number of ways where Aaron and Sandy are not in the same group<br>= 25200 | |
| **7(c)** | Let $A$ = {Aaron and Billy are together in a group}<br>Let $B$ = {Aaron and Sandy are in different group}<br>Step 1: Put Aaron and Billy in the same group: ${}^{3}C_{1}$<br>Step 2: Put Sandy in one of the other 2 groups: ${}^{2}C_{1}$<br>Step 3: Select 2 people from the remaining 9 people to complete the group Aaron and Billy are in: ${}^{9}C_{2}$<br>Step 4: Select 3 people from the remaining 7 people to complete the group Sand is in: ${}^{7}C_{3}$<br>Step 5: Select the remaining 4 people to form the 3rd group: ${}^{4}C_{4}$<br><br>By Principle of Multiplication,<br><br>$n(A\cap B)={}^{3}C_{1}\times{}^{2}C_{1}\times{}^{9}C_{2}\times{}^{7}C_{3}\times{}^{4}C_{4}$<br>$=3\times2\times36\times35\times1=7560$<br><br>From **(b)**, $n(B)=25,200$<br><br>**Method 1:**<br>$\mathrm{P}(A\mid B)=\dfrac{n(A\cap B)}{n(B)}=\dfrac{7560}{25200}=\dfrac{3}{10}$ or 0.3<br>**Method 2:**<br>From **(a)**, $n(S)=34650$<br>$\mathrm{P}(A\cap B)=\dfrac{7560}{34650}=\dfrac{12}{55}$<br>$\mathrm{P}(B)=\dfrac{25,200}{34650}=\dfrac{8}{11}$<br>$\mathrm{P}(A\mid B)=\dfrac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}=\dfrac{12}{55}\div\dfrac{8}{11}=\dfrac{3}{10}$ or 0.3 | As this result is a probability, students who managed to count wrongly (in the same way) for both the numerator and denominator could still have arrived at the correct answer.<br><br>Generally, the conditional probability was well-considered, and students could process the event in the numerator well.<br><br>There were a number of shorter solutions that were given full credit as well. |

**8**

Given the probability distribution of X:

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $3$ |
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| $P(X=x)$ | 0.1 | $a$ | 0.3 | $b$ | $c$ |

$$E(X)=(-2)(\frac{1}{10})+(-1)(a)+(0)(\frac{3}{10})+(1)(b)+(3)(c)$$

$$\frac{9}{10}=-\frac{2}{10}-a+b+3c$$

$$-a+b+3c=\frac{11}{10} \text{ ..................................... (1)}$$

$$E(X^2)=(-2)^2(\frac{1}{10})+(-1)^2(a)+(0)^2(\frac{3}{10})+(1)^2(b)+(3)^2(c)$$

$$=\frac{4}{10}+a+b+9c$$

$$\text{Var}(X)=E(X^2)-[E(X)]^2$$

$$\frac{299}{100}=\left(\frac{4}{10}+a+b+9c\right)-\left(\frac{9}{10}\right)^2$$

$$a+b+9c=\frac{34}{10} \text{ ............................... (2)}$$

$$\sum P(X=x)=1$$

$$\frac{1}{10}+a+\frac{3}{10}+b+c=1$$

$$a+b+c=\frac{6}{10} \text{ ..... (3)}$$

Solving (1), (2) and (3), we have

$a=\frac{1}{10}$ or 0.1, $b=\frac{3}{20}$ or 0.15, $c=\frac{7}{20}$ or 0.35

A small number of students squared the probability instead of $x$ in computing $E(X^2)$.

Most did not apply the fact that $\sum P(X=x)=1$. As a result, the 3 unknowns cannot be solved.

Many input E(X) = 1.1 instead of E(X) = 0.9 into the statement $\text{Var}(X)=E(X^2)-[E(X)]^2$.

**9(a)**

The probability that an orange is rotten is a constant of 0.01.

Whether an orange is rotten is independent of whether any other oranges are rotten.

The most common mistake is using the term "probability" instead of "event" when stating the assumption for independence, an error that has been emphasized repeatedly.

Instead of stating the "probability that an orange is rotten", quite a number stated "probability that the number of oranges …", whereas some stated "probability that all oranges …"

Some stated the assumptions needed for sampling to be random.

Some stated that there is only 2 outcomes "success" and "failure" for each trial which is a fact and not an assumption.

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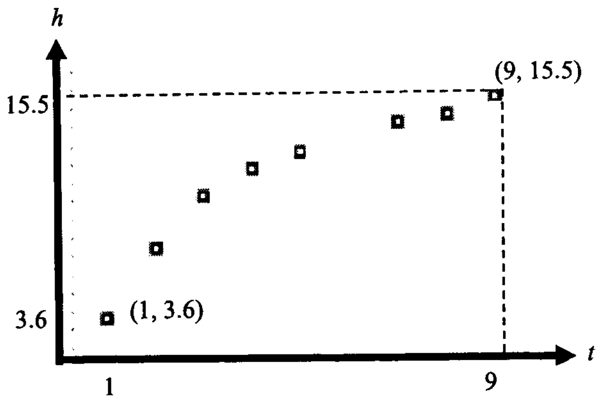
| **9(b)** | Let the random variable $X$ denote the number of rotten oranges in a box of 20 oranges.<br>$X \sim \mathrm{B}(20, 0.01)$<br>$\mathrm{P}(2 \le X \le 5) = \mathrm{P}(X \le 5) - \mathrm{P}(X \le 1) = 0.0169$ | Wrote $X \sim \mathrm{B}(0.01, 20)$ instead of $X \sim \mathrm{B}(20, 0.01)$.<br><br>Computed $\mathrm{P}(2 < X < 5)$ instead.<br><br>Many did not know that $\mathrm{P}(2 \le X \le 5) = \mathrm{P}(X \le 5) - \mathrm{P}(X \le 1)$ |
| **9(c)** | $\mathrm{E}(X) = (20)(0.01) = 0.2$ | Used $p = 0.1$ or $p = 0.0169$.<br><br>Did not know that $\mathrm{E}(X) = np$.<br><br>For some reason, some felt compelled to round off the answer for E($X$) into an integer.<br><br>Despite having obtained the answer E($X$) = 0.2, some still did not realise that this was the answer and went further to multiply 0.2 with some other numbers such as 20. |
| **9(d)** | Let the random variable $Y$ denote the number of substandard boxes out of the entire truckload of 10 boxes.<br>$Y \sim \mathrm{B}(10, p)$ where $p = 0.0169$<br>$\mathrm{P}(Y \le 2) = 0.999$ (3 s.f.) | Many did not get the right answer here because $p = 0.0169$ was wrongly computed in 9(b). |
| **9(e)** | Let $A$ = {he rejects the entire truckload}<br><br>$\mathrm{P}(A) = \mathrm{P}(X \ge 2) + \mathrm{P}(X = 1)\mathrm{P}(X \ge 2)$<br>$= [1 - \mathrm{P}(X \le 1)] + \mathrm{P}(X = 1)[1 - \mathrm{P}(X \le 1)]$<br>$= 0.0196$ | Probabilities were computed using random variable $Y$ in 9(d) instead of $X$.<br><br>In the case of opening 2 boxes, this case was computed using conditional probability.<br><br>Some simply could not figure out the 2 cases. Some broke down into 3 cases. |
| **10(a)** |  | Treated last 3 points as a point of inflexion when the last point is merely a point that is slightly higher (an overestimate).<br><br>Many still did not indicate the scales on both axes. Some wrote the scale on one axis but not the other. Some just labelled the first and the last points but it is still important to show the scales on the axes. |

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| | From the scatter diagram, there appears to be a non-linear relationship between $h$ and $t$. Thus linear model $h=a+bt$ is not suitable.<br><br>As $t$ increases, $h$ increases at a decreasing rate ($h$ increases by decreasing amounts), whereas the model $h=a+bt^2$, $(b>0)$ $h$ increases at an increasing rate as $t$ increases. Thus, relationship between $h$ and $t$ should not be modelled by $h=a+bt^2$ (where $b>0$). | Explained using the values of $r$ for each model. Some computed the values of $r$ whereas some did not even show the values of $r$. However, the question asked candidates to explain by referring to the scatter diagram that was sketched.<br><br>The condition $b>0$ is essential and important but has been left out when quoting the model $h=a+bt^2$. Without computing the values of $r$, the scatter diagram could well be perceived as $h=a+bt^2$, $b<0$.<br><br>Some claim that the model $h=a+bt^2$, $b>0$ must first decrease and then increase, which is not necessarily so for a restricted range of values for $t$.<br><br>Many described a concave upward curve as exponential or exponentially increasing. |
| **10(b)** | For $h=c+d\sqrt{t}$ : $r=0.97364=0.974$ (3 s.f.)<br>For $h=c+d\ln t$: $r=0.99658=0.997$ (3 s.f.)<br><br>The product moment correlation coefficient of the model $h=c+d\ln t$ <u>is closer to one</u>, hence, $h=c+d\ln t$ is a better model.<br><br>$h=3.890166185+5.311381407\ln t$<br>$h=3.890+5.311\ln t$ (3 d.p) | Many chose a model without computing and comparing the values of $r$ for both models, but by some alternative argument. As a result, many chose model $A$ for a variety of reasons including carelessness, despite having computed the values of $r$.<br><br>For those who have chosen model B, the equation of the regression line was wrong possibly due to entering wrong values for the data into the GC. Some mixed up the values of c and d, giving the answer $h=3.890\ln t+5.311$. |
| **10(c)** | When $t=6$,<br>$h=3.890+5.311\ln 6=$ 13.4<br><br>This estimate is likely to be reliable as $t=6$ lies within the $t$ data range $(1\le t\le 9)$ and $r=0.997$ is close to 1 indicating a strong linear relationship between $h$ and ln $t$. | Wrong values of $h$ was computed mainly because of wrong model chosen in 10(b) or wrong equation.<br><br>A common mistake was computing $h=3.890+5.311(6)$ instead of $h=3.890+5.311\ln 6$.<br><br>Almost every candidate left out the condition that $r$ is very close to 1. |

<table>
<tr>
<td>10(d)</td>
<td>As $t \to \infty$, $\ln t \to \infty$, $h \to \infty$.<br>The model is not suitable in the long run as the height of bamboo stalk will not increase indefinitely.<br><br>Alternative answer:<br><br>The model is only appropriate for the range $1 \le t \le 9$. In the long run, the model is unsuitable as $t > 9$.</td>
<td>Reasons given for the model being suitable or in some cases unsuitable, are contradictory. For example, “the model is suitable because the bamboo will not grow infinitely tall”.<br><br>Many thought that the logarithmic function has a horizontal asymptote and thus, the model is suitable.</td>
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<tr>
<td>10(e)</td>
<td>$H = 3.890 + 5.311\ln t$ where $H$ is the predicted value of height of bamboo stalk using the least squares regression line:</td>
<td>Many did not know how to compute $S$. For some who did know how to, did so inaccurately.</td>
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<td></td>
<td colspan="2">
<table>
<tr><td>$t$</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>$h$</td><td>3.6</td><td>7.4</td><td>10.2</td><td>11.7</td><td>12.6</td><td>14.1</td><td>14.5</td><td>15.5</td></tr>
<tr><td>$H$</td><td>3.89</td><td>7.5713</td><td>9.7247</td><td>11.253</td><td>12.438</td><td>14.225</td><td>14.934</td><td>15.559</td></tr>
</table>
<br>$S = \sum (h - H)^2 = 0.77$ (2 dp)
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<td>10(f)</td>
<td>For $H' = p + q\ln t$, $\sum (h - H')^2 \ge \sum (h - H)^2 = S$ since $S$ is the <u>minimum sum of squared residuals</u> as $H$ is calculated from the least squares regression line.<br>$\therefore \sum (h - H')^2$ will be greater than (or equal to) $S$.</td>
<td>Many answered, “less” and a handful stated “same”.<br><br>Many used the term “lesser”. “Less” is used for comparing quantity whereas “lesser” is used for comparing quality.<br><br>Explanation was cumbersome instead of stating the key term, “least squares regression line”.</td>
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<td>11(a)</td>
<td>$\mu$ represent the <u>population mean</u> time taken to test an electric circuit board.<br><br>To test $\quad H_0 : \mu = 5.82$<br>Against $\quad H_1 : \mu < 5.82$ at 5% significance level</td>
<td>Some students did not define $\mu$, as required by the question. The word ‘population’ must be seen in defining $\mu$.<br><br>Incorrect hypothesis:<br>(a) $H_0 : \mu_0 = 5.82$<br>$H_1 : \mu_1 < 5.82$<br>(testing $\mu_0$ and $\mu_1$ instead of $\mu$)<br><br>(b) $H_0 : \mu = 5.74$<br>$H_1 : \mu < 5.74$<br>(confused between population mean 5.82 and sample mean 5.74)<br>It is important to read carefully to distinguish between population mean & sample mean for hypothesis question.</td>
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| **11(b)** | Under $H_0$, $\bar{X} \sim N\left(5.82, \frac{0.63^2}{150}\right)$ approx. by Central Limit Theorem since $n = 150$ is large.<br><br>Value of test statistic, $z = \frac{5.74-5.82}{\sqrt{\frac{0.63^2}{150}}} = -1.56$ (3 sf)<br>$p-\text{value} = P(\bar{X} > 5.74) = 0.0599 > 0.05$<br>$\therefore$ Do not reject $H_0$.<br>There is insufficient evidence at 5% level of significance to conclude that the inspector carries out the test more quickly. | <u>Common mistakes</u><br>• Putting the sample mean value 5.74 in the distribution. Should be $\bar{X} \sim N\left(5.82, \frac{0.63^2}{150}\right)$.<br>• Writing $X$ instead of $\bar{X}$<br>• Writing 0.63 instead of $0.63^2$<br>• Finding $s^2 = \frac{150}{149}\left(0.63^2\right)$ (This is not needed since the population variance $0.63^2$ is known)<br>• Failing to state CLT<br>• Some students left out the 5% sig. level at the conclusion part.<br>• Some conclusion were over-assertive, stating that 'there is sufficient evidence that mean time taken is 5.82', instead of 'insufficient evidence that mean time is less than 5.82.' |
| **11(c)** | There is no need for the laboratory supervisor to know anything about the population distribution of $X$. Since $n$ is large, by Central Limit Theorem, $\bar{X}$ will be approximately normally distributed. | Many wrote that 'by CLT, the population distribution will be normally distributed'.<br>It must be stated clearly that '$\bar{X}$ will be normally distributed.' |
| **11(d)** | (1) The time taken to test an electric circuit board follows a normal distribution (since sample size of 20 is small).<br><br>(2) The population standard deviation of the time taken to test an electric circuit board is still 0.63. | As the question already states that random sample is taken, there is no need to assume that the time taken is independent of each other.<br><br>It must be stated clearly that 'the population distribution **of *X*** follows a normal distribution'. There were many incorrect responses that wrote 'distribution of mean time taken follows a normal distribution'. |

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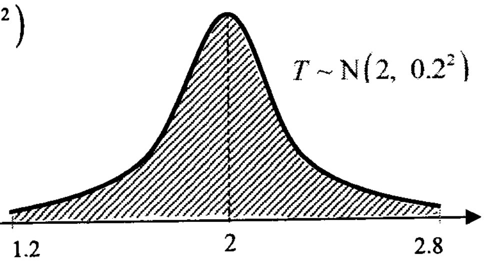
| **11(e)** | To test $H_0 : \mu = \mu_0$<br>Against $H_1 : \mu \neq \mu_0$ at 1% level of significance<br><br>Under $H_0$, $\overline{X} \sim N\left(\mu_0, \frac{0.63^2}{20}\right)$<br>Value of test statistic, $z = \frac{5.88-\mu_0}{\sqrt{\frac{0.63^2}{20}}}$<br>In order not to reject $H_0$,<br>$-2.57583 < \frac{5.88-\mu_0}{\sqrt{\frac{0.63^2}{20}}} < 2.57583$<br>$-2.57583\left(\frac{0.63}{\sqrt{20}}\right) < 5.88-\mu_0$ and<br>$5.88-\mu_0 < 2.57583\left(\frac{0.63}{\sqrt{20}}\right)$<br>$\Rightarrow \mu_0 < 6.24$ and $5.52 < \mu_0$<br>$\Rightarrow 5.52 < \mu_0 < 6.24$ | Many were confused with what to test in the hypotheses.<br><br>Common mistakes<br>• $H_0 : \mu_0 = 5.82$<br>$H_1 : \mu_0 \neq 5.82$<br><br>• Writing the range of values as $-0.12533 < z < 0.12533$. Incorrect critical values due to incorrect interpretation $P(-0.12533 < z < 0.12533) = 0.01$<br><br>• Writing the range of values as $z < -2.57583$ or $z > 2.57583$.<br>Note that '$H_0$ is **not** rejected' does not correspond to tail-ends |
| **12(a)** | $T \sim N(2,\ 0.2^2)$<br><br><br>Note: $P(1.2 < T < 2.8) = 0.99994$ | Since the interval (1.2, 2.8) is within $4\sigma$ from the mean 2 (note that $P(1.2 < T < 2.8) = 0.99994$), the markings for 1.2 and 2.8 should be drawn such that the curve is almost parallel and very close to the horizonal axis. Only very negligible unshaded region should be seen under the graph. |
| **12(b)** | Let the random variable $T$ denote the time taken in minutes to categorise an incoming call.<br>Let the random variable $X$ denote the time taken in minutes to resolve a routine call.<br>Let the random variable $Y$ denote the time taken in minutes to resolve a complex call.<br><br>$T \sim N(2, 0.2^2)$, $X \sim N(5, k)$, $Y \sim N(20, 6^2)$<br>$\therefore T + X \sim N(7,\ k + 0.2^2)$<br><br>Given: $P(T + X > 8) = 0.254$<br>$\Rightarrow P\left(Z > \frac{8-7}{\sqrt{k+0.2^2}}\right) = 0.254$<br>$\Rightarrow \frac{1}{\sqrt{k+0.2^2}} = 0.66196$<br>$\therefore k = 2.24$ (to 3 s.f.) | Mostly done well, except for minority who failed to standardize to show the result. |

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| **12(c)** | $\bar{Y} = \frac{Y_1 + Y_2 + ... + Y_n}{n} \sim N\left(20, \frac{6^2}{n}\right)$<br>Given: $P(\bar{Y} > 24) \le 0.01$<br>$\Rightarrow P\left(Z > \frac{24-20}{\sqrt{\frac{6^2}{n}}}\right) \le 0.01$<br>$\Rightarrow \frac{24-20}{\sqrt{\frac{6^2}{n}}} \ge 2.32635$<br>$\Rightarrow \frac{4\sqrt{n}}{6} \ge 2.32635$<br>$\Rightarrow n \ge 12.18$<br>Since $n \le 20$, Set of values of $n = \{13 \le n \le 20, n \in Z^+\}$<br><br>**Alternatively (by GC – table),**<br><table><tr><td>$n$</td><td>$P(\bar{Y} > 24)$</td></tr><tr><td>12</td><td>$0.0105 > 0.01$</td></tr><tr><td>13</td><td>$0.0081 < 0.01$</td></tr><tr><td>14</td><td>$0.0063 < 0.01$</td></tr></table>Since $n \le 20$, Set of values of $n = \{13 \le n \le 20, n \in Z^+\}$ | Many mis-read the question and consider the distribution of $T+Y$, instead of $\bar{Y}$.<br><br>For those who considered the distribution of $Y_1 + Y_2 + ... + Y_n$, $P(\bar{Y} > 24) \le 0.01$ is to be interpreted as $P(Y_1 + Y_2 + ... + Y_n > 24n) \le 0.01$ |
| **12(d)** | Let $W = 0.8(Y_1 + Y_2) - 2X$<br>$E(W) = 0.8(2)(20) - 2(5) = 22$<br>$Var(W) = 0.8^2(2)(6^2) + 2^2(2.24) = 55.04$<br>$W \sim N(22, 55.04)$<br>$P(W > 0) = 0.998$ (3 s.f.) | Some students factored in the reduced time for the Mean and forgot to do that for the Variance.<br><br><u>Common mistakes</u><br>• Writing $W = 0.8(Y+Y) - 2X$<br>• Writing $Var(0.8Y) = 0.8Var(Y)$ |