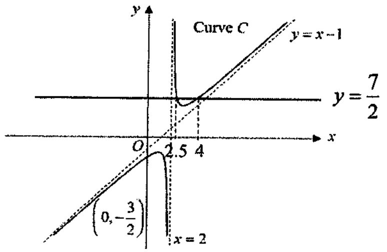
Qn	Solution
1 [4]	Since θ is sufficiently small, $\sin\theta \approx \theta$. **Method 1: Binomial Series (MF27)** $\mathrm{f}(\theta) \approx \frac{1}{3-\theta}$ $= (3-\theta)^{-1}$ $= \left[3\left(1-\frac{1}{3}\theta\right)\right]^{-1}$ $= 3^{-1}\left[1+\left(-\frac{1}{3}\theta\right)\right]^{-1}$ $= \frac{1}{3}\left[1+\left(-\frac{1}{3}\theta\right)\right]^{-1}$ $= \frac{1}{3}\left[1+(-1)\left(-\frac{1}{3}\theta\right)+\frac{(-1)(-2)}{2!}\left(-\frac{1}{3}\theta\right)^2+\ldots\right]$ $= \frac{1}{3}\left(1+\frac{1}{3}\theta+\frac{1}{9}\theta^2+\ldots\right)$ $\approx \frac{1}{3}+\frac{1}{9}\theta+\frac{1}{27}\theta^2$ (shown) $\therefore p=\frac{1}{3}, q=\frac{1}{9}$ and $r=\frac{1}{27}$. **Method 2: Repeated Differentiation** $\mathrm{f}(\theta) \approx (3-\theta)^{-1}$ $\mathrm{f}'(\theta) \approx -(3-\theta)^{-2}(-1) = (3-\theta)^{-2}$ $\mathrm{f}''(\theta) \approx -2(3-\theta)^{-3}(-1) = 2(3-\theta)^{-3}$ Sub $\theta=0$: $\mathrm{f}(0)=3^{-1}=\frac{1}{3}$ $\mathrm{f}'(0)=(3-0)^{-2}=\frac{1}{9}$ $\mathrm{f}''(0)=2(3-0)^{-3}=\frac{2}{27}$ Using MF27, $\mathrm{f}(\theta) \approx \frac{1}{3}+\frac{1}{9}x+\frac{\frac{2}{27}}{2!}x^2+\ldots$ $\mathrm{f}(\theta) \approx \frac{1}{3}+\frac{1}{9}x+\frac{1}{27}x^2+\ldots$ (shown) $\therefore p=\frac{1}{3}, q=\frac{1}{9}$ and $r=\frac{1}{27}$.

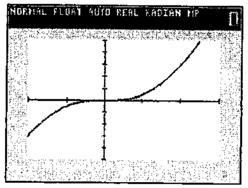
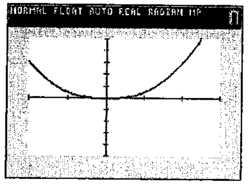
Qn	Solution
2(i) [3]	From the question, the equation of the curve is $y=\frac{x^2+px+q}{x-2}$. Since the vertical asymptote is $x=2 \Rightarrow r=2$. C passes through the point $(0,-1.5)$. $y=\frac{x^2+px+q}{x-2} \Rightarrow -1.5=\frac{0+0+q}{0-2} \Rightarrow q=3$ Hence, the equation of C is $y=\frac{x^2+px+3}{x-2}$ ----- (1) C has asymptotes with equations $y=x-1$ and $x=2$. $\Rightarrow y=x-1+\frac{a}{x-2}$, where a is a constant. $\therefore$ The equation of C is $y=x-1+\frac{a}{x-2}=\frac{(x-1)(x-2)+a}{x-2} \Rightarrow y=\frac{x^2-3x+2+a}{x-2}$ -- (2) Comparing (2) and (1), $p=-3$.
2(ii) [1]	 For $\mathrm{f}(x)=\frac{x^2-3x+3}{x-2}>\frac{7}{2}$, from graph, $2<x<2.5$ or $x>4$.

Qn	Solution
3(a) [2]	$\int \frac{x}{\sqrt{x^2-4}}\,\mathrm{d}x$ $=\frac{1}{2}\int 2x(x^2-4)^{-\frac{1}{2}}\,\mathrm{d}x$ $=\frac{1}{2}\left[\frac{(x^2-4)^{\frac{1}{2}}}{\frac{1}{2}}\right]+C$ $=\sqrt{x^2-4}+C$
3(b) [3]	Let $u=\ln(2x)$ $\quad \frac{\mathrm{d}v}{\mathrm{d}x}=x^2$ $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{x}$ $\quad v=\frac{x^3}{3}$ $\int x^2\ln(2x)\,\mathrm{d}x$ $=\left(\frac{x^3}{3}\right)\ln(2x)-\int\frac{1}{x}\left(\frac{x^3}{3}\right)\mathrm{d}x$ $=\frac{1}{3}x^3\ln(2x)-\frac{1}{3}\int x^2+\mathrm{d}x$ $=\frac{1}{3}x^3\ln(2x)-\frac{1}{9}x^3+C$
3(c) [2]	$\int\frac{\sin 2x}{1+\cos^2 x}\,\mathrm{d}x$ $=-\int\frac{-2\cos x\sin x}{1+\cos^2 x}\,\mathrm{d}x$ $=-\ln\left\|1+\cos^2 x\right\|+C$ $=-\ln\left(1+\cos^2 x\right)+C$

Qn	Solution
4(i) [3]	Let x, y and z be the usual selling price of Blend A, B and C coffee beans respectively. $x+y+z=176$----(2) $0.85x+0.90y+0.95z=161.80$----(2) $0.93x+0.95y+0.98z=169.56$----(3) From GC: Hence $x=\$32$, $y=\$44$, $z=\$100$
4(ii) [3]	Price difference: $169.56-161.80=7.76$ To make it more attractive: Additional loyalty discount > price difference $\left(\frac{x}{100}\right)(32)+\left(\frac{x}{100}\right)(44)>7.76$ $x>10.2$ $x=11$ Note: $x\%=\frac{x}{100}$

Qn	Solution
5(i) [4]	Let $u = x^2 + 3 \Rightarrow \frac{du}{dx} = 2x$ $\int \frac{x^3}{\sqrt{x^2+3}}\, dx$ $= \int \frac{x^2}{\sqrt{x^2+3}} (x\, dx)$ $= \int \frac{u-3}{\sqrt{u}} \left(\frac{du}{2}\right)$ $= \frac{1}{2} \int u^{\frac{1}{2}} - 3u^{-\frac{1}{2}}\, du$ $= \frac{1}{2}\left(\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - 3\frac{u^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} \right) + C$ $= \frac{1}{3}u^{\frac{3}{2}} - 3u^{\frac{1}{2}} + C$ $= \frac{1}{3}\sqrt{\left(x^2+3\right)^3} - 3\sqrt{\left(x^2+3\right)} + C$ (shown) Or Let $u = x^2 + 3 \Rightarrow \frac{du}{dx} = 2x$ $\int \frac{x^3}{\sqrt{x^2+3}}\, dx$ $= \int \frac{(u-3)^{\frac{3}{2}}}{\sqrt{u}} \left(\frac{du}{2x}\right)$ Continue to replace x in terms of u if it can not be done easily with 1 step $= \int \frac{(u-3)^{\frac{3}{2}}}{\sqrt{u}} \left(\frac{du}{2(u-3)^{\frac{1}{2}}}\right)$ $= \int \frac{u-3}{\sqrt{u}} \left(\frac{du}{2}\right)$ $= \frac{1}{2} \int u^{\frac{1}{2}} - 3u^{-\frac{1}{2}}\, du$ $= \frac{1}{2}\left(\frac{u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - 3\frac{u^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} \right) + C$ $= \frac{1}{3}\left(x^2+3\right)^{\frac{3}{2}} - 3\left(x^2+3\right)^{\frac{1}{2}} + C$ (shown)

5(ii) [3]	$\int_{-1}^{3} \left\lvert \frac{3x^3}{\sqrt{x^2+3}} \right\rvert dx$ $= \int_{-1}^{0} \left\lvert \frac{3x^3}{\sqrt{x^2+3}} \right\rvert dx + \int_{0}^{3} \left\lvert \frac{3x^3}{\sqrt{x^2+3}} \right\rvert dx$ $= \int_{-1}^{0} \frac{-3x^3}{\sqrt{x^2+3}}\, dx \quad + \int_{0}^{3} \frac{3x^3}{\sqrt{x^2+3}}\, dx$ [original curve reflected in the x-axis] [Part of original curve] $= -\left[\sqrt{\left(x^2+3\right)^3} - 9\sqrt{\left(x^2+3\right)}\right]_{-1}^{0} + \left[\sqrt{\left(x^2+3\right)^3} - 9\sqrt{\left(x^2+3\right)}\right]_{0}^{3}$ $= -\left[\left(\sqrt{27} - 9\sqrt{3}\right) - \left(\sqrt{64} - 9\sqrt{4}\right)\right] + \left[\left(\sqrt{12^3} - 9\sqrt{12}\right) - \left(\sqrt{3^3} - 9\sqrt{3}\right)\right]$ $= -\left[\left(3\sqrt{3} - 9\sqrt{3}\right) - (8-18)\right] + \left[\left(12\sqrt{12} - 9\sqrt{12}\right) - \left(3\sqrt{3} - 9\sqrt{3}\right)\right]$ $= -\left[10 - 6\sqrt{3}\right] + \left[\left(3\sqrt{12}\right) + 6\sqrt{3}\right]$ $= -\left[10 - 6\sqrt{3}\right] + \left[\left(3\sqrt{4(3)}\right) + 6\sqrt{3}\right]$ $= -\left[10 - 6\sqrt{3}\right] + \left[6\sqrt{3} + 6\sqrt{3}\right]$ $= -10 + 18\sqrt{3}$ Use GC:

Let $f(x) = \frac{3x^3}{\sqrt{x^2+3}}$	Let $f(x) = \left\lvert \frac{3x^3}{\sqrt{x^2+3}} \right\rvert$
	
$-1 \le x < 0$, we have $f(x) < 0$ $0 \le x \le 3$, we have $f(x) \ge 0$	$\int_{-1}^{0} \frac{-3x^3}{\sqrt{x^2+3}}\, dx$ [Reflection in x-axis] $+\int_{0}^{3} \frac{3x^3}{\sqrt{x^2+3}}\, dx$ [Part of original curve]

Qn | **Solution**

6(i)
[3]

Area :

$$S = 2\times\left[\frac{1}{2}\times\pi\left(\frac{x}{2}\right)^2\right]+2xy$$

$$S = \frac{\pi x^2}{4}+2xy$$

Perimeter:

$$25 = 2\times\left[\frac{1}{2}\times 2\pi\left(\frac{x}{2}\right)\right]+2x+2y$$

$$2y = 25-\pi x-2x$$

$$S = \frac{\pi x^2}{4}+(25-\pi x-2x)x$$

$$S = \frac{\pi x^2}{4}+25x-\pi x^2-2x^2$$

$$S = 25x-2x^2-\frac{3}{4}\pi x^2 \quad \text{(shown)}$$

6(ii)
[4]

$$S = 25x-2x^2-\frac{3}{4}\pi x^2$$

$$\frac{\mathrm{d}S}{\mathrm{d}x} = 25-4x-\frac{3}{2}\pi x$$

At stationary point, $\frac{\mathrm{d}S}{\mathrm{d}x} = 0$

$$25-4x-\frac{3}{2}\pi x = 0$$

$$50-8x-3\pi x = 0$$

$$(8+3\pi)x = 50$$

$$x = \frac{50}{8+3\pi}$$

$$\frac{\mathrm{d}S}{\mathrm{d}x} = 25-4x-\frac{3}{2}\pi x$$

$$\frac{\mathrm{d}^2S}{\mathrm{d}x^2} = -4-\frac{3}{2}\pi = -8.71 \quad <0$$

S is maximum area

Qn | **Solution**

7(i)
[2]

GP: $a = 500,\ r = 0.6$

$u_0 = 500$

$u_1 = 0.6\times 500 = 300$

$u_2 = 0.6\times(0.6\times 500) = 500\times 0.6^2 = 180$ (verified)

7 (ii)
[2]

Week n	New case, u_n	Total Cumulative, S_n
0	500	500
1	500×0.6^1	$500+500\times 0.6^1$
2	500×0.6^2	$500+500\times 0.6^1+500\times 0.6^2$
…	…	…
n	500×0.6^n	$500+500\times 0.6^1+500\times 0.6^2$ $+\ldots+500\times 0.6^n$

$$S_n = 500+500\times 0.6^1+500\times 0.6^2+\ldots+500\times 0.6^n$$

$$S_n = 500\left(1+0.6+0.6^2+\ldots+0.6^n\right)$$

$$S_n = 500\left[\frac{1\left(1-0.6^{n+1}\right)}{1-0.6}\right]$$

$$S_n = 1250\left(1-0.6^{n+1}\right) \quad \text{(shown)}$$

7
(iii)
[1]

Method 1: GC

$1250\left(1-0.6^{n+1}\right) > 1240$

From GC

n	$1250\left(1-0.6^{n+1}\right)$
8	1237.4
9	1242.4 > 1240

Method 2: Solving Inequality

$$S_n = 1250\left(1-0.6^{n+1}\right)$$

$$1250\left(1-0.6^{n+1}\right) > 1240$$

$$0.6^{n+1} < 1-\frac{1240}{1250}$$

$$n > \frac{\ln\left(\frac{1}{125}\right)}{\ln(0.6)}-1$$

$$n > 8.45$$

$$n_c = 9$$

$\ln(0.6) < 0$, handle the inequality sign accordingly

7 (iv) [2]	$S_n = 1250\left(1-0.6^{n+1}\right)$ $n \to \infty,\ 0.6^{n+1} \to 0,\ S_n \to 1250$ Since the maximum cumulative infection is 1250 < 1280, the patient capacity will be sufficient.

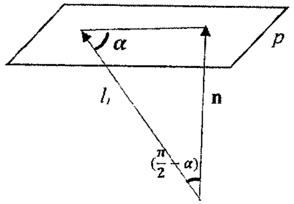
Qn	Solution
8 (i) [2]	**Method 1** $y = e^{2\sin^{-1}x} \quad \Rightarrow \quad \ln y = 2\sin^{-1}x$ Differentiating both sides wrt x: $\frac{1}{y}\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$ $\sqrt{1-x^2}\frac{dy}{dx} = 2y$ (shown) **Method 2** $y = e^{2\sin^{-1}x}$ $\frac{dy}{dx} = e^{2\sin^{-1}x} \times \frac{2}{\sqrt{1-x^2}}$ $\sqrt{1-x^2}\frac{dy}{dx} = 2e^{2\sin^{-1}x}$ $\sqrt{1-x^2}\frac{dy}{dx} = 2y$ (shown)
8 (ii) [4]	$\left(1-x^2\right)\left(\frac{dy}{dx}\right)^2 = 4y^2$ Differentiating both sides wrt x: $\left(1-x^2\right)\times 2\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)+(-2x)\left(\frac{dy}{dx}\right)^2 = 8y\frac{dy}{dx}$ ------------------------------ ***Alternatively,*** $\left(1-x^2\right)^{\frac{1}{2}}\frac{dy}{dx} = 2y$ Differentiating both sides wrtx: $\left(1-x^2\right)^{\frac{1}{2}}\frac{d^2y}{dx^2}+\frac{1}{2}\left(1-x^2\right)^{-\frac{1}{2}}(-2x)\frac{dy}{dx} = 2\frac{dy}{dx}$ ------------------------------

	When $x=0$, $y = e^{2\sin^{-1}0} \quad \Rightarrow \quad y=1$ $\sqrt{1-0^2}\frac{dy}{dx} = 2(1) \quad \Rightarrow \quad \frac{dy}{dx} = 2$ $\left(1-0^2\right)\times 2(2)\left(\frac{d^2y}{dx^2}\right)+(0)(2)^2 = 8(1)(2) \quad \Rightarrow \quad \frac{d^2y}{dx^2} = 4$ ------------------------------ ***Alternatively,*** $\left(1-0^2\right)^{\frac{1}{2}}\frac{d^2y}{dx^2}+\frac{1}{2}\left(1-0^2\right)^{-\frac{1}{2}}(0)(2) = 2(2) \quad \Rightarrow \quad \frac{d^2y}{dx^2} = 4$ ------------------------------ $\therefore$ The Maclaurin series for y: $y = 1+2x+4\left(\frac{x^2}{2!}\right)+\ldots$ $\Rightarrow y = 1+2x+2x^2+\ldots$
8 (iii) [2]	$\int_0^{0.1} xe^{2\sin^{-1}x}\,dx \approx \int_0^{0.1} x\left(1+2x+2x^2\right)\,dx = 0.00571667$ $= 0.005717$ (4 s.f.)

Qn	Solution
9 **(i)** **[2]**	$\overrightarrow{AB}$ is parallel to $\begin{pmatrix}k\\1\\2\end{pmatrix}$ $\overrightarrow{OB}-\overrightarrow{OA}=\lambda\begin{pmatrix}k\\1\\2\end{pmatrix}$ $\begin{pmatrix}17\\10\\19\end{pmatrix}-\begin{pmatrix}1\\2\\3\end{pmatrix}=\lambda\begin{pmatrix}k\\1\\2\end{pmatrix}$ $\lambda\begin{pmatrix}k\\1\\2\end{pmatrix}=\begin{pmatrix}16\\8\\16\end{pmatrix}=8\begin{pmatrix}2\\1\\2\end{pmatrix}$ By comparison: when $\lambda=8$, $k=2$ (shown)
9 **(ii)** **[4]**	Observe the l_1 and l_2 are not parallel. l_1 and l_2 either are skewed lines or intersect at a point. We need to further check if the lines intersect. $l_1:\ \underset{\sim}{r}=\begin{pmatrix}1\\2\\3\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\2\end{pmatrix},\lambda\in\mathbb{R}$ $l_2:\ \underset{\sim}{r}=\begin{pmatrix}3\\4\\6\end{pmatrix}+\mu\begin{pmatrix}2\\0\\1\end{pmatrix},\mu\in\mathbb{R}$ **<u>Method 1: Use GC</u>** $\begin{pmatrix}1+2\lambda\\2+\lambda\\3+2\lambda\end{pmatrix}=\begin{pmatrix}3+2\mu\\4\\6+\mu\end{pmatrix}$ $2\lambda-2\mu=2$ $\lambda=2$ $2\lambda-\mu=3$ From GC, $\lambda=2$ and $\mu=1$. The system of eqns are consistent and solution is unique. Both lines intersect at position vector, The lines intersect. Sub $\mu=1$ into l_2. $\begin{pmatrix}3\\4\\6\end{pmatrix}+(1)\begin{pmatrix}2\\0\\1\end{pmatrix}=\begin{pmatrix}5\\4\\7\end{pmatrix}$ **<u>Method 2: Solve manually</u>** If both lines intersect at point, there is an unique solution $\begin{pmatrix}1+2\lambda\\2+\lambda\\3+2\lambda\end{pmatrix}=\begin{pmatrix}3+2\mu\\4\\6+\mu\end{pmatrix}$ Only 1 unknown to start with! Compare j component $2+\lambda=4$ $\lambda=2$ Compare i component $1+2(2)=3+2\mu$ $\mu=1$ Note: To check if the system in consistent When $\lambda=2$ and $\mu=1$, $\text{LHS}=\begin{pmatrix}1+2[2]\\2+[2]\\3+2[2]\end{pmatrix}=\begin{pmatrix}5\\4\\7\end{pmatrix}=\begin{pmatrix}3+2[1]\\4\\6+[1]\end{pmatrix}=\text{RHS}$ Both lines intersect at position vector, $\begin{pmatrix}5\\4\\7\end{pmatrix}$
9(iii) **[We** **3]**	Given α the acute angle between l_1 and p (refer to diagram below) $\cos\left(\frac{\pi}{2}-\alpha\right)=\left\lvert\frac{\begin{pmatrix}2\\1\\2\end{pmatrix}\bullet\begin{pmatrix}1\\0\\1\end{pmatrix}}{\left(\sqrt{2^2+1+2^2}\right)\left(\sqrt{1^2+0+1^2}\right)}\right\rvert$ $\sin\alpha=\frac{4}{3\sqrt{2}}=\frac{2\sqrt{2}}{3}$

$\sin^2\alpha + \cos^2\alpha = 1$

$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\left(\frac{2\sqrt{2}}{3}\right)^2} = \frac{1}{3}$



9
(iv)
[3]

Method 1: Intersection of a Line and a Plane

Point F, $\overrightarrow{OF}$ lies on ℓ_{BF} :

$\underset{\sim}{r} = \begin{pmatrix}17\\10\\19\end{pmatrix} + \alpha\begin{pmatrix}1\\0\\1\end{pmatrix}$, for some $\alpha \in \mathbb{R}$ - - - -(1)

Sub (1) into the eqn of plane p as
Point F is also on plane p :

$\begin{pmatrix}17+\alpha\\10\\19+\alpha\end{pmatrix} . \begin{pmatrix}1\\0\\1\end{pmatrix} = 16$

$17+\alpha+19+\alpha = 16$

$\alpha = -\frac{20}{2} = -10$

$\overrightarrow{OF} = \overrightarrow{OB} + \overrightarrow{BF}$

$\overrightarrow{OF} = \begin{pmatrix}17\\10\\19\end{pmatrix} + (-10)\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}7\\10\\9\end{pmatrix}$

Coordinates of point F is (7, 10, 9).

Method 2: Find $\overline{BF}$ by projecting $\overline{BD}$ onto n, where D is a point on plane *p*.

Obtain a specific point, D from plane p, $\overrightarrow{OD} = \begin{pmatrix}16\\0\\0\end{pmatrix}$.

$\overrightarrow{BF}$

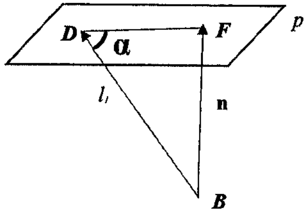
$= \left(\overrightarrow{BD}\cdot\underset{\sim}{n}\right)\underset{\sim}{n}$

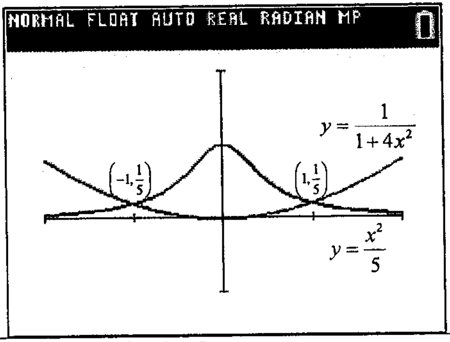
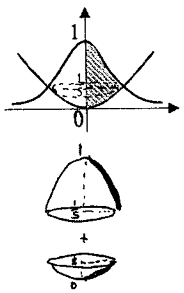
$= \frac{\left\{\begin{pmatrix}16\\0\\0\end{pmatrix} - \begin{pmatrix}17\\10\\19\end{pmatrix}\right\}\cdot\begin{pmatrix}1\\0\\1\end{pmatrix}}{\left|\sqrt{2}\right|} \frac{\begin{pmatrix}1\\0\\1\end{pmatrix}}{\left|\sqrt{2}\right|}$

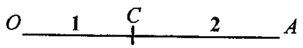
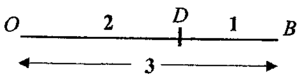
$= \frac{-20}{2}\begin{pmatrix}1\\0\\1\end{pmatrix} = -10\begin{pmatrix}1\\0\\1\end{pmatrix}$

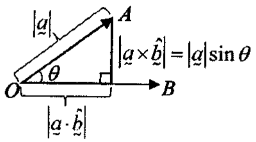
$\overrightarrow{OF} = \overrightarrow{OB} + \overrightarrow{BF}$

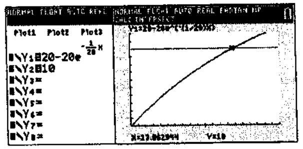
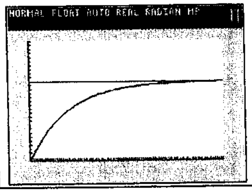
$\overrightarrow{OF} = \begin{pmatrix}17\\10\\19\end{pmatrix} + (-10)\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}7\\10\\9\end{pmatrix}$



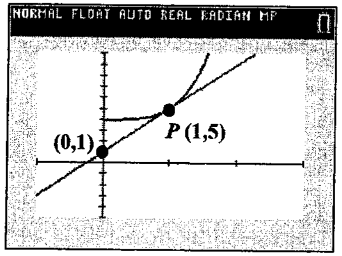
Qn	Solution
10(i) [2]	$y=\frac{1}{1+4x^2}$ - - - - - - - (1) $y=\frac{x^2}{5}$ - - - - - - - - - - (2) Using GC: The intersection points are $\left(-1,\frac{1}{5}\right)$, $\left(1,\frac{1}{5}\right)$.  NORMAL FLOAT AUTO REAL RADIAN MP
10(ii) [5]	Exact area of region R $=\int_{-1}^{1}\left(\frac{1}{1+4x^2}\right)-\left(\frac{x^2}{5}\right)\,dx$ $=\int_{-1}^{1}\frac{1}{4}\left(\frac{1}{\left(\frac{1}{2}\right)^2+x^2}\right)-\left(\frac{x^2}{5}\right)\,dx$ $=\left[\frac{1}{4}\left[\frac{1}{\left(\frac{1}{2}\right)}\tan^{-1}\left(\frac{x}{\frac{1}{2}}\right)\right]-\frac{1}{15}x^3\right]_{-1}^{1}$ $=\left[\left[\frac{1}{2}\tan^{-1}(2x)\right]-\frac{1}{15}x^3\right]_{-1}^{1}$ $=\left[\left(\frac{1}{2}\tan^{-1}(2)-\frac{1}{15}(1)^3\right)-\left(\frac{1}{2}\tan^{-1}(-2)-\frac{1}{15}(-1)^3\right)\right]$ $=\frac{1}{2}\tan^{-1}(2)-\frac{1}{15}+\frac{1}{2}\tan^{-1}(2)-\frac{1}{15}$ $=\tan^{-1}(2)-\frac{2}{15}$ units^2
10(iii) [5]	$y=\frac{x^2}{5}\Rightarrow x_1^{\,2}=5y$ $y=\frac{1}{1+4x^2}\Rightarrow x_2^{\,2}=\frac{1}{4}\left(\frac{1}{y}-1\right)$  Exact volume generated $=\pi\int_{0}^{\frac{1}{5}}x_1^{\,2}\,dy+\pi\int_{\frac{1}{5}}^{1}x_2^{\,2}\,dy$ $=\pi\int_{0}^{\frac{1}{5}}5y\,dy+\pi\int_{\frac{1}{5}}^{1}\frac{1}{4}\left(\frac{1}{y}-1\right)\,dy$ $=\pi\left[\frac{5}{2}y^2\right]_0^{\frac{1}{5}}+\frac{\pi}{4}[\ln y-y]_{\frac{1}{5}}^{1}$ $=\pi\left(\frac{1}{10}-0\right)+\frac{\pi}{4}\left[(\ln(1)-1)-\left(\ln\left(\frac{1}{5}\right)-\frac{1}{5}\right)\right]$ $=\frac{\pi}{10}+\frac{\pi}{4}\left[-\frac{4}{5}-\ln\left(\frac{1}{5}\right)\right]$ $=\frac{\pi}{4}\ln 5-\frac{\pi}{10}$ unit^3

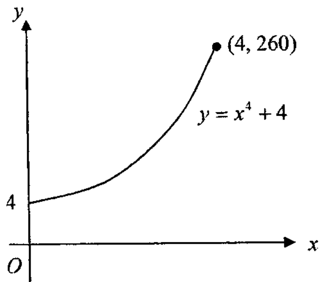
Qn	Solution
11(i) [2]	$\overrightarrow{OC}=\frac{1}{3}\overrightarrow{OA}$  $\overrightarrow{OC}=\frac{1}{3}\underset{\sim}{a}$ $\overrightarrow{OD}=\frac{2}{3}\overrightarrow{OB}$  $\overrightarrow{OD}=\frac{2}{3}\underset{\sim}{b}$
11(ii) [5]	$\overrightarrow{OE}=\underset{\sim}{r}=\overrightarrow{OA}+\lambda\overrightarrow{AD}$---(1) $\overrightarrow{OE}=\underset{\sim}{r}=\overrightarrow{OB}+\mu\overrightarrow{BC}$---(2) Solve (1) = (2): $\Rightarrow\overrightarrow{OA}+\lambda\overrightarrow{AD}=\overrightarrow{OB}+\mu\overrightarrow{BC}$ $\overrightarrow{OA}+\lambda\left(\overrightarrow{OD}-\overrightarrow{OA}\right)=\overrightarrow{OB}+\mu\left(\overrightarrow{OC}-\overrightarrow{OB}\right)$ $\underset{\sim}{a}+\lambda\left(\frac{2}{3}\underset{\sim}{b}-\underset{\sim}{a}\right)=\underset{\sim}{b}+\mu\left(\frac{1}{3}\underset{\sim}{a}-\underset{\sim}{b}\right)$ $(1-\lambda)\underset{\sim}{a}+\frac{2}{3}\lambda\underset{\sim}{b}=\frac{1}{3}\mu\underset{\sim}{a}+(1-\mu)\underset{\sim}{b}$ Comparing coefficient of $\underset{\sim}{a}:1-\lambda=\frac{1}{3}\mu$ $\Rightarrow\lambda+\frac{1}{3}\mu=1$ ---(3) Comparing coefficient of $\underset{\sim}{b}:\frac{2}{3}\lambda=1-\mu$ $\Rightarrow\frac{2}{3}\lambda+\mu=1$ ---(4) Solving (3) & (4), by GC: $\lambda=\frac{6}{7}$, $\mu=\frac{3}{7}$ Sub $\lambda=\frac{6}{7}$ into (1): $\overrightarrow{OE}=\underset{\sim}{a}+\frac{6}{7}\left(\frac{2}{3}\underset{\sim}{b}-\underset{\sim}{a}\right)$ $=\underset{\sim}{a}+\frac{4}{7}\underset{\sim}{b}-\frac{6}{7}\underset{\sim}{a}$ $=\frac{1}{7}\underset{\sim}{a}+\frac{4}{7}\underset{\sim}{b}$ (shown)

11 (iii) [3]	Area of $\Delta OAE=\frac{1}{2}\left\|\overrightarrow{OA}\times\overrightarrow{OE}\right\|$ $=\frac{1}{2}\left\|\underset{\sim}{a}\times\left(\frac{1}{7}\underset{\sim}{a}+\frac{4}{7}\underset{\sim}{b}\right)\right\|$ $=\frac{1}{2}\left\|\left(\underset{\sim}{a}\times\frac{1}{7}\underset{\sim}{a}\right)+\left(\underset{\sim}{a}\times\frac{4}{7}\underset{\sim}{b}\right)\right\|$ $=\frac{1}{2}\left\|\frac{1}{7}(\underset{\sim}{a}\times\underset{\sim}{a})+\frac{4}{7}(\underset{\sim}{a}\times\underset{\sim}{b})\right\|$ $=\frac{1}{2}\left\|\underset{\sim}{0}+\frac{4}{7}(\underset{\sim}{a}\times\underset{\sim}{b})\right\|\quad(\because\underset{\sim}{a}\times\underset{\sim}{a}=\underset{\sim}{0})$ $=\frac{2}{7}\left\|\underset{\sim}{a}\times\underset{\sim}{b}\right\|$, where $k=\frac{2}{7}$ (shown).
11 (iv) [1]	The length of projection of OA onto the line OB.
11(v) [3]	**Method 1** Perpendicular distance from A to OB $=\sqrt{(\|\underset{\sim}{a}\|)^2-(\|\underset{\sim}{a}\cdot\hat{\underset{\sim}{b}}\|)^2}$ $=\sqrt{(\|\underset{\sim}{a}\|)^2-\left(\frac{1}{2}\|\underset{\sim}{a}\|\right)^2}\quad\left(\|\underset{\sim}{a}\cdot\hat{\underset{\sim}{b}}\|=\|\underset{\sim}{a}\cdot\underset{\sim}{b}\|=\frac{1}{2}\|\underset{\sim}{a}\|\text{ (given)}\right)$ $=\sqrt{\|\underset{\sim}{a}\|^2-\frac{1}{4}\|\underset{\sim}{a}\|^2}$ $=\sqrt{\frac{3}{4}\|\underset{\sim}{a}\|^2}$ $=\frac{\sqrt{3}}{2}\|\underset{\sim}{a}\|$  **Method 2** Perpendicular distance $=\|\underset{\sim}{a}\|\sin\theta$ (need to find θ) From $\|\underset{\sim}{a}\cdot\underset{\sim}{b}\|=\frac{1}{2}\|\underset{\sim}{a}\|$, we have $\left\|\|\underset{\sim}{a}\|\|\underset{\sim}{b}\|\cos\theta\right\|=\frac{1}{2}\|\underset{\sim}{a}\|$ $\Rightarrow\cos\theta=\frac{1}{2}$ (since $\|\underset{\sim}{b}\|=1$) $\Rightarrow\theta=60°$ Perpendicular distance $=\|\underset{\sim}{a}\|\sin 60°=\frac{\sqrt{3}}{2}\|\underset{\sim}{a}\|$

Qn	Solution
12 (i) [1]	At any point in time, the inflow **rate** is equal to the outflow **rate** (both 5 litres per minute). Therefore, the volume of the mixture in the tank remains constant.
12 (ii) [1]	Salt inflow rate = Salt concentration of brine × Brine inflow rate $\frac{dQ_{in}}{dt}=\frac{dQ_{in}}{dV}\times\frac{dV}{dt}$ $=0.2\times5$ $=1\text{ kg / min}$ (shown)
12 (iii) [2]	Mass of salt in tank at time t is Q kg, Vol of mixture = 100 litres Salt concentration in tank at time t $=\frac{Q}{100}$ kg/litre Salt outflow rate = Salt Concentration in tank × Outflow rate of mixture $\frac{dQ_{out}}{dt}=\frac{dQ_{out}}{dV}\times\frac{dV}{dt}$ $=\frac{Q}{100}\times5$ $=\frac{Q}{20}$ $\frac{dQ}{dt}=\frac{dQ_{in}}{dt}-\frac{dQ_{out}}{dt}=1-\frac{Q}{20}$ (shown)
12 (iv) [5]	$\frac{dQ}{dt}=1-\frac{Q}{20}=\frac{20-Q}{20}$ $\int\frac{1}{20-Q}\,dQ=\int\frac{1}{20}\,dt$ $-\ln\lvert20-Q\rvert=\frac{1}{20}t+c$ $\ln\lvert20-Q\rvert=-\frac{t}{20}-c$ $\lvert20-Q\rvert=e^{-\frac{t}{20}-c}$ $20-Q=Ae^{-\frac{t}{20}}$ When $t=0$, $Q=0$ (initially pure water). $20-0=Ae^{0}\Rightarrow A=20$ $\therefore Q=20-20e^{-\frac{t}{20}}$
12 (v) [2]	Salt concentration in tank $=0.1$ kg/litre Mass of salt $=0.1\times100=10$ kg Sub $Q=10$: $10=20-20e^{-\frac{t}{20}}$ **Method 1 (GC Graph)** From GC, $t=13.863$ $t=13.9$ (3 s.f.)  Plot1 Plot2 Plot3 $Y_1=20-20e^{-\frac{1}{20}X}$ $Y_2=10$ $Y_3=$ $Y_4=$ $Y_5=$ $Y_6=$ $Y_7=$ $Y_8=$ CALC INTERSECT X=13.862944 Y=10 **Method 2 (Algebraic)** $10=20-20e^{-\frac{t}{20}}$ $e^{-\frac{t}{20}}=\frac{1}{2}$ $-\frac{t}{20}=\ln\frac{1}{2}$ $t=-20\ln\frac{1}{2}=13.9$ (3 s.f.)
12 (vi) [1]	$Q=20-20e^{-\frac{t}{20}}$ When $t\to\infty$, $e^{-\frac{t}{20}}\to0$, $Q\to20$ Q increases and **approaches** 20. 

Qn	Solution
1(a) [3]	**Method 1** $y=\mathrm{f}(x)\xrightarrow{x\to x-1}y=\mathrm{f}(x-1)\xrightarrow{x\to 2x}y=\mathrm{f}(2x-1)$ $(a,\ b)\qquad(a+1,\ b)\qquad\left(\frac{1}{2}(a+1),\ b\right)$ **Method 2** $y=\mathrm{f}(x)\xrightarrow{x\to 2x}y=\mathrm{f}(2x)\xrightarrow{x\to 2\left(x-\frac{1}{2}\right)}y=\mathrm{f}(2x-1)$ $(a,\ b)\qquad\left(\frac{1}{2}a,\ b\right)\qquad\left(\frac{1}{2}a+\frac{1}{2},\ b\right)$ P becomes $P'\left(\frac{1}{2}p+\frac{1}{2},\ 0\right)$, Q becomes $Q'\left(\frac{1}{2}q+\frac{1}{2},\ 0\right)$ and R becomes $R'\left(\frac{1}{2},\ r\right)$.
1(b) (i) [1]	The required area $=-I$ or $\lvert I\rvert$. Transformation: Reflection of the graph about the x-axis. The area in question remains the same.
1(b) (ii) [1]	The required area $=-\frac{1}{2}I$ or $\frac{1}{2}\lvert I\rvert$. Transformations: Scaling of the graph parallel to the y-axis by scale factor $\frac{1}{2}$. The area in question is halved. Translation of the graph in the negative x-direction by 3 units. The area in question remains the same.
1(c) [1]	$\int_0^q \mathrm{f}'(x)\,\mathrm{d}x=[\mathrm{f}(x)]_0^q=\mathrm{f}(q)-\mathrm{f}(0)$ $=0-r$ $=-r$

2(i) [3]	$x=\sqrt{t},\quad y=t^2+4$ $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{2\sqrt{t}},\quad \frac{\mathrm{d}y}{\mathrm{d}t}=2t$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2t}{\frac{1}{2\sqrt{t}}}=4t^{\frac{3}{2}}$ Note: $t=p$ The eqn of tangent to C at $P\left(\sqrt{p},p^2+4\right)$: $y-\left(p^2+4\right)=4p^{\frac{3}{2}}\left(x-\sqrt{p}\right)$ $y=4p^{\frac{3}{2}}x-3p^2+4$
2(ii) [2]	The eqn of tangent of C at P passes through $(0,1)$ At $(0,1)$: $1=4p^{\frac{3}{2}}(0)-3p^2+4$ $3p^2=3$ $p=1$ (reject -1 $\because p\geq 0$) When $p=1$ $x=1,y=5$ $P(1,5)$ 

2 **(iii)** **[2]**	
3(a) **[4]**	$z+2z^*+w=6+\mathrm{i}$ ... (1) $\mathrm{i}z-w=3 \Rightarrow w=\mathrm{i}z-3$ ... (2) Substitute (2) into (1): $z+2z^*+\mathrm{i}z-3=6+\mathrm{i}$ $(1+\mathrm{i})z+2z^*=9+\mathrm{i}$ Let $z=a+b\mathrm{i}$. Then, $z^*=a-b\mathrm{i}$. $(1+\mathrm{i})(a+b\mathrm{i})+2(a-b\mathrm{i})=9+\mathrm{i}$ $a+b\mathrm{i}+a\mathrm{i}-b+2a-2b\mathrm{i}=9+\mathrm{i}$ $(3a-b)+(a-b)\mathrm{i}=9+\mathrm{i}$ Comparing corresponding real and imaginary parts: $3a-b=9$ $a-b=1$ Solving the equations simultaneously: $a=4$ and $b=3$ $\Rightarrow z=4+3\mathrm{i}$ From (2), $w=\mathrm{i}(4+3\mathrm{i})-3=-6+4\mathrm{i}$ $\therefore z=4+3\mathrm{i}, w=-6+4\mathrm{i}$.
3(b) **[4]**	**Method 1: Use conjugate root and factorisation** All the coefficients of f(x) are real. Thus, $2-\mathrm{i}$ is also a root of $\mathrm{f}(x)=0$. A quadratic factor of $\mathrm{f}(x)$ is: $[x-(2+\mathrm{i})][x-(2-\mathrm{i})]=[(x-2)-\mathrm{i}][(x-2)+\mathrm{i}]$ $=(x-2)^2-\mathrm{i}^2$ $=x^2-4x+5$

	$\mathrm{f}(x)=x^3+mx^2+nx+5=(x+c)(x^2-4x+5)$, where c is a real constant. Comparing constant term: $5c=5 \Rightarrow c=1$ $\therefore \mathrm{f}(x)=0 \Rightarrow x=-1$ or $2\pm\mathrm{i}$ $\therefore$ The coordinates of P is $(-1, 0)$. **Method 2: Apply Factor Theorem by substituting the given root directly.** $2+\mathrm{i}$ is a root of $\mathrm{f}(x)=0$. $\mathrm{f}(2+\mathrm{i})=0$ $(2+\mathrm{i})^3+m(2+\mathrm{i})^2+n(2+\mathrm{i})+5=0$ $2+11\mathrm{i}+m(3+4\mathrm{i})+2n+5+n\mathrm{i}=0$ $7+3m+2n+(11+4m+n)\mathrm{i}=0$ Comparing corresponding real and imaginary parts: $7+3m+2n=0 \Rightarrow 3m+2n=-7$ $11+4m+n=0 \Rightarrow 4m+n=-11$ Solving the equations simultaneously: $m=-3$ and $n=1$ $\Rightarrow \mathrm{f}(x)=x^3-3x^2+x+5$ Using GC to solve $\mathrm{f}(x)=0$, $x=-1$ or $2\pm\mathrm{i}$. -- ***Alternatively (instead of GC),*** All the coefficients of f(x) are real. Thus, $2-\mathrm{i}$ is also a root of $\mathrm{f}(x)=0$. A quadratic factor of $\mathrm{f}(x)$ is: $[x-(2+\mathrm{i})][x-(2-\mathrm{i})]=[(x-2)-\mathrm{i}][(x-2)+\mathrm{i}]$ $=(x-2)^2-\mathrm{i}^2$ $=x^2-4x+5$ $\Rightarrow \mathrm{f}(x)=x^3-3x^2+x+5=(x+1)(x^2-4x+5)$. $\therefore \mathrm{f}(x)=0 \Rightarrow x=-1$ or $2\pm\mathrm{i}$

4(a) **[1]**

Given that $T_{n+2}=T_{n+1}+T_n$, and $T_1=3,\ T_2=1$

Sub $n=1,\ T_3=T_2+T_1=1+3=4$

Sub $n=2,\ T_4=T_3+T_2=4+1=5$

4(b) **[2]**

Given $r_n=\dfrac{T_{n+1}}{T_n}$

$$\Rightarrow r_{n+1}=\frac{T_{n+1+1}}{T_{n+1}}=\frac{T_{n+2}}{T_{n+1}}$$

$$r_{n+1}=\frac{T_{n+1}+T_n}{T_{n+1}}$$

$$r_{n+1}=1+\frac{T_n}{T_{n+1}}$$

$$r_{n+1}=1+\frac{1}{r_n}\ \text{(shown)}$$

4(c) **[3]**

Since r_n is convergent to L, it implies that as $n\to\infty$, $r_n\to L$ and $r_{n+1}\to L$.

Hence,

$$r_{n+1}=1+\frac{1}{r_n}\ \Rightarrow L=1+\frac{1}{L}\ \text{as } n\to\infty$$

$$L^2-L-1=0$$

$$L=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-1)}}{2(1)}$$

$$L=\frac{1\pm\sqrt{5}}{2}$$

$$L=\frac{1+\sqrt{5}}{2}\quad \left(\text{rej } \frac{1-\sqrt{5}}{2}\ \text{as } r_n>0\right)$$

4(d) **[2]**

Method 1

$$\sum_{k=1}^{n}u_k>930$$

$$\frac{n}{2}[2(5)+(n-1)(2)]>930$$

$$n(n+4)>930$$

From GC

n	$n(n+4)$
28	896
29	957

Least $n=29$

Method 2

$$\sum_{k=1}^{n}u_k>930$$

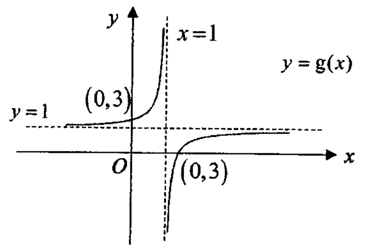
$$\sum_{k=1}^{n}(5+2(k-1))>930$$

From GC

n	$\sum_{k=1}^{n}(5+2(k-1))$
28	896
29	957

Least $n=29$

5(i) **[3]**



5(ii) **[1]**

Every horizontal line $y=k$, where $k\in\mathbb{R}$, cuts the graph of $y=\mathrm{g}(x)$ at most once. g is a one-one function.

$\therefore \mathrm{g}^{-1}$ exists.

5(iii) **[3]**

Let $y=\mathrm{g}(x)=\dfrac{x-3}{x-1}$.

$$y(x-1)=x-3$$

$$(y-1)x=y-3$$

$$x=\frac{y-3}{y-1}\ \Rightarrow\ \mathrm{g}^{-1}(y)=\frac{y-3}{y-1}$$

$$\therefore \mathrm{g}^{-1}(x)=\frac{x-3}{x-1}$$

From graph in part (i), $\mathrm{R_g}=(-\infty,\ 1)\cup(1,\ \infty)$

$\therefore \mathrm{D_{g^{-1}}}=\mathrm{R_g}=(-\infty,\ 1)\cup(1,\ \infty)$

5(iv) **[1]**

For $x\in \mathrm{D_g}$, $\mathrm{g}(x)=\dfrac{x-3}{x-1}$ and $\mathrm{g}^{-1}(x)=\dfrac{x-3}{x-1}$

$\therefore\ \mathrm{g}(x)=\mathrm{g}^{-1}(x)$ and hence g is self-inverse.

5(v) **[2]**

Since g is self-inverse, $\mathrm{g}^2(x)=\mathrm{g}^{-1}\mathrm{g}(x)=\mathrm{g}\mathrm{g}^{-1}(x)=x.$

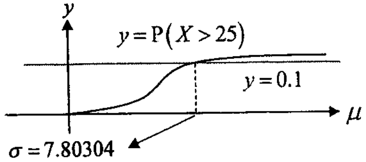
$\therefore \mathrm{g}^2(x)=-\mathrm{g}^{-1}(x)\ \Rightarrow x=-\dfrac{x-3}{x-1}$

$$x=-\frac{x-3}{x-1}$$

$$x^2-x=-x+3$$

$$x^2=3$$

$$x=\pm\sqrt{3}$$

6(i) **[3]**	Let X be the waiting time to get pastries from the popular bakery of a randomly chosen customer. $X \sim \mathrm{N}\left(\mu,\ \sigma^2\right)$ Given $\mathrm{P}(X<5)=\mathrm{P}(X>25)=0.10$. **Method 1** By symmetry, $\mu=\frac{5+25}{2}=15$ $\mathrm{P}(X>25)=0.10$ $\mathrm{P}\left(Z>\frac{25-15}{\sigma}\right)=0.10$ $\frac{10}{\sigma}=1.28155$ $\sigma=7.80305$ - - - - - - - - - - - - - - - - ***Alternatively,*** $\mathrm{P}(X>25)=0.10$ Using GC,  - - - - - - - - - - - - - - - - $\therefore \mu=15,\ \sigma=8$ (to the nearest minute). **Method 2** $\mathrm{P}(X>25)=0.10$ $\quad$ $\mathrm{P}(X<5)=0.10$ $\mathrm{P}\left(Z>\frac{25-\mu}{\sigma}\right)=0.10$ $\quad$ $\mathrm{P}\left(Z<\frac{5-\mu}{\sigma}\right)=0.10$ $\frac{25-\mu}{\sigma}=1.28155$ $\quad$ $\frac{5-\mu}{\sigma}=-1.28155$ $\mu+1.28155\sigma=25\ \ldots(1)$ $\quad$ $\mu-1.28155\sigma=5\ \ldots(2)$ Solving (1) and (2) simultaneously, $\mu=15,\ \sigma=7.80305$. $\therefore \mu=15,\ \sigma=8$ (to the nearest minute).

6(ii) **[1]**	Consider $X \sim \mathrm{N}\left(15,\ 8^2\right)$. $\mathrm{P}(X<0)=0.0303$. (or 0.02728 if we use $\sigma=7.80305$) The waiting times cannot be negative but $\mathrm{P}(X<0)$ is about 3%, which is not insignificant. Thus, the normal distribution is not a suitable model. **Note:** For normal distribution: $\mathrm{P}(\mu-3\sigma<X<\mu-3\sigma)\approx 0.997$ i.e. 99.7%. Hence, whenever the probability of obtaining unreasonable values of x exceeds 0.003, we can conclude that the normal distribution is not suitable
7(i) **[1]**	The **probability** of obtaining a card of **one colour** depends **solely on the colour** and **not any other factors**. Or The **probability** of obtaining a card of **one colour** is **exactly the same** as the probability of obtaining another card **of the same colour** in the same deck.
7(ii) **[4]**	**Method 1** $\mathrm{P}(G=2)=\frac{{}^{2}C_2\,{}^{10}C_1}{{}^{12}C_3}=\frac{1}{22}$ (shown) $\mathrm{P}(G=1)=\frac{{}^{2}C_1\,{}^{10}C_2}{{}^{12}C_3}=\frac{9}{22}$ $\mathrm{P}(G=0)=\frac{{}^{10}C_3}{{}^{12}C_3}=\frac{12}{22}=\frac{6}{11}$ - - - - - - - - - - - - - - - - ***Alternatively,*** $\mathrm{P}(G=0)=1-\frac{1}{22}-\frac{9}{22}=\frac{6}{11}$ - - - - - - - - - - - - - - - - **Method 2** $\mathrm{P}(G=2)=\left(\frac{2}{12}\right)\left(\frac{1}{11}\right)\left(\frac{10}{10}\right)\times\frac{3!}{2!}=\frac{1}{22}$ (shown) $\mathrm{P}(G=1)=\left(\frac{2}{12}\right)\left(\frac{10}{11}\right)\left(\frac{9}{10}\right)\times\frac{3!}{2!}=\frac{1}{22}$ $\mathrm{P}(G=0)=\left(\frac{10}{12}\right)\left(\frac{9}{11}\right)\left(\frac{8}{10}\right)=\frac{6}{11}$ - - - - - - - - - - - - - - - - ***Alternatively,*** $\mathrm{P}(G=0)=1-\frac{1}{22}-\frac{9}{22}=\frac{6}{11}$

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7(iii)
[2]

G (4 pt)	S (1 pt)	Score	Probability
0	3	3	$\frac{12}{22}=\frac{6}{11}$
1	2	6	$\frac{9}{22}$
2	1	9	$\frac{1}{22}$

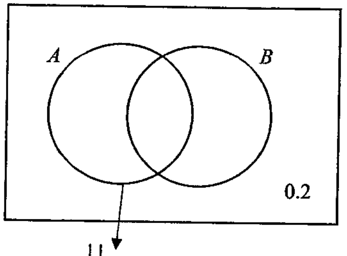
Probability Distribution of X

x	3	6	9
$\mathrm{P}(X=x)$	$\frac{12}{22}=\frac{6}{11}$	$\frac{9}{22}$	$\frac{1}{22}$

7(iv)
[1]

$$\mathrm{E}(\text{winnings})=10(3)\times\frac{6}{11}+10(6)\times\frac{9}{22}+10(9)\times\frac{1}{22}=45$$

$\therefore$ The expected winnings is \$45.



$\frac{11}{20}=0.55$

8(i)
(a)
[3]

$$\mathrm{P}(B|A)=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(A)}=\frac{4}{11}$$

$$\mathrm{P}(A\cap B)=\frac{4}{11}\mathrm{P}(A)$$

$$\mathrm{P}(A\cap B)=\frac{4}{11}\left(\frac{11}{20}\right)=0.2$$

$$\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$$

$$0.8=0.55+\mathrm{P}(B)-0.2$$

$$\mathrm{P}(B)=0.45$$

8(i)
(b)
[1]

Method 1

$\mathrm{P}(A\cap B')=\mathrm{P}(A\cup B)-\mathrm{P}(B)=0.8-0.45=0.35$

Method 2

$\mathrm{P}(A\cap B')=\mathrm{P}(A)-\mathrm{P}(A\cap B)=0.55-0.2=0.35$

8(ii)
[1]

From part **(i)(b)**, $\mathrm{P}(A\cap B')=0.35\neq 0$.

$\therefore$ A and B' are not mutually exclusive.

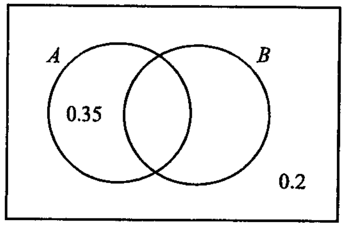
8(iii)
[1]

Since B and C are independent,

$\mathrm{P}(B\cap C)=\mathrm{P}(B)\mathrm{P}(C)=(0.45)(0.6)=0.27$

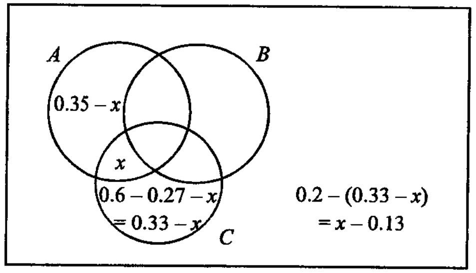
8(iv)
[2]

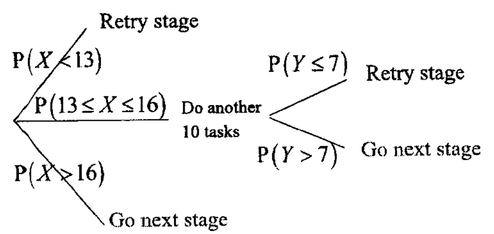
From our work so far, the relevant regions are shown below.



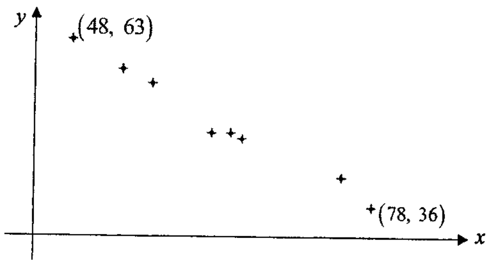
Method 1

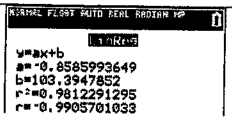
Let $\mathrm{P}(A\cap B'\cap C)=x$.



	Probability cannot be negative i.e. $0.35-x\ge 0 \quad \Rightarrow \quad x\le 0.35$ $x\ge 0$ $0.33-x\ge 0 \quad \Rightarrow \quad x\le 0.33$ $x-0.13\ge 0 \quad \Rightarrow \quad x\ge 0.13$ $\therefore 0.13\le \mathrm{P}(A\cap B'\cap C)\le 0.33.$
9(i) **[2]**	The two assumptions: 1. For each task, whether it is completed or not, is **independent** of that of the other tasks. 2. The **probability** that the player completes a task **is the same** for each task.
9(ii) **[2]**	$X\sim \mathrm{B}(20,\ 0.7)$ The required probability $=\mathrm{P}(13\le X\le 16)$ $=\mathrm{P}(X\le 16)-\mathrm{P}(X\le 12)$ $=0.66518$ $=0.665$ (3 s.f.)
9(iii) **(a)** **[3]**	$Y\sim \mathrm{B}(10,\ 0.8)$  P(go to the next stage) $=\mathrm{P}(X>16)+\mathrm{P}(13\le X\le 16)\mathrm{P}(Y>7)$ $=1-\mathrm{P}(X\le 16)+0.66518\left[1-\mathrm{P}(Y\le 7)\right]$ $=0.55795$ $=0.558$ (3 s.f.)

9(iii) **(b)** **[2]**	The required probability $=\mathrm{P}(X=15 \mid \text{don't progress to next stage})$ $=\dfrac{\mathrm{P}(X=15 \text{ and don't progress to next stage})}{\mathrm{P}(\text{don't progress to next stage})}$ $=\dfrac{\mathrm{P}(X=15 \text{ and retry the second stage})}{\mathrm{P}(\text{don't progress to next stage})}$ $=\dfrac{\mathrm{P}(X=15)\mathrm{P}(Y\le 7)}{\mathrm{P}(\text{don't progress to next stage})}$ $=\dfrac{0.17886\times 0.32220}{1-0.52209}$ $=0.13037$ $=0.130$ (3 s.f.)
10	Let X and Y be the mass of a randomly chosen apple and pear respectively. $X\sim \mathrm{N}(0.2,\ 0.05^2)\ ,\ \ Y\sim \mathrm{N}(0.3,\ 0.08^2)$
10(i) **(a)** **[1]**	The required probability $=\mathrm{P}(X>0.25)\mathrm{P}(Y<0.25)$ $=(0.15866)(0.26599)$ $=0.042188$ $=0.0422$ (3 s.f.)
10(i) **(b)** **[4]**	The required probability $=\mathrm{P}(\lvert X-Y\rvert\le 0.075)$ $=\mathrm{P}(-0.075\le X-Y\le 0.075)$ $\mathrm{E}(X-Y)=0.2-0.3=-0.1$ $\mathrm{Var}(X-Y)=0.05^2+0.08^2=0.0089$ $X-Y\sim \mathrm{N}(-0.1,\ 0.0089)$ The required probability $=0.36371=0.364$ (3 s.f.)
10(ii) **[4]**	The required probability $=\mathrm{P}\left[5(X_1+X_2+\ldots+X_{10})+7(Y_1+Y_2+\ldots+Y_5)>22\right]$ $=\mathrm{P}[C>22]$ $\mathrm{E}(C)=5\left[10(0.2)\right]+7\left[5(0.3)\right]=20.5$ $\mathrm{Var}(C)=5^2\left[10(0.05^2)\right]+7^2\left[5(0.08^2)\right]=2.193$ $C\sim \mathrm{N}(20.5,\ 2.193)$ The required probability $=0.15555=0.156$ (3 s.f.)

11(i) **[1]**	

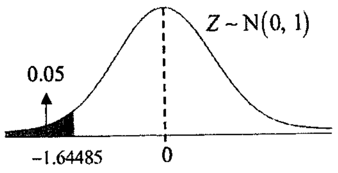
11(ii)
[2]
	 `y=ax+b`
	`a=-0.8585993649`
	`b=103.3947852`
	`r²=0.9812291295`
	`r=-0.9905701033`
	Using GC, $r = -0.991$ (3 s.f.).
	The **value of *r* is close to -1**. Thus, a **strong negative linear correlation** between the resting heart rate (x) and VO₂ max (y) is indicated.

11(iii) **[3]**	The regression line of y on x: $y = -0.85860x + 103.39$ $y = -0.859x + 103$ (3 s.f.) When $x = 60$, $y = -0.85860(60) + 103.39 = 51.874$ $\therefore$ The VO_2 max = 52 (to the nearest integer). Since $x = 60$ is within the given data range of x $(48 \le x \le 78)$ and the value of $\lvert r \rvert$ is close to 1. Thus, the estimate is reliable.
11(iv) **[1]**	When $x = 125$, $y = -3.935$. The predicted VO2 max is negative, which is not reasonable. Moreover, the resting heart rate of 125 is not reasonable for a typical person. Thus, extrapolation is not advisable.
11(v) **[3]**	The value of r remains the same. $r = -0.991$ (3 s.f.) Let the adjusted resting heart rate be w. $w = x + v \;\Rightarrow\; x = w - v$ $y = -0.85860(w - v) + 103.39$ $y = -0.85860w + 0.85860v + 103.39$ $y = -0.859w + 0.859v + 103$ (3 s.f.) $\therefore$ The required gradient $= -0.859$ (3 s.f.) and the required y-intercept $= 0.859v + 103$ (3 s.f.).

12(i) **[2]**	Unbiased estimate of population mean, $\bar{x} = \dfrac{\sum(x-200)}{50} + 200 = \dfrac{2159.62}{50} + 200 = 243.1924$ Unbiased estimate of population variance, $s^2 = \dfrac{1}{50-1}\left[\sum(x-200)^2 - \dfrac{\left(\sum(x-200)\right)^2}{50}\right]$ $= \dfrac{1}{49}\left[112081.09 - \dfrac{(2159.62)^2}{60}\right]$ $= 383.71$ $= 384$ (3 s.f.)
12(ii) **[5]**	Let X be the reaction time, in milliseconds, of a randomly chosen student, and μ be **the population mean** reaction time of the students at Supernova School. $\text{H}_0 : \mu = 250$ $\text{H}_1 : \mu < 250$ Under H_0, since ***n* = 50 is large**, by **Central Limit Theorem**, $\bar{X} \sim \text{N}\left(250,\ \dfrac{383.71}{50}\right)$ approximately. Use z-test at $\alpha = 0.01$. From GC, p-value $= 0.0069972 = 0.00700$ (3 s.f.). Since p-value $= 0.00700 < 0.01$, we reject H_0. There is sufficient evidence at 1% level of significance that the mean reaction time is reduced from 250 milliseconds.
12(iii) **[1]**	The sample size of 10 may not be sufficiently large for Central Limit Theorem to apply. $\bar{X}$ may not follow a normal distribution closely enough. Hence, the test in part **(ii)** may not be appropriate.
12(iv) **[4]**	Let Y be the reaction time, in milliseconds, of a randomly chosen student at Galaxy School. **Method 1** $\text{H}_0 : \mu = \mu_0$ $\text{H}_1 : \mu < \mu_0$ Under H_0, $\bar{Y} \sim \text{N}\left(\mu_0,\ \dfrac{153}{40}\right)$. Test statistic, $Z = \dfrac{\bar{Y} - \mu_0}{\sqrt{\dfrac{153}{40}}} \sim \text{N}(0,\ 1)$. -----

Corresponding test statistic value: $z = \dfrac{241 - \mu_0}{\sqrt{\dfrac{153}{40}}}$

Critical value: -1.64485
Rejection region: $z \le -1.64485$



Conclusion of test:

The reflex training programme was **not successful** i.e. H_0 is **not rejected** (test statistic value **does not lie in** critical region).

$$\Rightarrow \frac{241 - \mu_0}{\sqrt{\dfrac{153}{40}}} > -1.64485$$

$\mu_0 < 244.22$

$\mu_0 < 244$ (3 s.f.)

The test is on finding out if the mean reaction time is reduced.
$\therefore \mu_0 < 244$ (3 s.f.).