Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

MATHEMATICS (Higher 2) — 9758/01

Paper 1

16 September 2025
3 hours

Additional Materials : Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** the questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

1 It is given that $2xy\frac{dy}{dx} = x^2 - y^2$ where $x > 0$, $y < 0$. Using the substitution $u = \frac{y}{x}$, show that the differential equation can be transformed to $\frac{2u}{1-3u^2}\frac{du}{dx} = \frac{1}{x}$. Hence find the general solution of y in terms of x. [6]

2 A series is given by $\sum_{r=1}^{n} 2(4-3x)^r$ where x is constant.

(a) Explain why this is a geometric series. Determine the range of values of x for the sum to infinity of this series to exist. [3]

(b) Using $x = 1$, and given that $\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$, find $\sum_{r=0}^{n-1}\left[2(4-3x)^r + (r+1)(2r+5)\right]$, leaving your answer in the form of $n(an^2 + bn + c)$, where a, b and c are constants to be determined. [4]

3 **(a)** Find the first three non-zero terms in the Maclaurin series for $e^x \sin(x+\pi)$. [3]

(b) It is given that the three terms found in part **(a)** are equal to the first three terms in the series expansion of $ax(1+bx)^c$ for small x, where a, b and c are constants. Find the exact values of a, b and c. Use these values to find the coefficient of x^4 in the expansion of $ax(1+bx)^c$, giving your answer as a simplified rational number. [5]

4 A sequence of real numbers $x_1, x_2, x_3, \ldots$ satisfies the recurrence relation

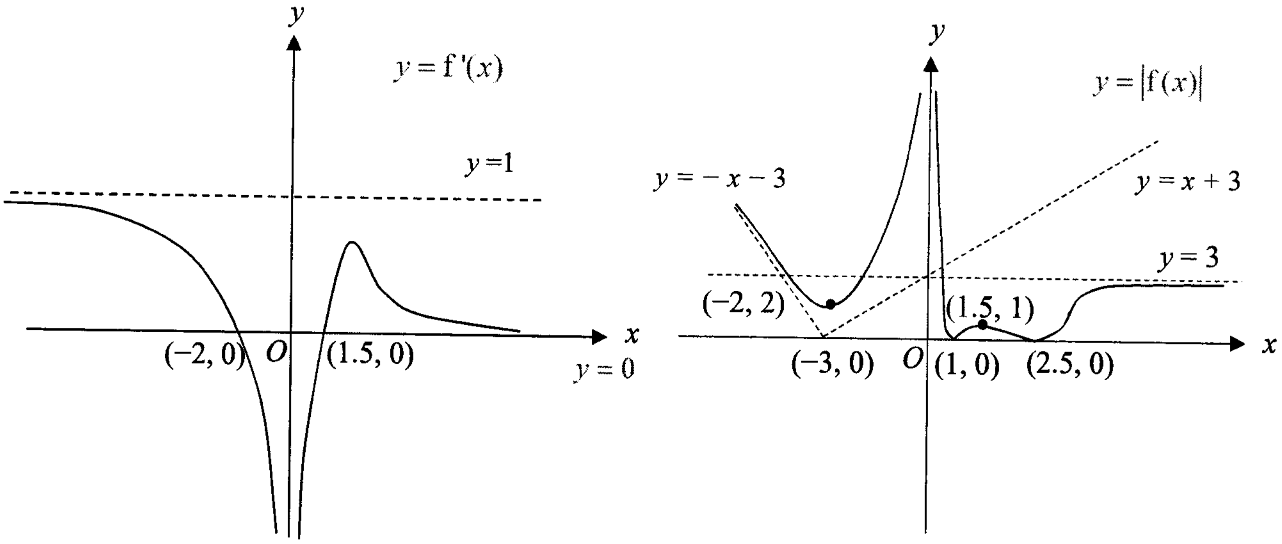
$$x_{n+1} = \frac{5x_n + 2}{2x_n + 3} \quad \text{for all } n \geq 1.$$

(a) Given that the sequence converges to l, find the possible exact values of l. [3]

(b) Describe how the sequence behaves when $x_1 = 3$. [1]

(c) Given that $x_5 = \frac{3503}{2158}$, find the value of x_1. [3]

5 The graphs of $y = \mathrm{f}'(x)$ and $y = |\mathrm{f}(x)|$ are shown below.



(a) State the nature of all turning point(s) of the graph of $y = \mathrm{f}(x)$. [2]

(b) State the range of values of x where f is decreasing. [2]

(c) Sketch the graph of $y = \mathrm{f}(x)$ indicating clearly the equations of the asymptotes, coordinates of the turning point(s) and the intersections with the axes. [3]

(d) On the copy of the graph of $y = |\mathrm{f}(x)|$ in the Printed Answer Booklet, sketch and label a line $y = kx + 3k$, where k is a constant. Hence state the range of values of k for which there is no real solution to the equation $|\mathrm{f}(x)| = kx + 3k$. [2]

6 A curve C is defined parametrically by

$$x = a\tan t, \qquad y = a\sec^2 t\sin t, \quad -\frac{\pi}{4} < t < \frac{\pi}{4},$$

where a is a positive constant.

(a) Show that $\dfrac{\tan t}{\sqrt{1+\tan^2 t}} = \sin t$. [1]

(b) By using part **(a)** or otherwise, find the cartesian equation of C in the form $y = \mathrm{f}(x)$, simplifying your answer. [2]

(c) Show that $\mathrm{f}(-x) = -\mathrm{f}(x)$. Hence sketch C. [2]

(d) Find the exact area of the region bounded by C, the x-axis and the lines $x = -A$ and $x = A$, where $0 < A < a$, leaving your answer in terms of A and a. [3]

7 The function f is defined by

$$\mathrm{f}: x \mapsto \mathrm{e}^{x} + \frac{1}{2x+2} \quad \text{for } x \in \mathbb{R},\ x \neq -1.$$

A function g, defined for $x \in \mathbb{R}, x \geq 1$, is such that $y \to \infty$ as x increases. It is also given that $\mathrm{g}(1) = -0.5$.

(a) Explain why the composite function fg exists and find the corresponding range of fg. [2]

(b) Given that $\mathrm{fg}(x) = \frac{x}{\sqrt{\mathrm{e}}} + \frac{1}{2\ln x + 1}$, find an expression for $\mathrm{g}(x)$ in terms of x. [2]

(c) The domain of f is now further restricted to $x > k$. State the least value of integer k for which the function f^{-1} exists. [1]

For the rest of the question, use the value of k found in part **(c)**.

(d) Without finding f^{-1},

(i) sketch, on the same diagram, the graphs of f, f^{-1} and ff^{-1} showing clearly the relationships between the graphs, [2]

(ii) find the gradient of the tangent to the graph of $y = \mathrm{f}^{-1}(x)$ at $x = \mathrm{e} + \frac{1}{4}$. [3]

8 A complex number z varies with t such that

$$z = 2\cos t + \mathrm{i}(3\sin t), \text{ where } 0 \leq t < 2\pi.$$

(a) By taking $x = \mathrm{Re}(z)$ and $y = \mathrm{Im}(z)$, sketch on an Argand diagram the curve that shows the positions of the points representing the complex number z. Find the cartesian equation of this curve. [3]

Two complex numbers z_1 and z_2 for two distinct values of t are such that $|z_1| = |z_2|$ and $0 < \arg(z_1) < \frac{\pi}{2}$.

(b) By referring to the Argand diagram in part **(a)**, find the possible values of $\arg(z_1 + z_2)$. [2]

It is given further that z_1 and z_2 are roots to the quadratic equation $z^2 + \alpha z + \beta = 0$.

(c) Explain whether it is necessary for α to be real. [2]

(d) Given that α is not real and $|z_1| = |z_2| = \frac{\sqrt{26}}{2}$, find the values for α and β. [4]

9 The point A has coordinates $(1, -2, 4)$ and the plane π_1 has equation $2x - y + 2z = 5$.

(a) Find the exact shortest distance between A and π_1. [2]

The plane π_2 has equation $x+3y-az=3$, where a is a constant.

(b) Find the vector equation of line l, which is the line of intersection of π_1 and π_2, in terms of a. [4]

The plane π_3 has equation $bx-2y+4z=3$, where b is a constant.

(c) Show that $(a-6)(b-4)=0$ if l is parallel to π_3. [2]

(d) State the conditions that a and b must follow for the three planes to form a triangular prism, where all the planes are non-parallel and they do not have any point in common. Justify your answer. [4]

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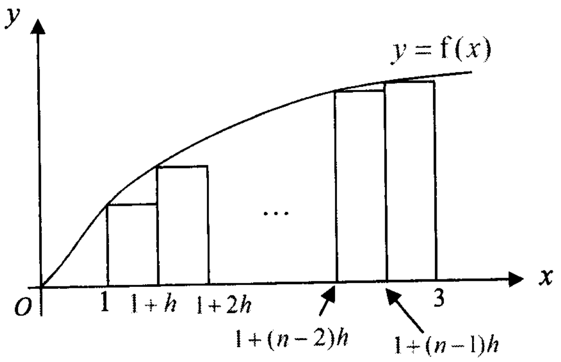


The diagram above shows a sketch of the graph of $y = f(x)$ for $x \geq 0$. There are n rectangles each of width h drawn under the curve for $1 \leq x \leq 3$. Each rectangle when rotated through 2π radians about the x-axis, will result in a cylindrical disc. The total volume of the n cylindrical discs V_1, can be used to estimate the volume V, which is the actual volume generated when the region bounded by the curve, the lines $x = 1$ and $x = 3$, and the x-axis is rotated through 2π radians about the x-axis.

(a) Show that V_1, the total volume of the n cylindrical discs, is given by $V_1 = \pi h \sum_{r=0}^{n-1} [f(1+rh)]^2$.
State the value of h in terms of n. [3]

(b) From the above diagram, it can be observed that $V_1 < V$. Write down V_2, a similar expression as V_1 where $V_2 > V$. [1]

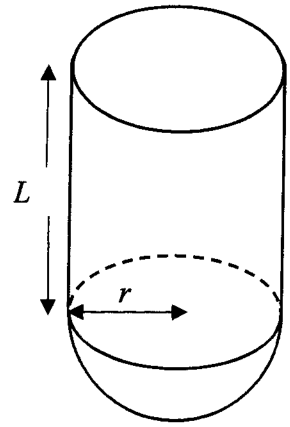
It is now given that $f(x) = \dfrac{x^2}{\sqrt{1+e^x}}$ for $x \geq 0$.

(c) Find the value of $\lim_{n \to \infty} V_1$. [2]

(d) Find the area of the region bounded by $y = f(x)$ and another curve with equation $(x-1)^2 + (y-3)^2 = 9$, for $y \leq 3$. [4]

11 [The surface area and volume of a sphere are given by $4\pi r^2$ and $\frac{4}{3}\pi r^3$ respectively.]

An aquarist who has a fixed budget of $\$k$ wants to make a goldfish sanctuary. His design of the tank consists of a circular cover, a cylinder of length L cm and a hemispherical base of radius r cm neatly fitted together. The costs of the material per cm^2 used for the cover, the cylindrical surface and hemispherical base are \$20, \$10 and \$30 respectively. Assume that the tank is made of material with negligible thickness.

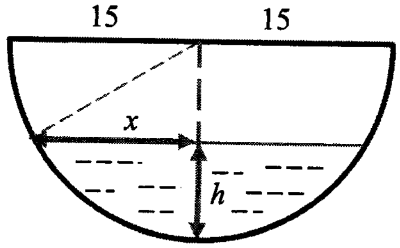


(a) Show that the volume of the tank is V cm^3, where $V = \frac{k}{20}r - \frac{10}{3}\pi r^3$. [3]

(b) As r varies, find the cost of the material used to make the hemispherical base in terms of k when V is at its maximum value. You need to show that V is a maximum. [3]

The aquarist has fixed the radius of the cylinder to be 15 cm. Initially, there is some water in the hemispherical part of the tank. However, due to a defect in the tank, water is leaking at a constant rate of 20 cm^3 per minute.

The depth of the water at the hemispherical bottom is denoted by h cm, and the radius of the water surface is x cm. The volume of water in the hemispherical bottom is given by W cm^3, where $W = \frac{\pi h(3x^2 + h^2)}{6}$.



(c) By formulating a relationship between x and h, find, when $h = 3$, the rate of change of

(i) the depth of water, [3]

(ii) the radius of the water surface. [3]

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DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

MATHEMATICS (Higher 2)

9758/02

Paper 2

19 September 2025

3 hours

Additional Materials : Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Write your Name, Index Number and Class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1 A curve C has equation $y = x + \dfrac{9}{x+1}$.

(a) Using differentiation, find the range of values of x such that C concaves upwards. [2]

(b) Sketch C. [3]

2 (a) Use the substitution $t = e^x$ to show that $\int e^{2x} \tan^{-1}(e^x)\,dx = \int t \tan^{-1}(t)\;dt$. [2]

(b) Hence find the exact value of $\int_0^1 e^{2x} \tan^{-1}(e^x)\,dx$. [5]

3 A curve C has equation $x^3 + y^3 - 5xy = 0$ where $x > 0$.

(a) Find $\dfrac{dy}{dx}$ in terms of x and y. [2]

(b) Find the coordinates of the point on C at which the normal is parallel to the y-axis. [3]

(c) Determine the nature of the stationary point. [2]

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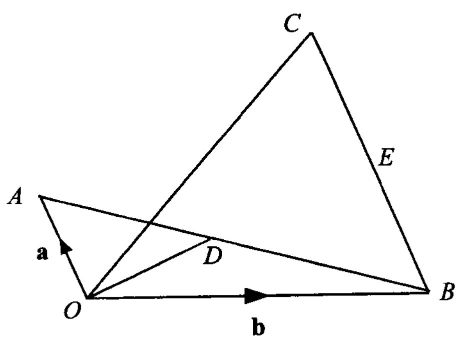


Relative to the origin O, the points A, B, and C have position vectors **a**, **b** and $3\mathbf{a}+\mathbf{b}$ respectively. The point D lies on AB such that $AD = kAB$, where $0 < k < 1$.

(a) Find a vector equation of the line OD in terms of k, **a** and **b**. [2]

(b) The point E is the midpoint of BC. Find the value of k if O, D and E are collinear. [4]

It is given that $|\mathbf{a}| = 1, |\mathbf{b}| = 2$ and $|3\mathbf{a} - 2\mathbf{b}| = \sqrt{31}$.

(c) By considering the scalar product $(3\mathbf{a}-2\mathbf{b})\cdot(3\mathbf{a}-2\mathbf{b})$, find the numerical value of $\mathbf{a}\cdot\mathbf{b}$ and hence determine the angle between **a** and **b**. [4]

(d) Give a geometrical interpretation of $|\mathbf{a}\cdot\mathbf{b}|$. [1]

5 A heated metal block at $160°C$ is left to cool in a laboratory with an ambient temperature of $30°C$. The temperature, T in $°C$, of the metal block t minutes after it is left in the laboratory is model as a function $T(t)$. The rate of cooling is proportional to the difference between the object's temperature and the ambient temperature, with a positive constant of proportionality k.

(a) Write down a differential equation for the situation and find the expression of $T(t)$ in terms k. [3]

An experiment to study the cooling of the heated metal block starts at 10:00 am. However, a protective thermal casing is used to delay the cooling of the metal block for 5 minutes, resulting in the temperature of the metal block remaining constant at $160°C$ during this time. At 10:05 am, the casing is removed and cooling begins immediately. At 10:15 am, the temperature of the metal block is measured to be $100°C$.

(b) In order to use the solution obtained in part **(a)** to model the cooling process now, the time t needs to be replaced with $t+a$, where a is a constant. State the value of a. [1]

(c) Due to the protective thermal casing, the temperature of the metal block $T_p(t)\,°C$, t minutes after 10:00 am can be modelled by the following piecewise function

$$T_p(t)=\begin{cases}160 & \text{for } 0\le t<5,\\ T(t+a) & \text{for } t\ge 5.\end{cases}$$

Using your answers from parts **(a)** and **(b)**, show that $T(t+a)=30+177\mathrm{e}^{-0.0619t}$. [3]

(d) Find the time it takes for the metal block to cool to $60°C$. [1]

(e) Sketch the graph of $T_p(t)$ against t for $t\ge 0$. [2]

Section B: Probability and Statistics [60 marks]

6 The events A, B and C are such that $\mathrm{P}(A)=0.8$, $\mathrm{P}(B)=0.2$ and $\mathrm{P}(C)=0.6$. It is also known that $\mathrm{P}(A\cap B)=\mathrm{P}(B)$ and $\mathrm{P}(B\cap C)=0.1$.

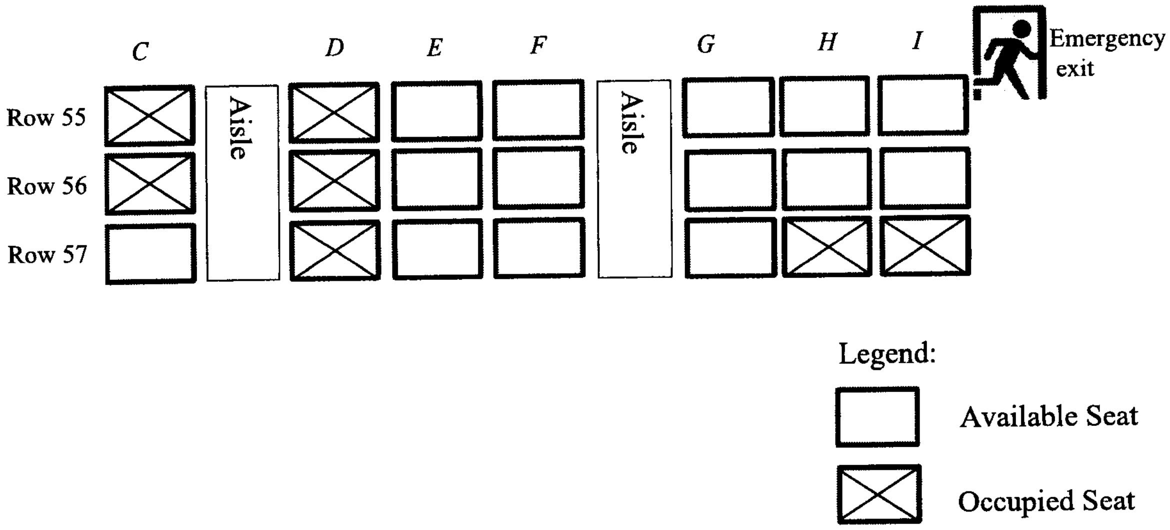
(a) Find exactly the maximum and minimum possible values of

(i) $\mathrm{P}(A\cap B'\cap C)$, [3]

(ii) $\mathrm{P}\left(A\cap B'\cap C \mid A\cup C\right)$. [2]

(b) It is given further that A and C are independent, find the value of $\mathrm{P}(A\cap B'\cap C)$. [2]

7 Twelve students from three CCAs organised an overseas learning trip to Shanghai. It comprises five students from Tennis, four students from Bowling and three students from Softball. The available seats for the flight for them to choose from are shown on the diagram as follows:



(a) In how many different ways can the students be seated if there is a particular student who must be seated nearest to the emergency exit? [2]

(b) Find the number of ways they can be seated if two particular students do not want to be seated on any of the aisle seats. [3]

(c) Find the number of ways they can be seated if students of the same CCA must be seated together either front and back or left and right, and cannot be separated by an aisle. For example, the three students from Softball can be seated at Row 56 Seat G, H and Row 57 Seat G, but not Row 56 Seat H, I and Row 57 Seat G. [3]

8 The bag contains 6 red discs, 6 green discs, 6 yellow discs and 6 blue discs. The discs are indistinguishable aside from their colour. 4 discs are taken one after another from the bag without replacement. The random variable X is the number of different colours obtained among the 4 discs.

(a) Show that $\mathrm{P}(X=2)=\dfrac{465}{1771}$. [3]

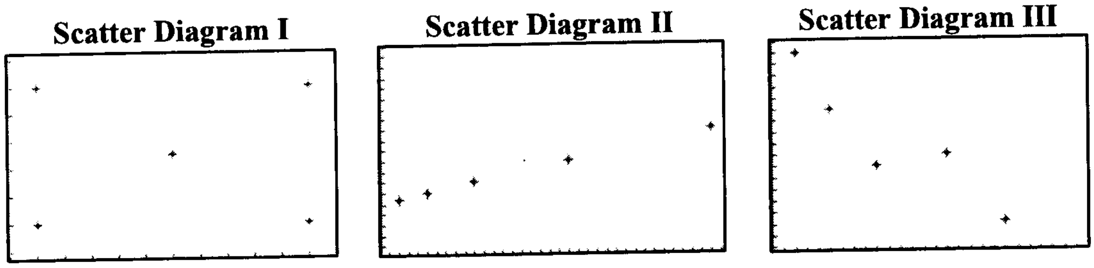
(b) Hence determine the probability distribution of X. [3]

Joe plays a game that comprises 8 rounds of drawing 4 discs from the bag. For each round, he wins \$10 if he obtains at most 2 discs of different colours, otherwise he loses \$5. After each round, all the discs are returned to the bag before the next round.

(c) Find the probability that Joe wins some money at the end of the game. [2]

(d) Explain, with working, whether it is worthwhile for Joe to play this game to win money. [2]

9 **(a)** The following scatter diagrams with 5 data points each have product moment correlation coefficients −0.7, 0, 1, not necessarily in the given order. State the product moment correlation coefficients for the scatter diagrams **I**, **II** and **III**. [1]



(b) A private developer hires workers for renovation works for its townhouse projects. The renovation works required for each townhouse in a certain project are identical. The private developer wishes to investigate how the number of working days taken to complete the renovation works for a townhouse for a certain project, y, varies with the number of workers hired to renovate a townhouse, x. The relevant data collected by the private developer is given in the following table.

x	3	5	8	10	13	15	18	20
y	92	50	33	25	21	19	17	16

(i) Draw a scatter diagram for these values. Use your diagram to comment on whether a linear model would be appropriate to model the relationship between x and y. [2]

(ii) By referring to the scatter diagram in part **(b)(i)** and the context of the question, choose and explain which one of the following models **(A)**, **(B)** and **(C)** below, where a and b are positive constants, is the most appropriate for modelling the relationship between x and y.

(A) $y = a - b\ln x$

(B) $y = a - bx^2$

(C) $y = a + \frac{b}{x^2}$ [2]

Use the most appropriate model from part **(b)(ii)** for the rest of the question.

(iii) Find the equation for the relationship between x and y, giving the numerical values of the coefficients and the product moment correlation coefficient for this model. [2]

(iv) Estimate the number of working days required to complete the renovation works for a townhouse when 9 workers are hired and comment on the reliability of your estimate. [2]

(v) The private developer pays each worker a fixed amount of \$120 for a day's work. Estimate the number of workers the developer should hire to minimise the total amount of workers' wages the developer will have to pay if he can only hire a maximum of 20 workers. [2]

10 In this question you should state the parameters of any distributions that you use.

To regulate the use of its facilities by its paying members, a gym provides a guideline of a maximum usage time of 90 minutes per visit per paying member. However, the gym's records show that the usage time, X minutes, of its paying members follows the distribution $\mathrm{N}(95,\ \sigma^2)$, where σ is the standard deviation. Only 30% of its paying members adhere to the guideline.

(a) Show that $\sigma = 9.53$, correct to 3 significant figures. [2]

(b) The gym closes at 11 pm. John, a paying member, arrives at 9.15 pm. Find the probability that he cannot finish his workout. [1]

(c) Ten paying members are randomly chosen. Find the probability that the usage time of the ninth paying member is the third and last one that adhere to the guideline. [2]

To grow its membership, the gym allows each paying member to sign up a friend as a trial member. For a limited period, the trial member can use the gym for free. Over time, it is found that the usage time, W minutes, of its trial members follows the distribution $\mathrm{N}(85,\ 80)$.

(d) Sketch the distribution of X and W on the same diagram. [2]

(e) Find the probability that the total usage time spent by two randomly chosen paying members differs from twice the time spent by a randomly chosen trial member by at least 15 minutes. [4]

(f) State an assumption needed for your calculation in part **(e)** to be valid. [1]

11 Mr Dough's factory produces packets of flour for sale in local supermarkets packed in 2 sizes – Regular-sized packets and Large-sized packets. Automated machines weigh and pack the flour. A recent major power outage resulted in all his machines being needed to be recalibrated upon restarting.

Regular-sized packets are supposed to have a mean mass of 1 kg. Mr Dough suspects that his machines now pack Regular-sized packets that are underweight. He took a sample of his Regular-sized packets and measured their masses, x g (correct to nearest 5 g). The results are recorded as follows:

Mass, x (g)	**No. of Regular-sized packets**
980	5
985	6
990	7
995	7
1000	9
1005	6
1010	5
1015	3
1020	2

(a) Determine the unbiased estimate of the population variance of the mass of Regular-sized packets. [1]

(b) Explain whether Mr Dough should conduct a one-tail or a two-tail test. [1]

(c) Carry out the test at 5% level of significance and give your conclusion in context. Explain whether it is necessary to assume that the mass of Regular-sized packets of flour follow a normal distribution. [5]

Large-sized packets of flour are known to have a normal distribution with a mean mass of 2 kg. Mr Dough also suspects that his machines are now not packing the Large-sized packets according to the required mass correctly. A sample of 15 Large-sized packets of flour gave a total mass of 30075 g.

Assuming Large-sized packets of flour have a population variance of σ^2 g^2, an appropriate test was carried out which gave a p-value of 0.0529.

(d) Showing all necessary workings, prove that $\sigma = 10.0$ correct to 3 significant figures. [2]

Another packet of Large-sized packet of flour with mass h g is then added to the sample of 15. The combined sample of Large-sized packets resulted in a rejection of the claim that the mean mass of a Large-sized packet of flour is 2 kg at 4% level of significance.

(e) Using $\sigma = 10.0$, find the possible range of values of h, giving your answer to the nearest gram. [3]