EJC 9758 2025 Prelim P1 Solutions

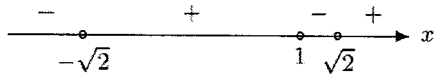
<table>
<tr><th></th><th>Solution</th></tr>
<tr><td>1(a)</td><td>

$x+1>\dfrac{1}{x-1}$

$x+1-\dfrac{1}{x-1}>0$

$\dfrac{(x+1)(x-1)-1}{x-1}>0$

$\dfrac{(x+\sqrt{2})(x-\sqrt{2})}{x-1}>0$



$-\sqrt{2}<x<1$ or $x>\sqrt{2}$

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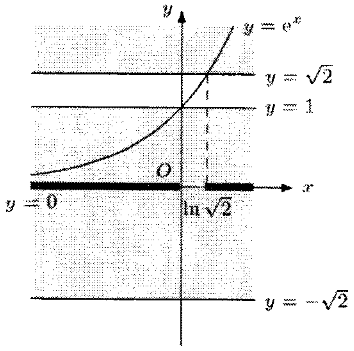
<tr><td>(b)</td><td>



$x<0$ or $x>\ln\sqrt{2}=\dfrac{1}{2}\ln 2$

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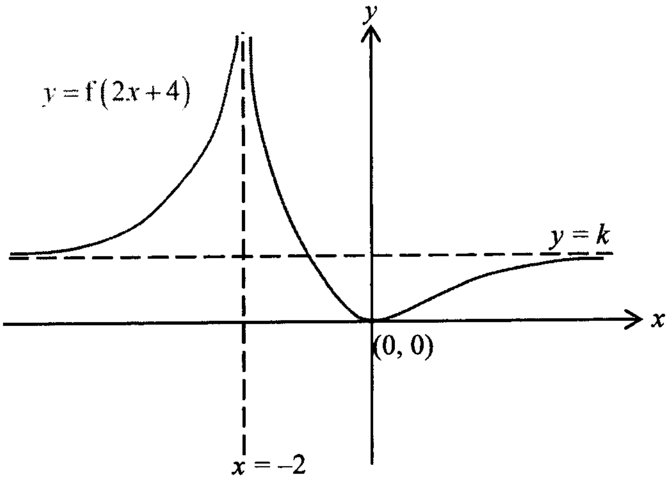
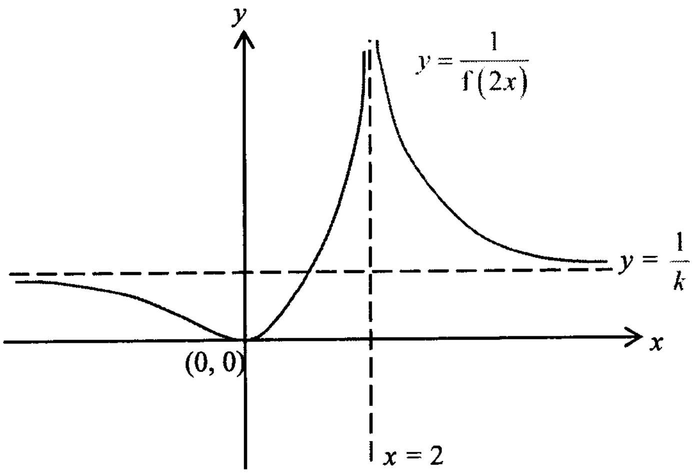
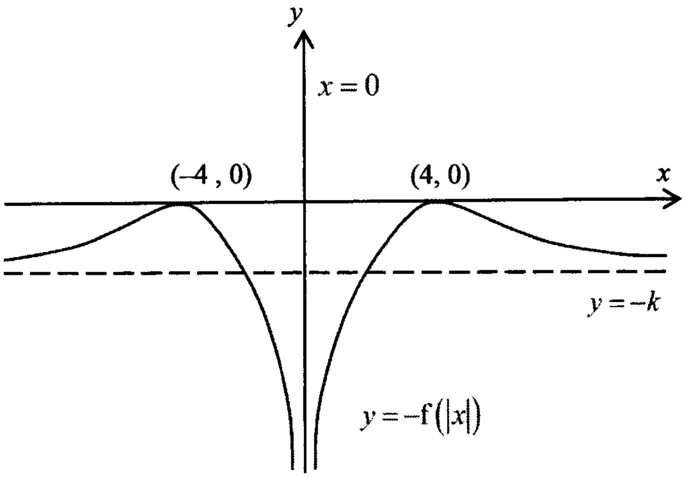
	Solution
2	Perimeter of outline = 180 $3a+2a+2b+\frac{1}{2}\pi(a)=180$ $\left(5+\frac{\pi}{2}\right)a+2b=180$ $\Rightarrow b=90-\left(\frac{5}{2}+\frac{\pi}{4}\right)a$ Area enclosed by wire, $A=3ab+\frac{1}{2}\pi\left(\frac{a}{2}\right)^2$ $=3a\left[90-\left(\frac{5}{2}+\frac{\pi}{4}\right)a\right]+\frac{\pi}{8}a^2$ $=270a-\frac{5\pi}{8}a^2-\frac{15}{2}a^2$ $=270a-\left(\frac{5\pi}{8}+\frac{15}{2}\right)a^2$ $\frac{dA}{da}=270-\left(\frac{5\pi}{4}+15\right)a$ When $\frac{dA}{da}=0$, $a=\frac{270}{\frac{5\pi}{4}+15}$ or 14.265 $\frac{d^2A}{da^2}=-\left(\frac{5\pi}{4}+15\right)<0$ Hence the area is maximum when $a=\frac{270}{\frac{5\pi}{4}+15}$. Maximum area $=270\left(\frac{270}{\frac{5\pi}{4}+15}\right)-\left(\frac{5\pi}{8}+\frac{15}{2}\right)\left(\frac{270}{\frac{5\pi}{4}+15}\right)^2$ $=1925.82\text{ cm}^2$

	Solution
3(a)	$AC=\sqrt{13^2-5^2}=12$ $\cos\angle BAC=\frac{AC}{AX}=\frac{12}{13}$
(b)	$AX=\frac{12}{\cos(\angle BAC-\alpha)}$ $=\frac{12}{\cos\angle BAC\cos\alpha+\sin\angle BAC\sin\alpha}$ $=\frac{12}{\frac{12}{13}\cos\alpha+\frac{5}{13}\sin\alpha}$ $=\frac{156}{12\cos\alpha+5\sin\alpha}$ (shown)
(c)	Using small angle approximation, $AX\approx\frac{156}{12\left(1-\frac{\alpha^2}{2}\right)+5\alpha}$ $=156\left[12\left(1+\frac{5}{12}\alpha-\frac{\alpha^2}{2}\right)\right]^{-1}$ $=156[12]^{-1}\left[1+\left(\frac{5}{12}\alpha-\frac{\alpha^2}{2}\right)\right]^{-1}$ $=13\left[1-\left(\frac{5}{12}\alpha-\frac{\alpha^2}{2}\right)+\left(\frac{5}{12}\alpha-\frac{\alpha^2}{2}\right)^2+\ldots\right]$ $=13\left[1-\frac{5}{12}\alpha+\frac{\alpha^2}{2}+\left(\frac{5}{12}\alpha\right)^2+\ldots\right]$ $\approx 13-\frac{65}{12}\alpha+\frac{1261}{144}\alpha^2$ $p=-\frac{65}{12},\quad q=\frac{1261}{144}$ (shown)

	Solution
4 **(a)**	Translation of 2 units in the negative x-direction. 
(b)	
(c)	

	Solution

5(a)

$$S_2 = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{7}{12}$$

Area under curve between $x = 1$ and $x = 2$ is $\int_1^2 \frac{1}{x}\,\mathrm{d}x = \left[\ln x\right]_1^2 = \ln 2$

Since the two rectangles lie under the curve, the area of the two rectangles must be less than the area under curve, so

$$\frac{7}{12} < \ln 2$$

(b)

We can use any integer larger than 2 for *n*.

Some possible answers are:

n	S_n
3	$\frac{37}{60}$
4	$\frac{533}{840}$
5	$\frac{1627}{2520}$
6	$\frac{15797}{27720}$

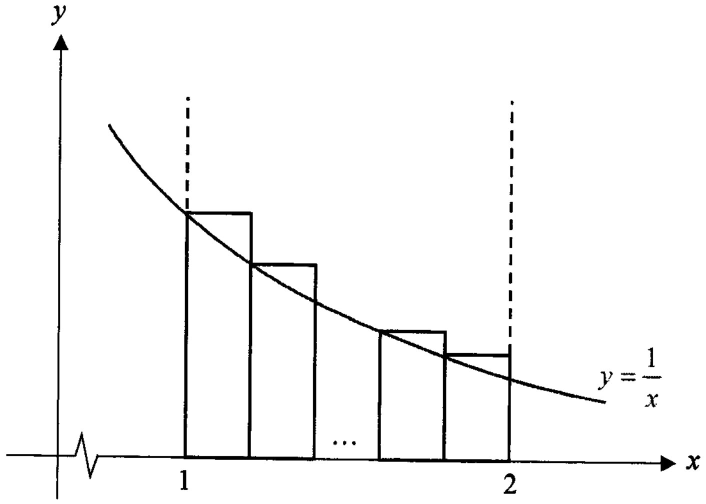
(c)

$$S_n = \left(\frac{1}{n}\right)\left(\frac{1}{\left(\frac{n+1}{n}\right)}\right) + \left(\frac{1}{n}\right)\left(\frac{1}{\left(\frac{n+2}{n}\right)}\right) + \cdots + \left(\frac{1}{n}\right)\left(\frac{1}{\left(\frac{2n}{n}\right)}\right)$$

$$= \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n-1} + \frac{1}{2n}$$

$$= \sum_{r=1}^{n} \frac{1}{n+r}$$

(d)



Consider n rectangles of equal width in the same interval, drawn above the curve.

Then

$\ln 2 <$ Area of rectangles

$$=\left(\frac{1}{n}\right)\left(\frac{1}{\frac{n}{n}}\right)+\left(\frac{1}{n}\right)\left(\frac{1}{\frac{n+1}{n}}\right)+\ldots+\left(\frac{1}{n}\right)\left(\frac{1}{\frac{2n-1}{n}}\right)$$

$$=\frac{1}{n}+\frac{1}{n+1}+\ldots+\frac{1}{2n-1}$$

$$=\frac{1}{n}+\quad\left(S_n-\frac{1}{2n}\right)$$

$$=S_n+\frac{1}{n}-\frac{1}{2n}$$

$$=S_n+\frac{1}{2n}$$

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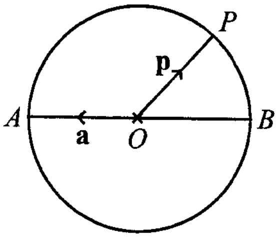
<tr><td>6(a)</td><td>



$\overrightarrow{AP}=\mathbf{p}-\mathbf{a}$

$\overrightarrow{BP}=\mathbf{p}+\mathbf{a}$

$\overrightarrow{AP}.\overrightarrow{BP}=(\mathbf{p}-\mathbf{a}).(\mathbf{p}+\mathbf{a})$

$=|\mathbf{p}|^2-|\mathbf{a}|^2$

$=|\mathbf{a}|^2-|\mathbf{a}|^2$ (since OA and OP are radius, $\therefore |\mathbf{p}|=|\mathbf{a}|$)

$=0$

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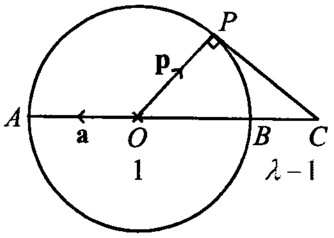
<tr><td>(b)</td><td>



By Ratio Theorem,

$\mathbf{b}=\dfrac{(\lambda-1)\mathbf{a}+\mathbf{c}}{\lambda}$

$\mathbf{c}=\lambda\mathbf{b}-(\lambda-1)\mathbf{a}$

$\mathbf{c}=\lambda(-\mathbf{a})-(\lambda-1)\mathbf{a}$

$=(1-2\lambda)\mathbf{a}$

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<tr><td>(c)</td><td>

$\overrightarrow{PC}=(1-2\lambda)\mathbf{a}-\mathbf{p}$

$\overrightarrow{OP}.\overrightarrow{PC}=0$

$\mathbf{p}.\left((1-2\lambda)\mathbf{a}-\mathbf{p}\right)=0$

$(1-2\lambda)\mathbf{p}.\mathbf{a}-\mathbf{p}.\mathbf{p}=0$

$(1-2\lambda)|\mathbf{p}||\mathbf{a}|\cos 120^\circ-|\mathbf{p}|^2=0$

$-\dfrac{1}{2}(1-2\lambda)|\mathbf{a}|^2-|\mathbf{a}|^2=0$

$-\dfrac{1}{2}(1-2\lambda)=1$

$\lambda=\dfrac{3}{2}$

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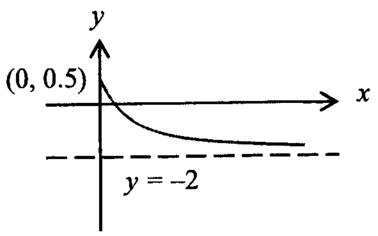
	Solution
7(a)	$u_2 = \frac{u_1 + u_3}{2} = \frac{9+b}{2}$
(b)	$v_2 = \pm\sqrt{9b} = \pm 3\sqrt{b}$
(c)	$u_2 - v_2 = 8$ $v_2 = u_2 - 8$ $v_2^{\ 2} = (u_2 - 8)^2$ $9b = \frac{b^2 - 14b + 49}{4}$ $b^2 - 50b + 49 = 0$ $(b-1)(b-49) = 0$ $b = 1$ or $b = 49$ If $b = 49$, $\lvert r \rvert = \sqrt{\frac{49}{9}} = \frac{7}{3}$ but we need $\lvert r \rvert < 1$ since the sequence is convergent. So $b = 1$. $u_2 = \frac{9+1}{2} = 5$ $v_2 = 5 - 8 = -3$

	Solution
8(a)	$(a+bi)^2 = 2i$ $a^2 - b^2 + 2abi = 2i$ $a^2 - b^2 = 0 - (1)$ $2ab = 2 - (2)$ From (2), $a = \frac{1}{b}$ $(\frac{1}{b})^2 - b^2 = 0$ $1 - b^4 = 0$ $(1+b^2)(1-b^2) = 0$ $b^2 = 1$ ($\because$ b is real) So $b=1, a=1$ or $b=-1, a=-1$ (reject since $0 \le \arg(w) \le \frac{\pi}{2}$) $w = 1+i$
(b)	Since $1+i$ is a root, we can write $z^3 - z^2 + (1-i)z + s = \left[z-(1+i)\right](z^2+Bz+C)$ for some (possibly complex) constants B and C. Comparing coefficients, z^2 : $-1 = -1 - i + B$ ⇒ $B = i$ z : $1 - i = -B - Bi + C$ ⇒ $C = 0$ constant: $s = -(1+i)C$ ⇒ $s = 0$ $z^2 + iz = 0$ $z(z+i) = 0$ $z = 0$ or $z = -i$
(c)	Replace z with iv: $(iv)^3 - (iv)^2 + (1-i)(iv) + s = 0$ $-iv^3 + v^2 + (1+i)v + s = 0$ Hence, $z = iv$ ⇒ $v = -iz$ $v = -i(1+i) = 1-i$, $v = -i(0) = 0$ or $v = -i(-i) = -1$

	Solution
9(ai)	From GC, coordinates are $(-1,10)$ and $(3,-22)$
(aii)	Largest possible value is $a=-1$ Domain of f^{-1} is equal to range of f, which is $(-\infty,10]$
(bi)	$g^2(x)=\dfrac{1-2\left(\dfrac{1-2x}{x+2}\right)}{\left(\dfrac{1-2x}{x+2}\right)+2}$ $=\dfrac{x+2-2(1-2x)}{1-2x+2(x+2)}$ $=\dfrac{5x}{5}$ $=x$
(bii)	$g^n(x)=\begin{cases} x & \text{if } n \text{ is even} \\ \dfrac{1-2x}{x+2} & \text{if } n \text{ is odd} \end{cases}$
(ci)	$R_h=[0,\infty)$ $D_g=\mathbb{R}\backslash\{-2\}$ OR $D_g=(-\infty,-2)\cup(-2,\infty)$ Since $R_h \subseteq D_g$, then gh exists.
(cii)	From graph of $y=g(x)$ with domain restricted to $[0,\infty)$  $R_{gh}=\left(-2,\tfrac{1}{2}\right]$

	Solution
10(a)	$\int_0^4 e^{\lvert x-1\rvert}\,dx$ $=\int_0^1 e^{1-x}\,dx+\int_1^4 e^{x-1}\,dx$ $=\left[-e^{1-x}\right]_0^1+\left[e^{x-1}\right]_1^4$ $=-1+e+e^3-1$ $=e^3+e-2$
(b)	$\int \sin^2 3x+\tan^2 3x\,dx$ $=\int \frac{1-\cos 6x}{2}+\sec^2 3x-1\,dx$ $=\int -\frac{1}{2}\cos 6x+\sec^2 3x-\frac{1}{2}\,dx$ $=-\frac{1}{12}\sin 6x+\frac{1}{3}\tan 3x-\frac{1}{2}x+C$
(c)	Since $\frac{dx}{du}=6u^5$, $\int\frac{1}{\sqrt{x}+\sqrt[3]{x}}\,dx$ $=\int\frac{1}{u^3+u^2}\cdot 6u^5\,du$ $=6\int\frac{u^3}{u+1}\,du$ $=6\int u^2-u+1-\frac{1}{u+1}\,du$ $=6\left[\frac{1}{3}u^3-\frac{1}{2}u^2+u-\ln(u+1)\right]+C$ $=2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\ln\left(\sqrt[6]{x}+1\right)+C$

	Solution
11(a)	$\frac{\mathrm{d}y}{\mathrm{d}t}=-k\sqrt{y}$ $\frac{1}{\sqrt{y}}\frac{\mathrm{d}y}{\mathrm{d}t}=-k$ $\int\frac{1}{\sqrt{y}}\,\mathrm{d}y=\int -k\,\mathrm{d}t$ $2\sqrt{y}=-kt+C$ When $t=0$, $y=h$: $C=2\sqrt{(h)}+k(0)=2\sqrt{h}$ $2\sqrt{y}=-kt+2\sqrt{h}$ $\sqrt{y}=\sqrt{h}-\frac{kt}{2}$
(b)	When $y=0$, $\sqrt{h}-\frac{kt}{2}=0$ $\sqrt{h}=\frac{kt}{2}$ $t=\frac{2\sqrt{h}}{k}$
(c)	Since $\cos t$ varies between -1 and 1, $\frac{1+\cos t}{2}$ varies between 0 and 1. So the flow is now slower.
(d)	$\frac{\mathrm{d}y}{\mathrm{d}t}=-k\left(\frac{1+\cos t}{2}\right)\sqrt{y}$ $\frac{1}{\sqrt{y}}\frac{\mathrm{d}y}{\mathrm{d}t}=-\frac{k(1+\cos t)}{2}$ $\int\frac{1}{\sqrt{y}}\,\mathrm{d}y=-\frac{k}{2}\int 1+\cos t\,\mathrm{d}t$ $2\sqrt{y}=-\frac{k}{2}(t+\sin t)+D$ When $t=0$, $y=h$: $D=2\sqrt{(h)}+\frac{k}{2}(0)=2\sqrt{h}$ $2\sqrt{y}=-\frac{k}{2}(t+\sin t)+2\sqrt{h}$ $\sqrt{y}=\sqrt{h}-\frac{kt+k\sin t}{4}$

(e)	When $h=10$ and $k=0.1$, time taken in the original model is 63.25 (seconds). For the revised model, from GC, time taken is 126.08 (seconds). $\frac{126.08...}{63.25...}=1.99$ so the ratio is 1.99.

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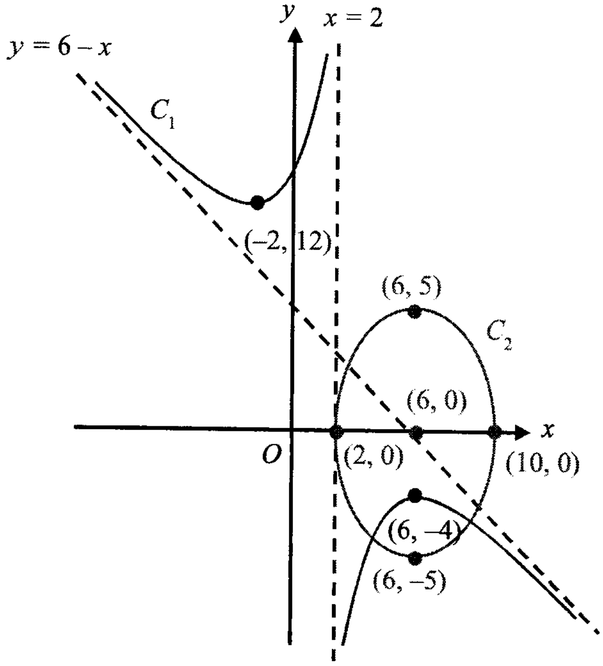
	Solution
1(a)	Differentiating C w.r.t. x, $3x^2+3y^2\frac{\mathrm{d}y}{\mathrm{d}x}-3x\frac{\mathrm{d}y}{\mathrm{d}x}-3y=0$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3y-3x^2}{3y^2-3x}=\frac{y-x^2}{y^2-x}$ When $y=0$, $x^3=k$, $x=k^{\frac{1}{3}}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=x=k^{\frac{1}{3}}$ $y=k^{\frac{1}{3}}\left(x-k^{\frac{1}{3}}\right)=k^{\frac{1}{3}}x-k^{\frac{2}{3}}$
(b)	For tangent to be parallel to y-axis, $\frac{\mathrm{d}y}{\mathrm{d}x}$ must be undefined. Hence, $y^2-x=0$ $x=y^2$ Substituting into C and $k=3$, $y^6+y^3-3y^3-k=0$ $y^6-2y^3-3=0$ Factorising, $\left(y^3-3\right)\left(y^3+1\right)=0$ $y^3=3$ or $y^3=-1$ $y=3^{\frac{1}{3}}$ or $y=-1$

	Solution
2(a)	Let L be the limit of the sequence $u_{n+1}=2u_n\left(1-u_n\right)$. $L=2L\left(1-L\right)$ $2L^2-L=0$ $L\left(2L-1\right)=0$ $L=0$ or $L=\frac{1}{2}$
(b)	When $u_1=0$, $u_2=0$, $u_3=0$, sequence is convergent. When $u_1=0.01$, $u_2=0.0198$ (exact), $u_3=0.0388$ (3 s.f.), sequence is convergent. When $u_1=-0.01$, $u_2=-0.0202$ (exact), $u_3=-0.0412$ (3 s.f.), sequence is not convergent.

	Solution
3(a)	
(b)	x-coordinate of points of intersection are 4.6391, 7.7172 to 5 sf Equation of ellipse: $y^2 = 25\left(1 - \frac{(x-6)^2}{16}\right)$ Required volume $= \pi \int_{4.6391}^{7.7172} 25\left(1 - \frac{(x-6)^2}{16}\right) - \left(\frac{x^2 - 8x + 28}{2 - x}\right)^2 \, dx$ $= 58.7 \text{ units}^3$

	Solution
4(a)	Normal vector of $p = \begin{pmatrix}0\\5\\2\end{pmatrix}\times\begin{pmatrix}5\\0\\t\end{pmatrix}=\begin{pmatrix}5t\\10\\-25\end{pmatrix}=5\begin{pmatrix}t\\2\\-5\end{pmatrix}$ plane p: $\mathbf{r}\bullet\begin{pmatrix}t\\2\\-5\end{pmatrix}=0$ Since l_1 and p are parallel, $\begin{pmatrix}t\\3-t^2\\1\end{pmatrix}\bullet\begin{pmatrix}t\\2\\-5\end{pmatrix}=0$ $t^2+6-2t^2-5=0$ $t^2=1$ $t=1$ or $t=-1$ Since point A is not on p, $\begin{pmatrix}1\\2\\1\end{pmatrix}\bullet\begin{pmatrix}t\\2\\-5\end{pmatrix}\neq 0$ $t+4-5\neq 0$ $t\neq 1$ $\therefore t=-1$ (Shown)
(b)	Let θ be the acute angle between l_1 and l_2 $\theta=\cos^{-1}\left\lvert\frac{1}{\sqrt{6}}\begin{pmatrix}-1\\2\\1\end{pmatrix}\bullet\frac{1}{\sqrt{6}}\begin{pmatrix}1\\2\\1\end{pmatrix}\right\rvert$ $=\cos^{-1}\left\lvert\frac{4}{6}\right\rvert$ $=0.841$ or 48.2°
(c)	$\lvert\mathbf{a}\cdot\mathbf{n}\rvert$ is the perpendicular distance from A to p $\lvert\mathbf{a}\cdot\mathbf{n}\rvert=\left\lvert\begin{pmatrix}1\\2\\1\end{pmatrix}\bullet\frac{1}{\sqrt{30}}\begin{pmatrix}-1\\2\\-5\end{pmatrix}\right\rvert$ $=\left\lvert\frac{-2}{\sqrt{30}}\right\rvert$ $=\frac{2}{\sqrt{30}}$

(d) Let N be the foot of perpendicular from A to p

$$\overrightarrow{ON}=\begin{pmatrix}1\\2\\1\end{pmatrix}-s\begin{pmatrix}-1\\2\\-5\end{pmatrix}=\begin{pmatrix}1+s\\2-2s\\1+5s\end{pmatrix}$$

Since N is on p,

$$\begin{pmatrix}1+s\\2-2s\\1+5s\end{pmatrix}\bullet\begin{pmatrix}-1\\2\\-5\end{pmatrix}=0$$

$$-1-s+4-4s-5-25s=0$$

$$s=-\frac{1}{15}$$

$$\therefore\overrightarrow{ON}=\frac{1}{15}\begin{pmatrix}14\\32\\10\end{pmatrix}$$

Let B be the point of reflection of A in p

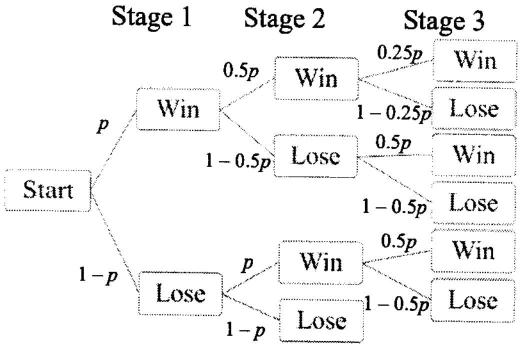
$$\overrightarrow{OB}=2\overrightarrow{ON}-\overrightarrow{OA}$$

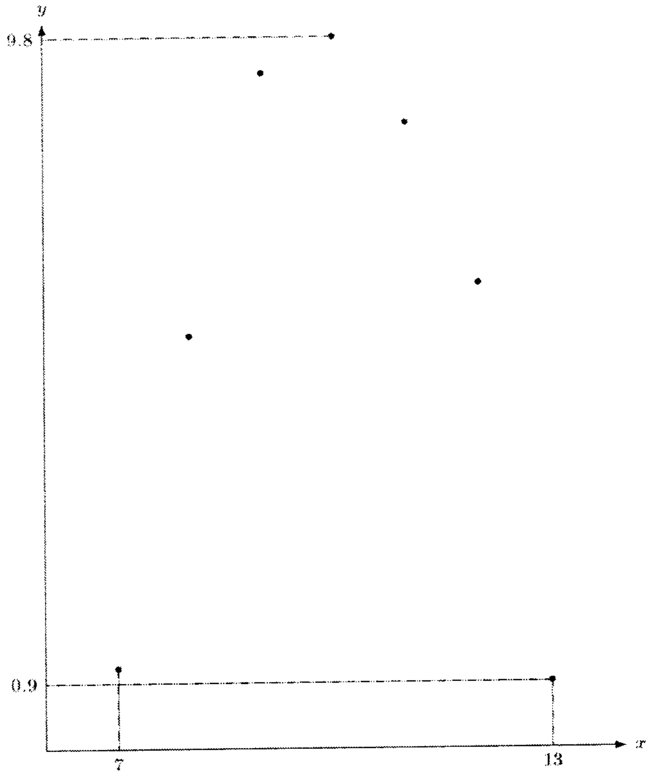
$$=\frac{2}{15}\begin{pmatrix}14\\32\\10\end{pmatrix}-\begin{pmatrix}1\\2\\1\end{pmatrix}$$

$$=\frac{1}{15}\begin{pmatrix}13\\34\\5\end{pmatrix}$$

line of reflection of l_2 in p: $\mathbf{r}=k\begin{pmatrix}13\\34\\5\end{pmatrix},k\in\mathbb{R}$

	Solution
5(a)	Number of ways to choose the 6 glasses = total number of ways – number of ways with 1 missing colour $= \binom{8}{6} - \binom{4}{1} = 24$ Required number of ways $= 24\times(6-1)! = 2880$
(b)	Number of ways = $4\times 4 = 16$
(c)	Let A be the event that the blue glasses are adjacent. Let B be the event that the yellow glasses are adjacent. Then $\mathrm{n}(A') = 4!\times\binom{5}{2}\times 2! = 480$ [Explanation: arrange the four non-blue glasses (4!), then slot in the blue glasses (${}^5C_2\times 2!$)] Also $\mathrm{n}(A'\cap B) = 3!\times 2!\times {}^4C_2\times 2! = 144$ [Explanation: group the two yellow glasses (2!), arrange this group with the red and green glass (3!), then slot in the blue glasses (${}^4C_2\times 2!$)] So the required number of ways is $\mathrm{n}(A') - \mathrm{n}(A'\cap B) = 336$.

	Solution
6(a)	
(b)	$p(0.5p)(0.5^2 p) > 0.1$ $p^3 > 0.8$ $p > 0.928$
(c)	$\mathrm{P}(\text{wins exactly 2 stages out of 3} \mid \text{proceeds to Stage 3})$ $= \dfrac{\mathrm{P}(\text{wins exactly 2 stages out of 3} \cap \text{proceeds to Stage 3})}{\mathrm{P}(\text{proceeds to Stage 3})}$ $= \dfrac{0.9(0.45)(1-0.225)+0.9(0.55)(0.45)+0.1(0.9)(0.45)}{1-(0.1)(0.1)}$ $= \dfrac{0.577125}{0.99} = 0.583$

	Solution
7(a)	$r = 0.00208$
(b)	 There is a non-linear (curved) relationship between x and y.
(c)	$r = -0.997$. This means there is a (very) strong negative linear correlation between y and $(x-10)^2$.
(d)	Equation is $y = -0.987(x-10)^2 + 9.92$ When $x = 17$, $y = -38.4$ This estimate is not valid because $x = 17$ lies outside the valid range of $7 \le x \le 13$

Solution

8(a)

$\mathrm{E}(X-3)=\mathrm{E}(X)-3=4-3=1$

$\mathrm{Var}(X-3)=\mathrm{E}\left((X-3)^2\right)-\left[\mathrm{E}(X-3)\right]^2=40-1=39$

$\mathrm{Var}(X)=\mathrm{Var}(X-3)=39$

Alternatively,

$\mathrm{E}\left((X-3)^2\right)=\mathrm{E}\left(X^2-6X+9\right)=40$

$\mathrm{E}\left(X^2\right)-6\mathrm{E}(X)+9=40$

$\mathrm{E}\left(X^2\right)=40+6\times4-9=55$

$\mathrm{Var}(X)=\mathrm{E}\left(X^2\right)-\left[\mathrm{E}(X)\right]^2=55-16=39$

(b)

y	0	1	2	3
$\mathrm{f}(y)$	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$
$\mathrm{P}(Y=y)$	$\left(\frac{2}{3}\right)^3$	${}^3C_1\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)$	${}^3C_2\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^2$	$\left(\frac{1}{3}\right)^3$
$\mathrm{f}(y)\times\mathrm{P}(Y=y)$	$\frac{8}{27}$	$\frac{2}{9}$	$\frac{2}{27}$	$\frac{1}{108}$

$\mathrm{E}\left[\mathrm{f}(Y)\right]=\sum_{y}\left[\mathrm{f}(y)\times\mathrm{P}(Y=y)\right]$

$=\frac{8}{27}+\frac{2}{9}+\frac{2}{27}+\frac{1}{108}$

$=\frac{65}{108}$ (or 0.602 to 3 s.f.)

(c)

Since sum of probabilities is 1,

$p+q+0.15+r=1$

$p+q+r=0.85$ (1)

From $\mathrm{E}(W)=-0.2$,

$-3p-q+0.15+2r=-0.2$

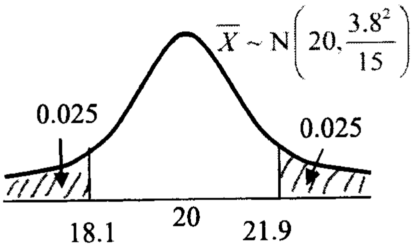
$-3p-q+2r=-0.35$ (2)

From $\mathrm{E}\left(W^3\right)=-3.2$,

$-27p-q+0.15+8r=-3.2$

$-27p-q+8r=-3.35$ (3)

From GC, solving simultaneous linear equations, $p=0.2, q=0.35, r=0.3$

	Solution
9(a)	The probability that any student is selected is the same, and the selection of any student is independent of the selection of other students.
(b)	Let X be the queue time at the canteen during peak hours of a randomly chosen EJC student and let μ be the population mean. $\text{H}_0 : \mu = 20, \quad \text{H}_1 : \mu \neq 20$ Assume that the queue times are normally distributed. Under H_0, $\bar{X} \sim \text{N}\left(20, \frac{3.8^2}{15}\right)$ From GC,  Given that H₀ is not rejected at 5% level of significance, $18.1 < \bar{x} < 21.9$
(c)	$\bar{t} = \frac{\sum(t-20)}{50} + 20 = 19.3$ $s^2 = \frac{1}{n-1}\left(\sum(t-20)^2 - \frac{\left(\sum(t-20)\right)^2}{n}\right)$ $= \frac{1}{49}\left(489 - \frac{(-35)^2}{50}\right)$ $= \frac{929}{98}$ or $9.4796 \approx 9.48$ (3 s.f.)
(d)	Let μ be the population mean (of T). $\text{H}_0 : \mu = 20, \quad \text{H}_1 : \mu < 20$ Under H_0, since sample size n=50 is large, by Central Limit Theorem, $\bar{T} \sim \text{N}\left(20, \frac{9.4796}{50}\right)$ approximately. At the 10% level of significance, the critical region is $\bar{t} < 19.442$. From data, $\bar{t} = 19.3 < 19.442$ Hence, we reject H₀ and conclude at 10% level of significance that there is sufficient evidence that EuOrder has reduced the queue time at the canteen.

<table>
<tr><th></th><th>Solution</th></tr>
<tr><td>10(a)</td><td>Let X be the mass of a randomly chosen apple, in grams. Then $X \sim \mathrm{N}\left(100, 6^2\right)$.

$\mathrm{P}(X \le 105) = 0.79767 = 0.798 \quad (3 \text{ s.f.})$</td></tr>
<tr><td>(b)</td><td>Let Y be the mass of a prepared apple, in grams.
$\mathrm{E}(Y) = 0.7 \times 100 = 70$
$\mathrm{Var}(Y) = 0.7^2 \times 6^2 = 17.64$
Then, $Y \sim \mathrm{N}(70, 17.64)$.

$\mathrm{P}(Y > 78) = 0.028405 = 0.0284 \quad (3 \text{ s.f.})$</td></tr>
<tr><td>(c)</td><td>Let T be the total mass of 2 prepared apples, 1 peeled banana, 12 blackcurrants and milk, in grams.
$\mathrm{E}(T) = 2(70) + 130 + 12(2.5) + 50 = 350$
$\mathrm{Var}(T) = 2(17.64) + 8^2 + 12\left(0.6^2\right) = 103.6$
$T \sim \mathrm{N}(350, 103.6)$

$\mathrm{P}(T \ge m) = 0.9$
Using GC, $m = 336.96 = 337 \quad$ (nearest gram)</td></tr>
<tr><td>(d)</td><td>$\mathrm{P}(330 < T < 360) = 0.81236$
Let R be the number of Signature smoothies, out of 25, with masses between 330 and 360 grams.
Then, $R \sim \mathrm{B}(25, 0.81236)$
$\mathrm{P}(R \ge 20) = 1 - \mathrm{P}(R \le 19) = 0.677 \quad (3 \text{ s.f.})$</td></tr>
<tr><td>(e)</td><td>Let W be the total mass of n blackcurrants and milk, in grams.
$\mathrm{E}(W) = n(2.5) + 175 = 2.5n + 175$
$\mathrm{Var}(W) = n\left(0.6^2\right) = 0.36n$
$W \sim \mathrm{N}(2.5n + 175, 0.36n)$

$\mathrm{P}(W > 300) \ge 0.95$
Using GC,
<table>
<tr><th>n</th><th>$\mathrm{P}(W > 300)$</th></tr>
<tr><td>52</td><td>0.8761</td></tr>
<tr><td>53</td><td>0.957</td></tr>
</table>
Hence, the least value of $n = 53$.</td></tr>
</table>