ANGLO-CHINESE JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

Higher 2

MATHEMATICS

9758/01

Paper 1
QUESTION PAPER

26 August 2025
3 hr

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

1 **(a)** Without using a calculator, solve the inequality $\frac{x}{x+2} \geq \frac{2}{2-x}$. [4]

(b) Hence, solve the inequality $\frac{|x|}{|x|+2} \geq \frac{2}{2-|x|}$. [2]

2 The graph of $y = \frac{ax^2+bx+c}{x+d}$ has a vertical asymptote with equation $x=-1$ and an oblique asymptote with equation $y = 2x+2$. It is given that $c \neq 2$.

(a) Write down the value of d and show that $a=2$ and $b=4$. [3]

(b) Hence find the range of values of c if the graph has no stationary points. [2]

3 It is given that $y = x^{xy}$, where $x>0$, $y>0$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2(1+\ln x)}{1-\ln y}$. [3]

(b) Hence find the coordinates of the point on the curve $y = x^{xy}$ whose tangent is parallel to the y-axis. [2]

4 The region R is bounded by the curve with equation $y = x\ln x$, the lines $x = \mathrm{e}$, $x = \mathrm{e}^2$ and the x-axis. Find the exact volume of the solid formed when R is rotated about the x-axis by 2π radians. Give your answer in the form $\frac{\pi \mathrm{e}^3}{27}\left(a\mathrm{e}^3+b\right)$, where a and b are integers to be found. [6]

5 Relative to the origin O, the points A and B have non-zero and non-parallel position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. The plane p has equation $\mathbf{r}\cdot\mathbf{n}=0$.

(a) Given that $\mathbf{a}\cdot\mathbf{n} = \mathbf{b}\cdot\mathbf{n} \neq 0$, show that $\overrightarrow{AB}$ is perpendicular to $\mathbf{n}$. Hence, describe the geometrical relationship between $\overrightarrow{AB}$ and the plane p. [2]

(b) Write down the equation of a line parallel to $\overrightarrow{AB}$ that is contained in the plane p. [1]

(c) The point F is the foot of perpendicular from a point C with position vector $\mathbf{c}$ to the plane p. Find the position vector of point F, giving your answer in terms of $\mathbf{c}$ and $\mathbf{n}$. [3]

6

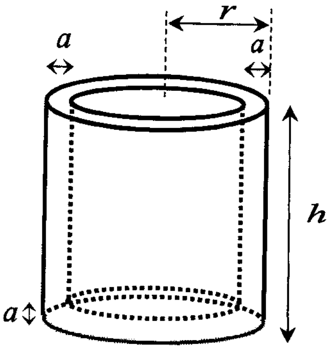


Figure 1

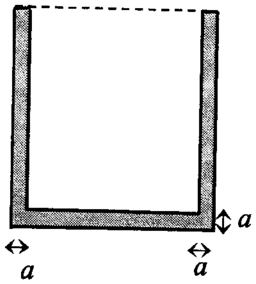


Figure 2

Figure 1 shows an open cylindrical tank. Figure 2 shows the cross-sectional view of the tank. The external radius of the tank is r cm, the external height is h cm and the tank is made with a material of thickness a cm throughout. The internal volume of the tank is fixed at 1000π cm^3.

(a) Show that the volume of the material needed to make the cylindrical tank is given by $V = k\pi\left[\frac{r^2}{(r-a)^2} - 1\right] + \pi r^2 a$, where k is a constant to be determined. [3]

(b) Find, in terms of a, the value of r that minimises V. (You need not show that your answer gives a minimum.) [3]

7 A freshly brewed cup of tea, initially at a temperature of 85°C, is left out to cool in a room with a room temperature of 25°C. After 20 minutes, the temperature of the tea is 55°C.

It is known that the rate of change of the temperature of the tea is proportional to the difference between the temperature of the tea and the room temperature.

(a) Given that θ is the temperature of the tea t minutes after the tea is left out to cool, show that $\theta = 25 + 60\mathrm{e}^{-kt}$, where k is a constant to be determined. [5]

(b) Sketch the graph of θ against t. [2]

8 The curve with equation $y=\mathrm{f}(x)$ for $x\le b$, where $\mathrm{f}(x)$ is a **quadratic** function, intersects the x-axis at the points $(a,0)$ and $(b,0)$, and the y-axis at the point $(0,c)$. The graph of $y=\mathrm{f}(x)$ is shown in Figure 1. The scales on the x- and y-axes are the same.

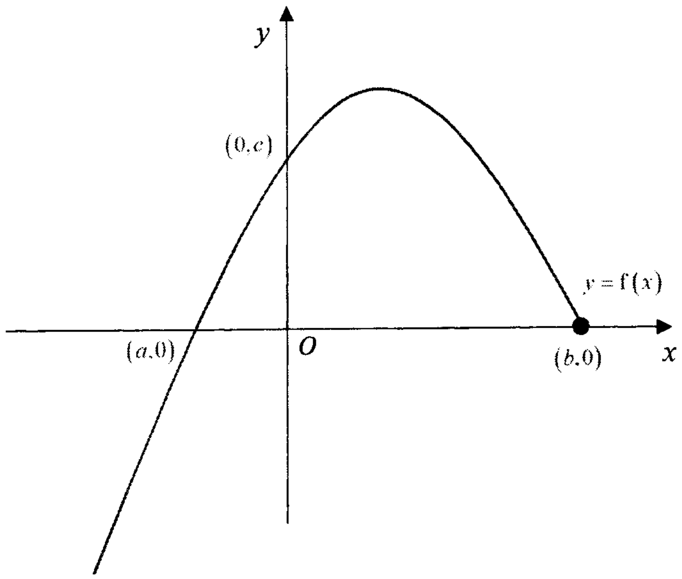


Figure 1

(a) **(i)** Sketch the graph of $y=\mathrm{f}(|x|)$, labelling the points where the graph intersects or touches the axes. [2]

(ii) Sketch the graph of $y=\dfrac{1}{\mathrm{f}(x)}$, labelling the points where the graph intersects or touches the axes, as well as the equations of any asymptotes. [2]

(iii) Describe fully a sequence of transformations which transforms the graph of $y=\mathrm{f}(2x+1)$ onto the graph of $y=\mathrm{f}(x)$. [2]

(b) The function $y=\mathrm{g}(x)$ is such that $\mathrm{g}(x)=\mathrm{f}(x)$ for $x\le k$, and $y=\mathrm{g}^{-1}(x)$ exists.

(i) Find the largest possible value of k in terms of a and b. [1]

(ii) Using the value of k found in **(b)(i)**, sketch the graph of $y=\mathrm{g}^{-1}(x)$ on Figure 1 in the Answer Booklet, labelling the intersections with the axes. Write down the range of g^{-1} in terms of a and b. [2]

(iii) Explain why the solution to $\mathrm{g}^{-1}(x)=x$ satisfies the equation $\mathrm{g}^{-1}(x)=\mathrm{g}(x)$. [1]

9 The planes π_1 and π_2 have equations $3x+c(y+z)-2=0$ and $\mathbf{r}=(\mathbf{i}+3\mathbf{j}-2\mathbf{k})+s(2\mathbf{i}-\mathbf{j}+3\mathbf{k})+t(\mathbf{i}-\mathbf{k})$ respectively, where c is a constant, and s and t are parameters. The point $A(1,3,-2)$ lies in both planes.

(a) Show that $c=-1$. [1]

(b) Show that the vector equation of the line of intersection of π_1 and π_2, line l, is given by $\mathbf{r}=\mathbf{i}+3\mathbf{j}-2\mathbf{k}+\alpha(\mathbf{i}-\mathbf{j}+4\mathbf{k})$, where α is a parameter. [3]

(c) Find the position vectors of the points on the line l which are a distance of $3\sqrt{2}$ from the point $B(2,-3,7)$. [4]

(d) Find the equation of the plane π_3 which is parallel to π_2 and contains the point B. Hence show that the distance between the planes π_2 and π_3 is $\frac{20}{3\sqrt{3}}$. [3]

10 **Do not use a calculator in answering this question.**

(a) One of the roots of the equation $\omega^4-2\omega^3+10\omega^2+p\omega+q=0$, where p and q are real, is $2+3\mathrm{i}$. Find the values of p and q and the other roots of the equation. [6]

(b) The complex numbers u and v are such that $u=-\sqrt{2}+\mathrm{i}\sqrt{2}$, and $|v|=3$ and $\arg v=\theta$, where $0<\theta<\frac{\pi}{4}$. The points A, B and C represents u, v and $u+v$ respectively on an Argand diagram.

(i) Find the modulus and argument of u. [2]

(ii) Sketch the points A, B and C on an Argand diagram. [2]

(iii) By finding the angle OAC in terms of θ or otherwise, show that $|u+v|^2=a+b\cos(\theta+K)$, where a, b and K are constants to be determined. [3]

11 The curve C is defined by the parametric equations $x = at^2$, $y = \frac{a}{t}$, $t \neq 0$, where a is a positive constant.

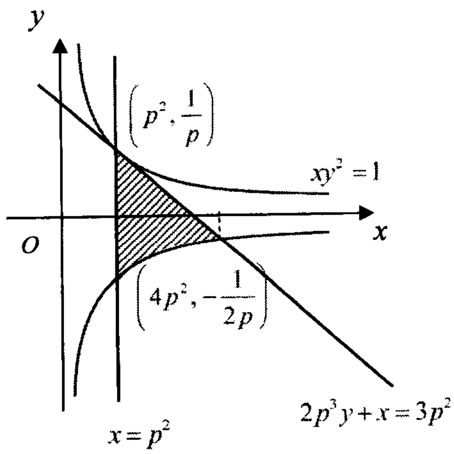
(a) Show that the equation of the tangent to the curve at the point $P\left(ap^2, \frac{a}{p}\right)$ is $2p^3y + x = 3ap^2$. [2]

(b) Find the equations of the tangents to C at the points where $x = a$. Find also the acute angle between these tangents. [3]

(c) **(i)** Show that $(q-p)^2(q+2p) = q^3 - 3p^2q + 2p^3$. [1]

(ii) The tangent to the curve at P cuts the curve again at $Q\left(aq^2, \frac{a}{q}\right)$. Find q in terms of p. [3]

The following diagram shows the graph of C for $a = 1$, and the Cartesian equation of the curve C is $xy^2 = 1$. The tangent to C at the point $\left(p^2, \frac{1}{p}\right)$ cuts the curve again at the point $\left(4p^2, -\frac{1}{2p}\right)$.



The region S is bounded by the tangent to C at the point $\left(p^2, \frac{1}{p}\right)$, the curve C and the line $x = p^2$.

(d) Find the area of S in terms of p. [3]

12 Sarah, a Singaporean Citizen, started a 4-year undergraduate Computing course at the National University of Singapore (NUS) in August 2021. The annual tuition fee, after the MOE Tuition Grant, is \$8,250. Sarah took a Tuition Fee Loan (TFL) covering 90% of her tuition fees. This loan is interest-free during her course of study.

Sarah decided to set aside \$$x$ per month from August 2021. This amount is increased by \$2 each subsequent month. (Sarah set aside \$$x$ in August 2021, \$$(x+2)$ in September 2021, ...) With the amount set aside, she repaid one-fifth of her total TFL at the end of July 2025.

(a) Find the value of x. [3]

Upon Sarah's graduation, the TFL's interest rate becomes effective at a fixed rate of 0.4% per month, calculated on the outstanding balance at the end of the month. The interest is first charged on 31st August 2025.

Sarah plans to make monthly repayments of \$$y$ to the bank on the 1st of every month, starting from 1st September 2025.

(b) Find the amount of outstanding loan after Sarah's first monthly repayment. [1]

(c) Show that the amount of outstanding loan after Sarah's nth monthly repayment is $23760(1.004)^n - 250y(1.004^n - 1)$. [3]

(d) Sarah wants to pay off her remaining loan by the end of 3 years. Find the value of y. [2]

Sarah decided on a different repayment strategy. For the first 24 months, she repays \$500 per month. After these 24 months, she doubles her monthly repayment to \$1000.

(e) Determine the month and year Sarah will clear her loan and calculate the amount of her final repayment. [4]

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MATHEMATICS

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29 August 2025
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Section A: Pure Mathematics [40 marks]

1 It is given that $\overrightarrow{OA} = 3\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{OB} = 5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$. Find the position vector of a point R on line OB such that AR is perpendicular to OB. Hence find the position vector of the point A', the reflection of the point A in the line OB. [4]

2 It is given that $\mathrm{f}(x) = \dfrac{1}{\sqrt{1-4x^2}}$ where $-\dfrac{1}{2} < x < \dfrac{1}{2}$.

(a) Using standard series from the List of Formulae (MF27), find the Maclaurin's expansion of $\mathrm{f}(x)$, up to and including the term in x^6. [2]

(b) Hence find the first four non-zero terms of the Maclaurin series for $\sin^{-1} 2x$. [4]

3 **(a)** For this question, you may use these results:

$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6} \quad \text{and} \quad \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}.$$

(i) Show that $\displaystyle\sum_{r=1}^{n} r(r+1)^2 = \frac{n(n+1)(n+2)(3n+5)}{12}$. [3]

(ii) Hence find $\displaystyle\sum_{r=5}^{n-1} (r+2)(r+3)^2$ in terms of n. [3]

(b) The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 2$, $u_{n+1} = \dfrac{1}{1-u_n}$, $n \geq 1$.

Find the values of u_2, u_3 and u_4. Hence find the value of u_{2025}. [3]

4 The function f is defined by $\mathrm{f}: x \mapsto \dfrac{1}{x^2+6x+5}$, for $x \geq -3$, $x \neq -1$.

(a) Find $\mathrm{f}^{-1}(x)$. [3]

(b) Find **algebraically** the range of f. [4]

The function g is defined by $\mathrm{g}: x \mapsto \mathrm{e}^x$, for $x \in \mathbb{R}$.

(c) Find the exact range of gf. [2]

5 **(a)** By using the substitution $x = a\tan\theta$, show that

$$\int \frac{1}{x\sqrt{x^2+a^2}}\,\mathrm{d}x = \frac{1}{a}\ln\frac{x}{\sqrt{x^2+a^2}+a} + c,$$

where $x > 0$ and $a > 0$. [4]

(b) At time t in a chemical reaction, x kg of substance X and y kg of substance Y are present. Initially, there is 4 kg of X and 5 kg of Y.

The variables x and y satisfy the equations $\dfrac{\mathrm{d}x}{\mathrm{d}t} = xy$ and $\dfrac{\mathrm{d}y}{\mathrm{d}t} = x^2$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y, and by solving this differential equation, show that $y = \sqrt{x^2+9}$. [4]

(ii) Obtain a differential equation in terms of x and t only and hence find an expression of t in terms of x. [4]

Section B: Probability and Statistics [60 marks]

6 **(a)** If $\mathrm{P}(B \mid A) = 0.2$, $\mathrm{P}(A \mid B) = 0.6$ and $\mathrm{P}(A' \cap B') = 0.3$, find $\mathrm{P}(A \cap B)$. [3]

(b) The events X, Y and Z are such that the events X and Z are mutually exclusive. Given further that $\mathrm{P}(X \cap Y) = 0.1$, $\mathrm{P}(X' \cap Y) = 0.35$, $\mathrm{P}(Y \cap Z) = 0.2$, and $\mathrm{P}(X \cup Y \cup Z) = 0.95$, find the minimum and maximum values of $\mathrm{P}(X)$. [3]

7 A gathering of 12 people break into 3 groups of 4 to play 3 different games: carrom, UNO and bridge.

(a) Find the number of ways these 12 people can be grouped. [1]

(b) Aaron and Sandy do not get along with each other and do not want to be in the same group. Find the number of ways these 12 people can be grouped with this restriction. [3]

(c) Given that Aaron and Sandy are not in the same group, find the probability that a third person Billy is in the same group as Aaron. [3]

8 The probability distribution of a discrete random variable X is given as follows:

x	-2	-1	0	1	3
$\mathrm{P}(X = x)$	0.1	a	0.3	b	c

Given that $\mathrm{E}(X) = 0.9$ and $\mathrm{Var}(X) = 2.99$, find the values of a, b and c. [5]

9 A fruit seller sources his oranges from a supplier with a large plantation where 1% of the oranges are rotten. The supplier picks the oranges regardless of whether the oranges are rotten or not, and packs 20 oranges in each box at random.

(a) The random variable X denotes the number of rotten oranges in a randomly chosen box. State in context two assumptions for X to follow a binomial distribution. [2]

Assume that X follows a binomial distribution for the rest of this question.

The fruit seller considers a box containing between 2 and 5 rotten oranges inclusive as substandard.

(b) Find the probability that a randomly chosen box is substandard. [1]

(c) Find the expected number of rotten oranges in a box. [1]

A truckload of 10 randomly chosen boxes of oranges is then sent to the fruit seller.

(d) Find the probability that not more than 2 boxes from the entire truckload are substandard. [2]

Upon receiving the oranges, the fruit seller inspects the boxes with these conditions:

- Accept the entire truckload if the first box opened has no rotten oranges.
- Reject the entire truckload if the first box opened has more than 1 rotten orange.
- If the first box has exactly 1 rotten orange, the fruit seller opens a second box.
- If the second box has at most 1 rotten orange, he will accept the entire truckload. Otherwise, he will reject the entire truckload.

(e) Find the probability that he will reject the entire truckload. [2]

10 Scientists are studying the growth of a particular species of bamboo. They want to understand how the height of the bamboo stalk changes over time. The table below shows the height, h inches, of a bamboo stalk at age, t weeks, after sprouting.

t	1	2	3	4	5	7	8	9
h	3.6	7.4	10.2	11.7	12.6	14.1	14.5	15.5

(a) Draw a scatter diagram of these data, and explain using the diagram why the relationship between h and t is not well-modelled by an equation of the form $h = a + bt$ or $h = a + bt^2$, where $b > 0$. [3]

(b) Consider the two models below:

$$\text{Model } A\text{: } h = c + d\sqrt{t},$$

$$\text{Model } B\text{: } h = c + d\ln t,$$

where c and d are constants. Explain which of Model A or Model B is a better fit to the data. State the equation of the least squares regression line in this case, giving your answer to 3 decimal places. [3]

(c) Use the equation of your regression line in **(b)** to estimate the length of a 6-week-old bamboo stalk. Comment on the reliability of this estimate. [2]

(d) Comment on the suitability of using this model in the long run. [1]

(e) Let H denote the height of the bamboo stalk calculated for each value of t in the table using the least squares regression line obtained in **(b)**. Find the value of S, where $S = \sum (h - H)^2$, giving your answer to 2 decimal places. [1]

(f) For each of the eight sample values of t, H' is given by $H' = p + q\ln t$, where p and q are constants. Would $\sum (h - H')^2$ be greater or less than S and why? [1]

11 In a laboratory, the time in minutes for an inspector to test an electrical circuit board is a continuous random variable X. The standard deviation of X is 0.63 and under ordinary conditions, the expected value of X is 5.82. As a result of the introduction of an incentive scheme, the laboratory supervisor claims that the inspector carries out the test more quickly. It is found that for a random sample of 150 tests, the mean time taken to test an electrical circuit board is 5.74.

(a) State appropriate hypotheses for the test of the claim, defining any symbols that you use. [2]

(b) Test whether the supervisor's claim is justified at 5% level of significance. [3]

(c) Explain whether there is a need for the laboratory supervisor to know anything about the population distribution of X. [1]

Two years later, a quality control officer took a random sample of 20 tests and found that the mean time taken to test an electrical circuit board is 5.88. He makes a claim that the mean time taken to test an electrical circuit board differs from μ_0 mins.

(d) State two assumptions the quality control officer has to make in order to carry out a hypothesis test for this claim. [2]

(e) Given that the officer concludes that there is no reason to reject the null hypothesis at 1% level of significance, find the range of values of μ_0. [3]

12 **In this question you should state the parameters of any distribution you use.**

A customer service call centre categorises incoming calls for follow-up resolution of the issues. The time taken to categorise an incoming call, T minutes, follows a normal distribution $N(2, 0.2^2)$. Incoming calls are categorised as routine or complex calls. The time taken to resolve a routine call, X minutes, follows a normal distribution $N(5, k)$ and the time taken to resolve a complex call, Y minutes, follows a normal distribution $N(20, 6^2)$. You may assume that any incoming call is first categorised as either routine or complex and is immediately followed up with the resolution of the issue. The duration of the call is taken to be the sum of the time taken to categorise the call and then to resolve the issue.

(a) Sketch the distribution for the time taken to categorise an incoming call, and shade clearly the area representing the probability that it took between 1.2 minutes and 2.8 minutes to categorise a randomly chosen incoming call. [2]

For a randomly chosen incoming call that is categorised as routine and then resolved, there is a probability of 0.254 that the duration of the call is more than 8 minutes.

(b) Show that $k = 2.24$ correct to 2 decimal places. [3]

(c) On a weekday morning where there are 20 incoming calls, n calls are categorised as complex calls. Given that there is a probability of at most 0.01 that the mean time taken to resolve the n complex calls exceeds 24 mins, find the set of values of n. [3]

There is a review of the resolution processes such that the time taken to resolve a complex call is now reduced by 20%.

(d) Find the probability that the time taken to resolve 2 complex calls is more than twice the time taken to resolve a routine call. [4]