Name:____________________________ Class:__________

JURONG PIONEER JUNIOR COLLEGE

JC2 Preliminary Examination 2025

MATHEMATICS
Higher 2

Paper 1

9758/01
3 Sept 2025
3 hours

Additional materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given by [ ] at the end of each question or part question.

This document consists of **7** printed pages and **1** blank page.

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1 The curve C has equation $y = \frac{1}{4x - x^2}$.

(i) Find $\frac{dy}{dx}$. Hence, find the x-coordinate, $x = x_1$, of the turning point on C and determine its nature. [3]

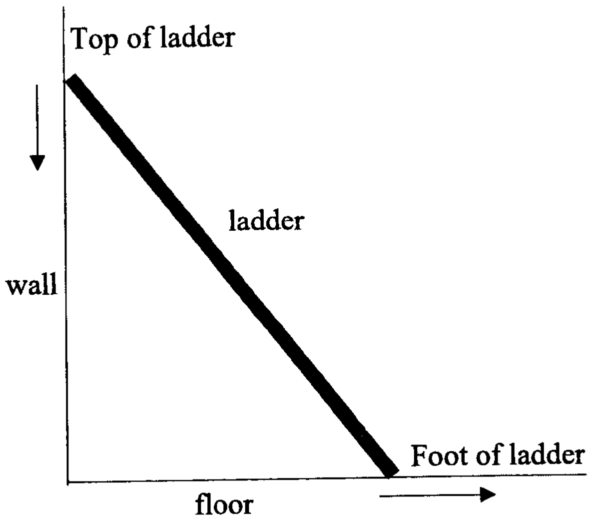
(ii) Using calculus, find the exact area of the region between C, the x-axis and the lines with equations $x = 1$ and $x = x_1$. [3]

2 (i) Find, in terms of a, the roots of the equation $\frac{1}{(x-a)^2} = |x - a|$. [3]

(ii) On the same axes, sketch the curves with equations $y = \frac{1}{(x-a)^2}$ and $y = |x - a|$, where $a > 1$.

Hence solve the inequality $\frac{1}{(x-a)^2} > |x - a|$. [3]

3



A ladder of length 3.12 m is sliding down a vertical wall such that the foot of the ladder is moving along the floor at a constant rate of 0.2 m/s (see diagram). Find the rate at which the top of the ladder is sliding down the wall when it is 1.2 m above the floor. [4]

4 Functions f and g are defined by

$$\mathrm{f}: x \mapsto [\ln(x-1)]^2 + 2, \quad x \geq a,$$
$$\mathrm{g}: x \mapsto 4 + 3x - x^2, \quad x \leq \frac{3}{2}.$$

(i) It is given that the function f^{-1} exists. State the smallest value of a. [1]

(ii) Find an expression for $\mathrm{g}^{-1}(x)$, stating its domain. [3]

Using the value of a found in part **(i)**,

(iii) determine whether the composite function $\mathrm{g}^{-1}\mathrm{f}^{-1}$ exists. [1]

5 (a) The curves C_1 and C_2 have equations $\frac{x^2}{9} - \frac{y^2}{4} = 1$ and $y^2 + x^2 = k^2$ respectively, where k is a positive constant.

(i) Sketch C_1 and C_2 on the same diagram, stating the coordinates of any points of intersection with the axes and the equations of any asymptotes. [3]

(ii) State the range of values of k for C_1 and C_2 to intersect. [1]

(iii) State the equations of the common lines of symmetry for both C_1 and C_2. [1]

(b) The function f, with domain the set of all real values, is given by

$$\mathrm{f}(x) = \begin{cases} -2x + 6 & \text{for } 0 < x \leq 3, \\ 3x - 9 & \text{for } 3 < x \leq 5, \end{cases}$$

and that $\mathrm{f}(x) = \mathrm{f}(x+5)$.

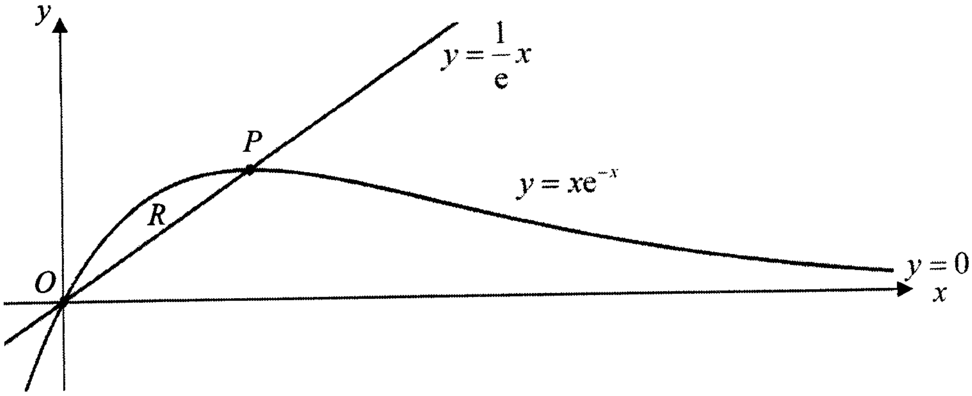
(i) Find $\mathrm{f}(46)$. [1]

(ii) Sketch the graph of $y = \mathrm{f}(x)$ for $-5 \leq x \leq 5$. [2]

(iii) Hence, state the roots of $\mathrm{f}(-x) = 0$ for $-5 \leq x \leq 5$. [1]

6 (i) Find $\int x^2 e^{-2x}\, dx$. [4]

(ii) The curve with equation $y = xe^{-x}$ and the line with equation $y = \frac{1}{e}x$ meet at the origin O and the point P with x-coordinate 1. The region R is bounded by the curve and the line (see diagram).



Find the exact volume of the solid formed when R is rotated through 360° about the x-axis. [4]

7 (a) (i) Find $\int \frac{x}{\sqrt{25-x^2}}\, dx$. [2]

(ii) Hence, given that $\int_{\alpha}^{4} \left| \frac{x}{\sqrt{25-x^2}} \right| dx = 3$, where $\alpha < 0$, find α algebraically. [3]

(b) Using the substitution $x = 4\tan\theta$, evaluate $\int_0^4 \sqrt{\frac{x^2}{16+x^2}}\, dx$ exactly. [5]

8 (a) The first three terms of a sequence are given by $u_1 = 9$, $u_2 = 27$ and $u_3 = 55$. Given that u_n is a quadratic polynomial in n, find u_n in terms of n. [3]

(b) It is given $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$ and $u_r = 2r^3 + 5$.

(i) Find $\sum_{r=1}^{n} u_r$. [2]

(ii) Hence, or otherwise, find $\sum_{r=2}^{n} \left(2(r+2)^3 + 5\right)$. [3]

(iii) Explain why the series $\sum_{r=1}^{\infty} u_r$ does not converge. [1]

9 It is given that $f(x)=\dfrac{1}{4+9x^2}$.

(i) Find $\int f(x)\,dx$. [2]

(ii) Find the binomial expansion for f(x), up to and including the term in x^4. Give the coefficients as exact fractions in their simplest form. [2]

(iii) Hence, find the Maclaurin series for $\tan^{-1}\dfrac{3x}{2}$. Give the coefficients as exact fractions in their simplest form. [3]

(iv) Use your series from part **(iii)** to estimate $\int_0^{0.5}\tan^{-1}\dfrac{3x}{2}\,dx$, correct to 3 decimal places. [1]

(v) Use your calculator to find $\int_0^{0.5}\tan^{-1}\dfrac{3x}{2}\,dx$, correct to 3 decimal places. [1]

(vi) Comparing your answers to parts **(iv)** and **(v)**, comment on the accuracy of your estimate in **(iv)** and how it can be improved. [2]

10 (a) Show that $y=\ln\left(\dfrac{e^2}{3x}\right)$ can be written in the form $y=a+b\ln(cx)$, where a, b and c are integers to be found. Hence, state a sequence of transformations which transform the graph of $y=\ln x$ onto the graph of $y=\ln\left(\dfrac{e^2}{3x}\right)$. [4]

(b) The curve $y=f(x)$ passes through the point P with coordinates (a, b), where $b\neq 0$. The tangent to the curve at P has gradient 5. When $y=f(x)$ is transformed onto the curve $y=g(x)$, P corresponds to the point R on $y=g(x)$. For each of the following curves, state the coordinates of R and find the gradient of the curve at R.

(i) $g(x)=2f(x-1)$ [3]

(ii) $g(x)=\dfrac{1}{f(x)}$ [2]

11 A company posted a video on a social media platform to advertise a new product. The video is uploaded at the start of 1st April and the number of daily views is recorded at the end of each day. Let u_n, where $n \geq 1$, denotes the number of daily views recorded each day. It is given that

$$u_{n+1} = (1+k)u_n,$$

where k is a positive constant.

(i) Explain why the sequence $\{u_n\}$ is a geometric progression. [1]

(ii) Given that the number of daily views recorded at the end of 1st April is 311 and the total number of daily views recorded from 1st April to 3rd April is 4043. Find the value of k. [3]

(iii) Explain why there is no limit to the total number of daily views in the long run. [1]

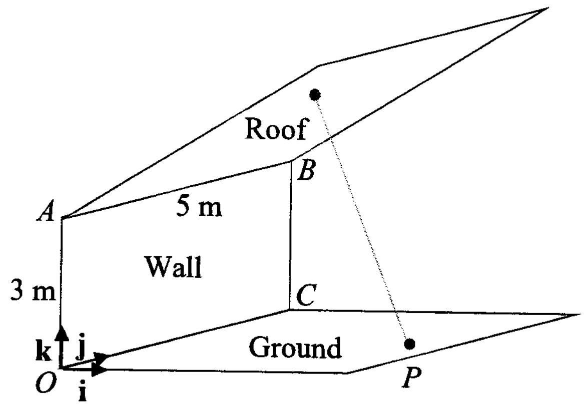
The company also looks at the number of comments being posted on the social media platform. The number of daily comments is recorded at the end of each day. It is given that v_r, where $r \geq 1$, denotes the number of daily comments recorded and it is defined by the following relation:

$$v_r = \begin{cases} u_r & \text{for } 1 \leq r \leq 4, \\ v_{r-1} + 80 & \text{for } r \geq 5. \end{cases}$$

(iv) Show that the total number of comments up to the rth day, where $r \geq 5$, is $40r^2 + 8117r - t$, where t is a constant to be determined. [3]

(v) Find the least number of days required for the total number of daily comments to exceed 100 000. [3]

12 A designer wants to construct an inclined roof that is attached to a vertical rectangular wall, positioned above a horizontal flat ground, as shown in the diagram below. Assume that the roof and wall are of negligible thickness.



Points (x, y, z) are defined relative to a corner of the wall at $O(0,0,0)$, where units are metres. Unit vectors $\mathbf{i}, \mathbf{j}$ and $\mathbf{k}$ are in the directions shown in the diagram. The other corners of the wall are *A*, *B* and *C*, such that *AB* is 5 m and *A* and *B* are 3 m vertically above *O* and *C* respectively. The roof is attached to the wall along *AB* and is modelled by a plane parallel to $4\mathbf{i}+5\mathbf{j}+\mathbf{k}$.

(i) Show that the roof is defined by the plane $x-4z=-12$. [3]

(ii) Find the obtuse angle between the wall and the roof, correct to the nearest degree. [3]

To support the roof, a wooden strut, of negligible thickness, is constructed. It has one end at point *P* with coordinates $(4,3,0)$ on the horizontal ground and the other end at the roof, such that the length of the wooden strut is a minimum.

(iii) Find the exact length of the wooden strut. [2]

(iv) A ray of light is emitted in the direction $7\mathbf{i}-4\mathbf{j}+4\mathbf{k}$ from a light source at *C*. Show that this light ray meets the wooden strut. Hence, find the coordinates of the point where the light ray meets the wooden strut. [4]

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Section A: Pure Mathematics [40 Marks]

1 To prepare for the General Election, a political party studies the effectiveness of spreading information on Instantglam, a social media platform, by uploading a particular video. The number of Instantglam users who have viewed the video increases at a rate that is directly proportional to the number of Instantglam users who have yet to view the video. The number of Instantglam users who have viewed the video t hours after the video is uploaded is x. The total number of Instantglam users is P, assumed to be fixed.

(i) Write down a differential equation and show that $x = P\left(1 - \mathrm{e}^{-kt}\right)$, where k is a positive constant. [4]

It is known that 12 hours after the video is uploaded, half the total number of Instantglam users have viewed the video.

(ii) Estimate to the nearest hour, the time needed for 80% of the total number of Instantglam users to have viewed the video. [3]

(iii) Sketch the graph of x against t. [2]

2 With reference to the origin O, the points A, P and Q have position vectors $\mathbf{a}$, $\mathbf{p}$ and $\mathbf{q}$ respectively. A straight line l through the point A is parallel to a unit vector $\mathbf{e}$.

(i) The point Q lies on l. Show that $(\mathbf{q} - \mathbf{a}) \times \mathbf{e} = \mathbf{0}$. [2]

(ii) Given that P is not on l, give the geometrical meaning of $|(\mathbf{p} - \mathbf{a}) \times \mathbf{e}|$. [1]

It is also given that $\mathbf{p} = 3\mathbf{q}$ and AQ is 2 units.

(iii) Using the results in **(i)** and **(ii)**, or otherwise, show that the area of triangle APQ can be written as $k|\mathbf{q} \times \mathbf{e}|$, where k is a constant to be determined. [4]

(iv) Hence, or otherwise, find the acute angle between l and PQ, given further that the area of triangle APQ is 3 units^2 and $|\mathbf{q}| = 2$. [3]

3 The curve C has parametric equations

$$x = \sin\theta + 1 \quad \text{and} \quad y = \sqrt{3}\cos\theta - 1\text{, where } -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}.$$

(i) Show that $\frac{dy}{dx} = -\sqrt{3}\tan\theta$. [2]

(ii) The tangent to C at point $T\left(\sin t + 1, \sqrt{3}\cos t - 1\right)$ makes an angle of $\frac{3\pi}{4}$ with the positive x-axis. Find the equation of the normal to C at T. [3]

(iii) Find the exact y-coordinate of the point on C where the tangent to C is parallel to the x-axis. [2]

(iv) Find the area of the quadrilateral bounded by the axes, the normal to C at T and the tangent in **(iii)**. [3]

4 **Do not use a calculator in answering this question.**

(a) One of the roots of the equation $5w^3 + pw^2 + 68w + q = 0$, where p and q are real, is $3 - \mathrm{i}$. Find the other roots of the equation and the values of p and q. [5]

(b) Two complex numbers are given by $z_1 = -1 + 2\mathrm{i}$ and $z_2 = 2 + \mathrm{i}$. Draw an Argand diagram showing z_1 and z_2, labelling the origin as O and the points representing z_1 and z_2 as Z_1 and Z_2 respectively. Given that $z_3 = z_1 + z_2$, mark the corresponding point Z_3 on your Argand diagram, showing clearly the geometrical relationship between Z_1, Z_2 and Z_3. [2]

(i) Find $\frac{z_1}{z_2}$ in the form $k\mathrm{i}$. [2]

(ii) Hence, or otherwise, show that $OZ_2Z_3Z_1$ is a square. [2]

Section B: Probability and Statistics [60 Marks]

5 In a strategy game called *Colour Clash*, players draw coloured orbs from a magical pouch to determine their elemental powers. The pouch contains:

- 5 red orbs (Level 1 to 5),
- 4 blue orbs (Level 1 to 4),
- 3 green orbs (Level 1 to 3).

Orbs of the same colour are non-identical with each same-coloured orb representing a different level.

A player randomly draws four orbs without replacement at the start of the game. Given that the order in which the orb is drawn is not relevant, find the number of ways this can be done such that

(i) there are no restrictions, [1]

(ii) there are at least two colours present, [2]

(iii) at least one orb of each colour is drawn. [3]

6 A particular streaming service platform claims that the average time a user spends watching content on the platform per visit is more than 15 minutes. A team of data analysts for the platform wants to verify this claim. They randomly select a sample of 80 users and record the amount of time each user spends watching content in a single visit. The sample mean of time spent is found to be 16 minutes. The population standard deviation of time spent is assumed to be σ minutes.

(i) Explain why the team should carry out a 1-tail test. [1]

A test at the 5% significance level indicates that the platform's claim is valid.

(ii) Explain why the team is able to carry out a hypothesis test without knowing anything about the distribution of the times spent by the users. [1]

(iii) Find the range of values of σ. [4]

(iv) Explain, in the context of the question, the meaning of 'at the 5% significance level'. [1]

7 In a game, a player selects one ball at random from a bag which contains four balls numbered '1', '1', '2' and '3'. The player notes the number on the selected ball, and then tosses that number of fair coins. The number of heads obtained is denoted by X and has a probability distribution as follows:

x	0	1	2	3
$P(X=x)$	a	$\frac{15}{32}$	b	$\frac{1}{32}$

(i) Find the exact values of a and b. [3]

(ii) Find $E(X)$ and show that $\text{Var}(X)=\frac{39}{64}$. [2]

(iii) The player plays 50 games. Find the probability that the average number of heads obtained per game exceed 1. [2]

8 A company makes portable speakers. Some speakers turn out to be faulty. Each carton has 24 portable speakers.

(i) State, in context, two assumptions needed for the number of faulty portable speakers in a randomly chosen carton to be well modelled by a binomial distribution. [2]

It is given that 2% of the portable speakers produced are faulty.

(ii) Find the probability that a randomly chosen carton contains at most one faulty portable speaker. [1]

(iii) A carton with more than one faulty portable speaker is considered as 'substandard'. Find the probability that, out of 50 cartons, there are no 'substandard' cartons. [2]

The company also makes flash drives packed in batches of one hundred. The probability that a flash drive being faulty is p, where $0<p<1$. For quality control purposes, a random sample of 3 flash drives is tested from each batch.

- If there are 2 or 3 faulty flash drives, the batch is rejected.
- If there is no faulty flash drive, the batch is accepted.
- If there is one faulty flash drive in the sample, another random sample of 3 flash drives is tested and the batch is accepted if there are no faulty flash drive in the second sample. Otherwise the batch is rejected.

(iv) Find the range of p such that we accept at least 95% of the batches based on this quality control method. [3]

9 The events A and B are such that $\mathrm{P}(B)=\frac{1}{3}$, $\mathrm{P}(A|B)=\frac{2}{5}$ and $\mathrm{P}(A\cup B)=\frac{3}{5}$.

(i) Find $\mathrm{P}(A)$ and hence, determine whether events A and B are independent. [4]

(ii) Find the exact value of $\mathrm{P}(A'\cap B)$. [1]

For a third event C, it is given that $\mathrm{P}(C)=\frac{1}{2}$ and that A and C are mutually exclusive.

(iii) Find exactly the greatest and least possible values of $\mathrm{P}(B'\cap C)$. [4]

10 **In this question, you should state clearly all the distributions that you use, together with the values of the appropriate parameters.**

A company produces two types of yarn balls with two machines. Machine A produces balls of 2-ply yarn. Over a period of time, it is found that 80% of the balls of yarn from Machine A have lengths less than 340 metres and 10% of the balls of yarn from Machine A have lengths less than 280 metres.

(i) Assuming that the lengths, in metres, of yarn in balls of 2-ply yarn from Machine A follows the distribution $\mathrm{N}(\mu,\sigma^2)$, find μ and show that $\sigma=28.3$. [3]

For the rest of the question let $\mu=300$ and $\sigma=20$.

Machine B produces balls of 3-ply yarn. The lengths, in metres, of yarn in balls of 3-ply yarn from Machine B follows the distribution $\mathrm{N}(220,6^2)$.

(ii) Five balls of 3-ply yarn from Machine B are randomly chosen. Find the probability that two of them have lengths more than 230 m and three of them have lengths less than 215 m. [2]

(iii) The probability that two randomly chosen balls of 2-ply yarn from Machine A and three randomly chosen balls of 3-ply yarn from Machine B have total length exceeding k metres is 0.385. Find the value of k, correct to 2 decimal places. [3]

(iv) A technician changed the settings on the two machines. The lengths of yarn in balls of 2-ply yarn from Machine A is reduced by 9% and the lengths of yarn in balls of 3-ply yarn from Machine B is reduced by 5%. Find the probability that the total length of yarn in three randomly chosen balls of 2-ply yarn from Machine A is within 50 metres of the total length of yarn in four randomly chosen balls of 3-ply yarn from Machine B. [4]

11 **(a)** Draw separate scatter diagrams, each with 7 points, all in the first quadrant, which represent the situation where the product moment correlation coefficient between variables x and y is

(i) 0, [1]

(ii) approximately 0.8. [1]

(b) Eight cities in a certain country are linked by rail to the capital city. The table below shows the distance of each city from the capital and the rail price from the city to the capital.

City	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Distance, d km	124	44	76	148	16	180	104	195
Rail price, $\$p$	156	53	99	169	23	177	138	178

(i) Draw a scatter diagram of the data. [2]

(ii) For a line of best fit $p = \mathrm{f}(d)$, the residual for a point (a, b) plotted on the scatter diagram is the vertical distances between $(a, \mathrm{f}(a))$ and (a, b). Explain why, in general, the sum of the squares of the residuals rather than the sum of the residuals is used. [1]

(iii) Using the scatter diagram in **(i)**, explain if a linear relationship between the distance and the rail price is appropriate. [1]

(iv) Suppose that the relationship between d and p are modelled by an equation of the form $p = s + t\ln d$, where s and t are constants. Find the equation of this regression line and use it to estimate the rail price for a city which is 210 km from the capital. [2]

(v) Comment on the reliability of the estimate in **(iv)**. [1]

(vi) Following an adjustment to the rail prices, all rail prices are increased by 10 dollars. Without any further calculations, state any change you would expect in the values of the constants s and t found in **(iv)**. [2]

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