EUNOIA JUNIOR COLLEGE
JC2 Preliminary Examination 2025
General Certificate of Education Advanced Level
Higher 2

CANDIDATE NAME			
CIVICS GROUP		INDEX NO.	

MATHEMATICS

Paper 1

9758/01

01 September 2025

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

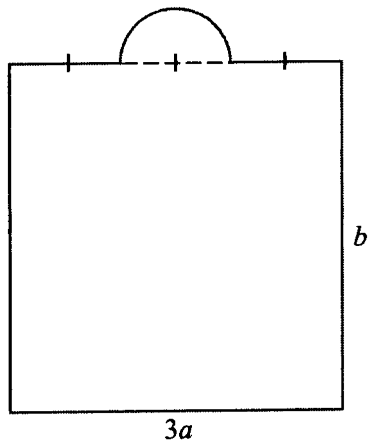
1 **(a)** Solve the inequality

$$x+1>\frac{1}{x-1}$$

giving your answer in exact form. [3]

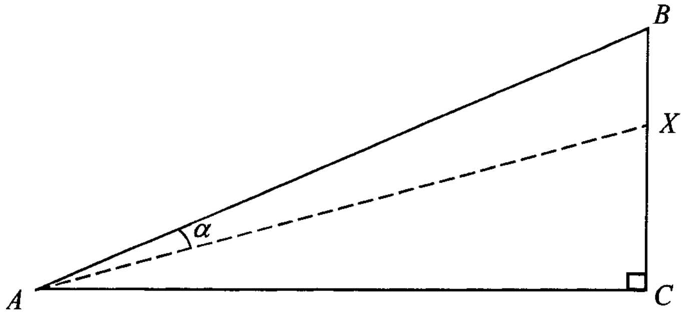
(b) Hence solve, in exact form, $e^x+1>\frac{1}{e^x-1}$. [2]

2 Pete has 180 cm of wire. He bends it to form the outline of his brand logo, which consists of a semicircle centered on top of a rectangle as shown in the diagram below. The width and length of the rectangle are $3a$ cm and b cm respectively. The diameter of the semicircle is one-third the width of the rectangle.



Find the maximum possible area enclosed by the wire, showing that it is a maximum value. Give your answer correct to 2 decimal places. [6]

3 In the right-angled triangle ABC, angle C is a right angle. $AB = 13$ cm, $BC = 5$ cm and X is a point on BC such that angle BAX is α radians.



(a) Find the exact value of $\cos\angle BAC$. [1]

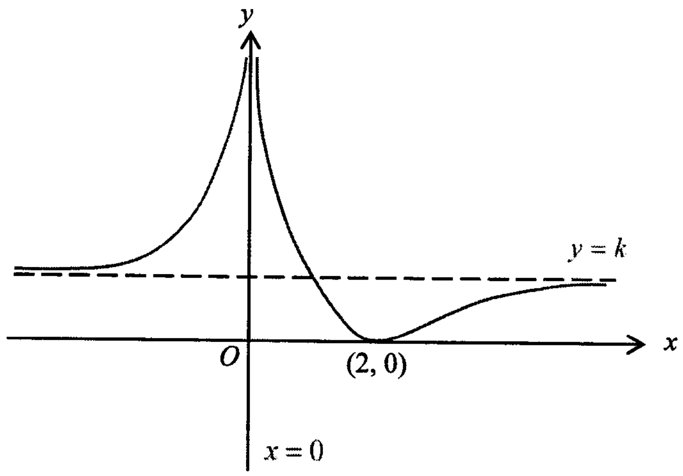
(b) By considering triangle AXC, or otherwise, show that $AX = \dfrac{156}{12\cos\alpha + 5\sin\alpha}$. [3]

(c) Given that α is a sufficiently small angle, show that

$$AX \approx 13 + p\alpha + q\alpha^2,$$

where p and q are constants to be determined exactly. [4]

4 The diagram below shows the graph of $y = \mathrm{f}(2x)$. The graph has a turning point at $(2, 0)$, and asymptotes with equations $x = 0$ and $y = k$.



(a) State a single transformation that will transform the graph of $y = \mathrm{f}(2x)$ onto the graph of $y = \mathrm{f}(2x+4)$. Hence sketch the graph of $y = \mathrm{f}(2x+4)$. [3]

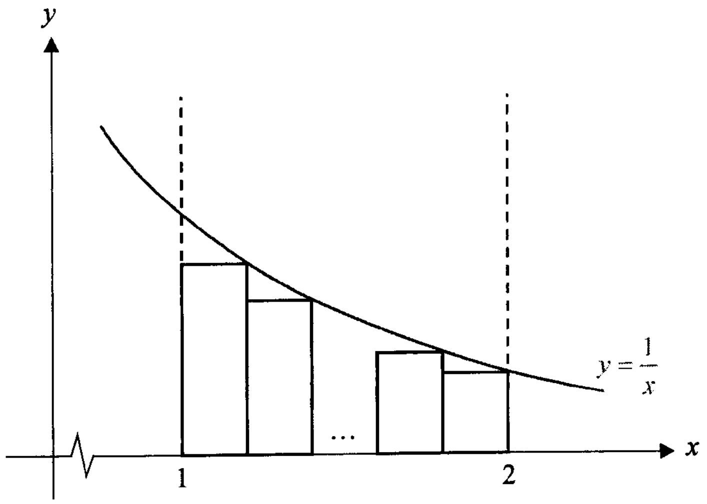
On separate clearly labelled diagrams, sketch the graphs of

(b) $y = \dfrac{1}{\mathrm{f}(2x)}$, [3]

(c) $y = -\mathrm{f}(|x|)$. [3]

5 The diagram shows the curve with equation $y = \frac{1}{x}$.

n rectangles of equal width are drawn under the curve between $x = 1$ and $x = 2$.



Let S_n be the **total** area of the n rectangles.

(a) Let $n = 2$. By considering S_2, show that $\frac{7}{12} < \ln 2$. State your reasoning clearly. [3]

(b) By considering S_n for a suitable value of n, find a rational number q such that $\frac{7}{12} < q < \ln 2$. [2]

(c) Express S_n in the form

$$\sum_{r=1}^{n} g(r),$$

where the function g is to be determined. [1]

(d) By considering another suitable set of n rectangles, show with the aid of a diagram that

$$\ln 2 < S_n + \frac{1}{2n}$$

where n is any given positive integer. [3]

6 There are three distinct points A, B and P. The point P lies on the circle with centre O and diameter AB. Relative to the origin O, the position vectors of points A and P are **a** and **p** respectively.

(a) By expressing $\overrightarrow{AP}$ and $\overrightarrow{BP}$ in terms of **a** and **p**, show that $\overrightarrow{AP}.\overrightarrow{BP}=0$. [3]

The point C with position vector **c** lies on AB produced such that $AC:AB$ is $\lambda:1$.

(b) Find **c** in terms of λ and **a**. [2]

(c) It is given that PC is a tangent to the circle and that angle AOP is $120°$. Using a suitable scalar product, find the value of λ. [4]

7 **Do not use a calculator in answering this question.**

An arithmetic sequence $u_1, u_2, u_3, \ldots$ has $u_1=9$ and $u_3=b$, where b is a constant.

(a) Find u_2 in terms of b. [2]

A geometric sequence $v_1, v_2, v_3, \ldots$ has $v_1=9$ and $v_3=b$.

(b) Find the possible values of v_2 in terms of b. [2]

It is now given that $u_2-v_2=8$, and the geometric sequence $v_1, v_2, v_3, \ldots$ is convergent.

(c) Find the value of b.
Hence find u_2 and v_2. [5]

8 **Do not use a calculator in answering this question.**

The complex number w is such that $w^2=2\text{i}$ and $0\le\arg(w)\le\frac{\pi}{2}$.

(a) Find w. [3]

(b) Given that one of the roots of the equation $z^3-z^2+(1-\text{i})z+s=0$ is w, find the other roots of the equation and the value of s. [5]

(c) Hence, find the roots of the equation $-\text{i}v^3+v^2+(1+\text{i})v+s=0$. [2]

9 **(a)** **(i)** State the coordinates of the turning points of the graph of $y = x^3 - 3x^2 - 9x + 5$. [1]

(ii) The function f is defined by

$$f(x) = x^3 - 3x^2 - 9x + 5,\ x \in \mathbb{R},\ x \leq a$$

where a is a constant. Find the largest possible value of a such that f^{-1} exists, and state the domain of f^{-1} in this case. [2]

(b) The function g is defined by

$$g(x) = \frac{1-2x}{x+2},\ x \in \mathbb{R},\ x \neq -2.$$

(i) Find $g^2(x)$. [2]

(ii) Hence find the possible expressions of $g^n(x)$, where n is a positive integer. [2]

(c) The function h is defined by

$$h(x) = x^2,\ x \in \mathbb{R},\ x \leq 0.$$

(i) Explain why the composite function gh exists. [2]

(ii) Find the range of gh. [2]

10 **(a)** Find $\int_0^4 e^{|x-1|}\,dx$, leaving your answer in exact form. [4]

(b) Find $\int \sin^2 3x + \tan^2 3x\ dx$. [3]

(c) Use the substitution $x = u^6$, where $u > 0$, to find $\int \frac{1}{\sqrt{x} + \sqrt[3]{x}}\,dx$. [5]

11 A cylindrical container of height h metres is originally completely filled with water. The water is flowing out through an opening at the base of the container into a pipe. The rate of flow of water through the opening at time t seconds is proportional to the square root of the height of the water in the container, y metres. Since the container is cylindrical, the amount of water in the container is also proportional to the height of water, so the differential equation relating y and t can be written as

$$\frac{dy}{dt} = -k\sqrt{y}$$

where k is a positive constant.

(a) Solve this differential equation to find $\sqrt{y}$ in terms of t, h and k. [4]

(b) Hence find the time taken for the container to empty, in terms of h and k. [2]

It is found that the water was draining too fast. To regulate the flow of water, a mechanical device is placed at the opening. This device slows the flow of water into the pipe by covering all or part of the opening, where the area covered varies with time. It is proposed to model this situation using the revised differential equation

$$\frac{dy}{dt} = -k\left(\frac{1+\cos t}{2}\right)\sqrt{y},$$

where k is the same constant as before.

(c) With reference to the possible values of $\cos t$, explain how the revised differential equation corresponds to a model with slower flow of water. [1]

(d) Given again that the container is originally completely filled with water, solve the revised differential equation to find $\sqrt{y}$ in terms of t, h and k. [3]

(e) In the case where $h = 10$ and $k = 0.1$, find the ratio of the time taken in the revised model to the time taken in the original model for the container to empty. Give your answer to 3 significant figures. [2]

END OF PAPER

EUNOIA JUNIOR COLLEGE

JC2 Preliminary Examination 2025

General Certificate of Education Advanced Level

Higher 2

CANDIDATE NAME

CIVICS GROUP

INDEX NO.

MATHEMATICS

9758/02

Paper 2

18 September 2025

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae (MF27)

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This document consists of **7** printed pages and **1** blank page.

Section A: Pure Mathematics [40 marks]

1 The curve C has equation $x^3 + y^3 - 3xy = k$, where k is a positive constant.

C intersects the x-axis at the point A.

(a) Find the equation of the tangent to C at A. Give your answer in the form $y = px + q$, where p and q are constants to be found in terms of k. [5]

(b) There are two points on C where the tangents are parallel to the y-axis. Given that $k = 3$, find the exact y-coordinates of these two points. [3]

2 A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = 2u_n(1 - u_n)$$

for all positive integers n.

(a) Find the possible limits of the sequence, if the sequence is convergent. [2]

(b) For each of the following, evaluate u_2 and u_3, and state whether the sequence is convergent:

- $u_1 = 0$,
- $u_1 = 0.01$,
- $u_1 = -0.01$. [6]

3 The curve C_1 has equation $y = \dfrac{x^2 - 8x + 28}{2 - x}$.

The curve C_2 has equation $\dfrac{(x-6)^2}{16} + \dfrac{y^2}{25} = 1$.

(a) Sketch C_1 and C_2 on the same diagram, stating the coordinates of any vertices, turning points, intersections with the x-axis and the equations of any asymptotes. [7]

(b) Find the volume of solid obtained when the smaller region bounded by C_1 and C_2 is rotated through 2π radians about the x-axis. [4]

4 Relative to the origin O, point A has position vector given by $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$. The line l_1 passes through the point A and is parallel to $\begin{pmatrix} t \\ 3-t^2 \\ 1 \end{pmatrix}$. The plane p has equation $\mathbf{r} = \lambda \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ t \end{pmatrix}$, $\lambda, \mu \in \mathbb{R}$ where t is a real constant. It is known that l_1 and p are parallel and l_1 is not on p.

(a) Show that $t = -1$. [4]

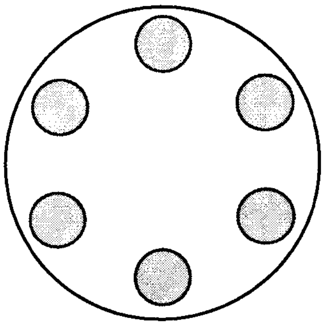
(b) The line l_2 passes through the origin and point A. Find the acute angle between l_1 and l_2. [2]

(c) The vector $\mathbf{n}$ is a unit vector normal to p. State the geometrical meaning of $|\mathbf{a}.\mathbf{n}|$ and find the exact value of $|\mathbf{a}.\mathbf{n}|$. [3]

(d) Find a vector equation of the line of reflection of l_2 in p. [4]

Section B: Probability and Statistics [60 marks]

5 Mary is setting up a tabletop display using 6 glasses, arranged around a circular tray, as shown in the diagram below. There are 4 different colours of glasses – Red, Blue, Green and Yellow. For each colour, Mary owns one tall and one short glass, so she has 8 glasses in total.



(a) In how many ways can Mary arrange 6 of the 8 glasses around the circular tray, such that at least one glass of each colour is used? [3]

(b) Mary decides to give away one tall glass and one short glass to support a charity drive. In how many ways can she choose the two glasses to give away? [1]

(c) Mary decides to give away the red tall glass and green short glass. She now wants to arrange the remaining 6 glasses in a straight row, such that no two adjacent glasses are of the same colour. Find the number of different arrangements. [3]

6 An online game consists of three stages. All players start with Stage 1, and no matter whether they win or lose, they proceed to Stage 2. Players are allowed to proceed to Stage 3 only if they have won at least one of the first two stages.

Ming's probability of winning Stage 1 is p.

If Ming wins a stage, his probability of winning the next stage is halved. If he loses a stage, his probability of winning remains unchanged in the next stage.

(a) Draw a probability tree diagram to represent all possible outcomes for Ming over the three stages. [3]

(b) Find the range of possible values of p if the probability that Ming wins all three stages is more than 0.1. [2]

(c) It is now known that $p = 0.9$. Given that Ming proceeds to Stage 3, find the probability that he wins exactly two stages out of the three. [4]

7 An investigation into the relationship between two variables x and y results in the following data.

x	7	8	9	10	11	12	13
y	1.1	5.7	9.3	9.8	8.6	6.4	0.9

(a) Calculate the product moment correlation coefficient between x and y. [1]

(b) Draw a scatter diagram of the data and comment on the relationship between x and y based on the scatter diagram. [3]

Several possible models for the relationship between x and y are proposed; one is chosen for further investigation.

(c) Calculate the product moment correlation coefficient between y and $(x-10)^2$, and comment on its value. [2]

(d) Find the equation of the regression line of y on $(x-10)^2$. Use the equation of the regression line to estimate the value of y when $x=17$ and comment on the reliability of this estimate. [3]

8 **(a)** X is a random variable such that $\mathrm{E}(X)=4$ and $\mathrm{E}\left[(X-3)^2\right]=40$. Find the value of $\mathrm{Var}(X)$. [3]

(b) Let Y be a discrete random variable. Let f be a function which is defined for all values that Y can take. Then $\mathrm{f}(Y)$ is also a random variable, and its expectation is given by

$$\mathrm{E}[\mathrm{f}(Y)]=\sum_y[\mathrm{f}(y)\times\mathrm{P}(Y=y)].$$

Suppose now that $Y\sim\mathrm{B}\left(3,\frac{1}{3}\right)$ and $\mathrm{f}(t)=\frac{1}{t+1}$. Find the value of $\mathrm{E}[\mathrm{f}(Y)]$. [3]

(c) W is a random variable with the following probability distribution:

w	-3	-1	1	2
$\mathrm{P}(W=w)$	p	q	0.15	r

It is given that $\mathrm{E}(W)=-0.2$ and $\mathrm{E}(W^3)=-3.2$. Find the values of p, q and r. [4]

9 The students in Eunoia Junior College (EJC) have been complaining of long queue times at the canteen during peak hours. It was claimed that the average queue time per student is 20 minutes. It is known that the standard deviation of the queue times is 3.8 minutes.

The admin manager of EJC wishes to test if the average queue time is in fact 20 minutes. She examines a random sample of 15 students to determine the average queue time.

(a) State what it means for a sample to be random in this context. [1]

(b) Given that the admin manager concludes that there is no reason to reject the null hypothesis at the 5% level of significance, find the range of possible values of the sample mean, and state an assumption needed for your calculations. [4]

To reduce the queue time, a lunch pre-order system, EuOrder, was introduced. The admin manager is tasked to find out if there is any improvement to the average queue time per student. She obtained a random sample of 50 students and recorded their queue times, t minutes, on a particular day. The results are summarised as follows:

$$\sum(t-20)=-35 \quad \sum(t-20)^2=489.$$

(c) Find unbiased estimates of the population mean and variance. [2]

(d) Test, at the 10% level of significance, the claim that EuOrder has reduced the average queue time at the canteen. [4]

10 **In this question you should state the parameters of any distributions you use.**

A drinks stall specialises in fruit smoothies, which contain apples. The apples used by the stall have masses, in grams, that follow the distribution $N(100, 6^2)$.

(a) Find the probability that the mass of a randomly chosen apple is no more than 105 grams. [1]

Before the apples can be blended, they need to be prepared by peeling them and removing their cores. This preparation process reduces the mass of each apple by 30%.

(b) Find the probability that the mass of a randomly chosen apple, after preparation, is more than 78 grams. [3]

The stall sells its Signature smoothie, which uses 2 prepared apples, 1 peeled banana and 12 blackcurrants; all of these are randomly chosen. The stall also adds a fixed 50 grams of milk before blending everything together. You should ignore any changes in mass from the blending process.

- The masses of peeled bananas, in grams, follow the distribution $N(130, 8^2)$.
- The masses of blackcurrants, in grams, follow the distribution $N(2.5, 0.6^2)$.

(c) 90% of the Signature smoothies have a mass of at least m grams. Find the value of m. [4]

(d) A teacher purchases 25 Signature smoothies for her form class. Find the probability that at least 20 of the Signature smoothies have masses between 330 and 360 grams. [3]

The stall also makes blackcurrant smoothies using n blackcurrants and a fixed 175 grams of milk.

(e) Find the smallest value of n such that the probability of the mass of a blackcurrant smoothie exceeding 300 grams is at least 0.95. [3]

END OF PAPER