H

millennia institute

CANDIDATE NAME

CLASS

ADMIN NUMBER

2025 Preliminary Examination

Pre-University 3

MATHEMATICS

9758/01

Paper 1

August/September 2025

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** the questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
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You are reminded of the need for clear presentation in your answers.

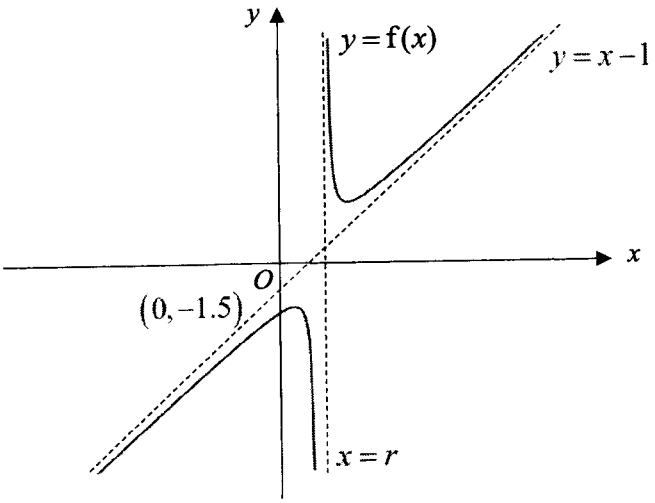
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **7** printed pages and **1** blank page.

1 In this question you may use expansions from the List of Formulae (MF27).

It is given that $\mathrm{f}(\theta)=\dfrac{1}{3-\sin\theta}$, for $\theta>0$. Show that when θ is sufficiently small, $\mathrm{f}(\theta)\approx p+q\theta+r\theta^2$, where p, q and r are exact constants to be determined. [4]

2 The diagram below shows the curve with equation $y=\mathrm{f}(x)$ where $\mathrm{f}(x)=\dfrac{x^2+px+q}{x-2}$ for some constants p and q. The curve passes through the point with coordinates $(0,-1.5)$ and has asymptotes $y=x-1$ and $x=r$ for some constant r.



(i) Write down the value of r and show that $p=-3$ and $q=3$. [3]

(ii) Solve the inequality $\mathrm{f}(x)>\dfrac{7}{2}$. [1]

3 **(a)** Find $\displaystyle\int \frac{x}{\sqrt{x^2-4}}\,\mathrm{d}x$. [2]

(b) Find $\displaystyle\int x^2\ln(2x)\,\mathrm{d}x$. [3]

(c) Find $\displaystyle\int \frac{\sin 2x}{1+\cos^2 x}\,\mathrm{d}x$. [2]

4 Three different blends of coffee beans – Blend A, Blend B and Blend C – are sold at both M Café and I Café. The usual selling prices of the blends are the same at both cafés. The total cost of buying one package each of Blend A, Blend B and Blend C is \$176. During a holiday sale, the two cafés offered the following discounts:

Café	**Discounts given for each package**			**Total price after the discount**
	Blend A	**Blend B**	**Blend C**	
M Café	15%	10%	5%	\$161.80
I Café	7%	5%	2%	\$169.56

(i) Find the usual selling price of each package of Blend A, Blend B and Blend C coffee beans respectively. [3]

Loyal customers of I Café receive an additional discount of x% on the usual selling price of the packages for Blend A and Blend B only. (This loyalty discount is calculated as x% of the usual price of each package and is subtracted from the sale prices above.)

I Café wants the total cost for its loyal customers to be less than the total cost at M Café when buying one of each package of the three blends.

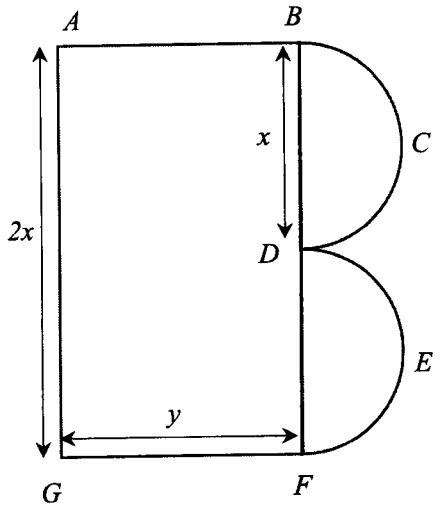
(ii) Form an inequality and find the smallest integer value of x. [3]

5 **(i)** Using the substitution $u = x^2 + 3$, show that

$$\int \frac{x^3}{\sqrt{x^2+3}}\,\mathrm{d}x = \frac{1}{3}\sqrt{\left(x^2+3\right)^3} - 3\sqrt{\left(x^2+3\right)} + C.$$ [4]

(ii) Hence find the exact value of $\int_{-1}^{3}\left|\frac{3x^3}{\sqrt{x^2+3}}\right|\mathrm{d}x$, leaving your answer in the form $a+b\sqrt{3}$ where a and b are integers to be determined. [3]

6 A company produces customised badges. The diagram below shows the design of one such badge which consists of a rectangle and two identical semicircles.



It is given that $AG = 2x$ cm, $FG = y$ cm, $BD = DF = x$ cm and the outer perimeter $ABCDEFGA$ of the badge is 25 cm.

(i) Show that the area of the badge, S cm^2 is given by the formula

$$S = 25x - 2x^2 - \frac{3}{4}\pi x^2 .$$ [3]

(ii) Use calculus to find the exact value of x which gives the maximum value of S. [4]

7 Epidemiologists are modelling the spread of an infectious disease in a town. The following assumptions are made:

- At Week 0, there are 500 new infections.
- In each subsequent week, the number of new infections is 60% the number of new infections in the previous week, due to ongoing community transmission.

Let u_n denote the number of new infections in Week n, for $n \geq 0$, $n \in \mathbb{Z}$.

(i) Write down the value u_1 and verify that $u_2 = 180$. [2]

(ii) If S_n denotes the total cumulative number of new infections after n weeks, show that $S_n = 1250\left(1 - 0.6^{n+1}\right)$. [2]

The local healthcare facility has a capacity of 1240 beds. You may assume that each infected person occupies one bed, and that none are discharged during the period being considered.

(iii) Find the smallest value of n such that the total number of infections up to and including Week n first exceeds 1240. [1]

The healthcare facility later increases its capacity to 1280 beds.

(iv) Based on the given model, comment on whether this new capacity is sufficient to accommodate all infected persons over a prolonged period of time. [2]

8 It is given that $y = \mathrm{e}^{2\sin^{-1}x}$.

(i) Show that $\sqrt{1-x^2}\,\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2y$. [2]

(ii) By further differentiation of the result in part **(i)**, find the Maclaurin series for y up to and including the term in x^2. [4]

(iii) Using your result from part **(ii)**, find an approximate value for $\int_0^{0.1} x\mathrm{e}^{2\sin^{-1}x}\,\mathrm{d}x$, giving your answer to 4 significant figures. [2]

9 The coordinates of point A and B are (1, 2, 3) and (17, 10, 19) respectively. The line l_1 passes through both points A and B and is parallel to the vector $\begin{pmatrix} k \\ 1 \\ 2 \end{pmatrix}$, where k is a constant.

(i) Show that $k = 2$. [2]

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ where μ is a parameter.

(ii) Determine whether the lines l_1 and l_2 intersect. If they do, find the position vector of their point of intersection; otherwise, explain why they do not intersect. [4]

The plane p has equation $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 16$.

(iii) Let α be the acute angle between l_1 and p. Find the exact value of $\sin\alpha$ and $\cos\alpha$. [3]

(iv) Find the coordinates of point F, the foot of the perpendicular of the point B to the plane p. [3]

10 The curves C_1 and C_2 are given by the equations $y = \dfrac{1}{1+4x^2}$ and $y = \dfrac{x^2}{5}$ respectively.

(i) Find the coordinates of the intersection points for C_1 and C_2. [2]

The region R is bounded by C_1 and C_2.

(ii) Find the exact area of R. [5]

(iii) Find the exact volume generated when R is rotated through π radians about the y-axis. [5]

11 Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are non-zero and non-parallel vectors. The point C lies on OA such that $OC : CA = 1 : 2$. The point D lies on OB such that $OB : DB = 3 : 1$.

(i) Find the position vectors $\overrightarrow{OC}$ and $\overrightarrow{OD}$, giving your answers in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

(ii) Show that the point E where the lines AD and BC meet has position vector $\frac{1}{7}\mathbf{a}+\frac{4}{7}\mathbf{b}$. [5]

(iii) Show that the area of triangle OAE can be written as $k|\mathbf{a}\times\mathbf{b}|$, where k is a constant to be found. [3]

It is further given that $\mathbf{b}$ is a unit vector.

(iv) Give the geometrical meaning of $|\mathbf{a}\cdot\mathbf{b}|$. [1]

(v) If $|\mathbf{a}\cdot\mathbf{b}|=\frac{1}{2}|\mathbf{a}|$, find the perpendicular distance of point A to OB, leaving your answer in terms of $|\mathbf{a}|$. [3]

12 A tank initially contains 100 litres of pure water. A brine solution, with a salt concentration of 0.2 kg per litre, is pumped into the tank continuously at a constant rate of 5 litres per minute. At the same time, a well-stirred mixture of the tank's contents is continuously drained from the tank at the same rate of 5 litres per minute. At time t minutes, the mass of salt in the tank is denoted by Q kilograms.

(i) Explain briefly why the volume of the mixture in the tank remains constant. [1]

(ii) By considering the salt concentration of the brine solution, show that salt enters the tank at a constant rate of 1 kg per minute. [1]

(iii) Hence, by also considering the mass of salt leaving the tank, show that the rate of change of the mass of salt in the tank can be expressed as

$$\frac{dQ}{dt}=1-\frac{Q}{20}.$$ [2]

(iv) Solve the differential equation found in part **(iii)** to find Q in terms of t. [5]

(v) Determine the time taken for the salt concentration in the tank to reach 0.1 kg per litre. [2]

(vi) What happens to Q for large values of t? [1]

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Pre-University 3

MATHEMATICS

9758/02

Paper 2

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3 hours

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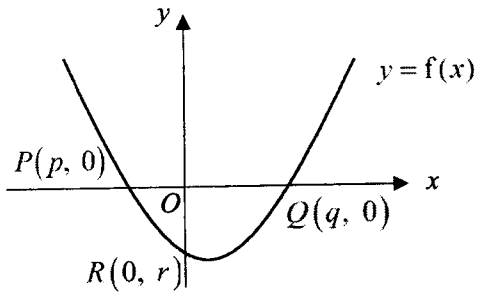
Section A: Pure Mathematics [40 marks]

1 The diagram shows the curve with equation $y = \mathrm{f}(x)$. The curve passes through the points $P(p, 0)$, $Q(q, 0)$ and $R(0, r)$.



(a) The curve $y = \mathrm{f}(x)$ is transformed onto the curve with equation $y = \mathrm{f}(2x-1)$. Find the coordinates of the points on the graph of $y = \mathrm{f}(2x-1)$ which correspond to the points P, Q and R on curve $y = \mathrm{f}(x)$. [3]

(b) It is given that $I = \int_p^q \mathrm{f}(x)\,\mathrm{d}x$. Find, in terms of I, the area of the finite region bounded by

(i) the curve with equation $y = -\mathrm{f}(x)$ and the x-axis, [1]

(ii) the curve with equation $y = \frac{1}{2}\mathrm{f}(x+3)$ and the x-axis. [1]

(c) Find the value of $\int_0^q \mathrm{f}'(x)\,\mathrm{d}x$, justifying your answer. [1]

2 A curve C has parametric equations $x = \sqrt{t}$, $y = t^2 + 4$, where $0 \le t \le 16$.

(i) Find the equation of the tangent to C at the point P with parameter p. [3]

(ii) Given that the tangent in part **(i)** passes through the point $(0,1)$, find the exact coordinates of the point P. [3]

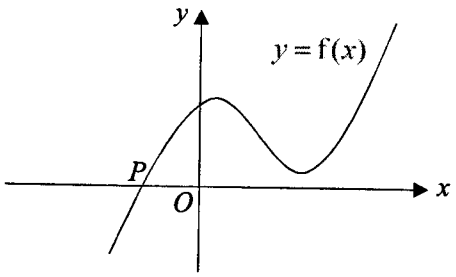
(iii) Sketch the curve C. [2]

3 **(a)** The complex numbers z and w satisfy the simultaneous equations

$$z+2z^*+w=6+\mathrm{i} \text{ and } \mathrm{i}z-w=3.$$

Find z and w, leaving your answers in the form $a+bi$, where a and b are real numbers. [4]

(b) The diagram shows the curve with equation $y=\mathrm{f}(x)$, where $\mathrm{f}(x)$ is given by x^3+mx^2+nx+5, and m and n are real constants. The curve crosses the x-axis at the point P.



One of the roots of the equation $\mathrm{f}(x)=0$ is $2+\mathrm{i}$. Determine the coordinates of the point P. [4]

4 A sequence is defined by $T_{n+2}=T_{n+1}+T_n$ with $T_1=3$, $T_2=1$ for $n\geq 1$. Another sequence is defined by $r_n=\dfrac{T_{n+1}}{T_n}$ for $n\geq 1$.

(a) Find T_4. [1]

(b) Show that $r_{n+1}=1+\dfrac{1}{r_n}$ for all for $n\geq 1$. [2]

(c) It is given that r_n converges to a limit L. Find the exact value of L. [3]

A third sequence u_n for $n\geq 1$, is an arithmetic sequence with first term 5 and common difference 2.

(d) Find the least value of n such that $\sum_{k=1}^{n} u_k > 930$. [2]

5 The function g is given by $\mathrm{g}: x \mapsto \dfrac{x-3}{x-1}$, for $x \in \mathbb{R}$, $x \neq 1$.

(i) Sketch the graph of $y = \mathrm{g}(x)$, stating clearly the coordinates of any points of intersection with the axes and the equation of any asymptotes. [3]

(ii) Explain if g^{-1} exists. [1]

(iii) Find $\mathrm{g}^{-1}(x)$ and state the domain of g^{-1}. [3]

A function f is said to be self-inverse if $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$ for all $x \in \mathbb{R}$ in the domain of f.

(iv) Show that g is self-inverse. [1]

(v) Without the use of a calculator, solve the equation $\mathrm{g}^2(x) = -\mathrm{g}^{-1}(x)$. [2]

Section B: Probability and Statistics [60 marks]

6 The waiting times for customers to get their pastries from a popular bakery is modelled by a normal distribution with mean μ minutes and standard deviation σ minutes. On average, 10% of customers wait for more than 25 minutes and the same percentage of customers wait for less than 5 minutes.

(i) Find the values of μ and σ, leaving your answers to the nearest minute. [3]

(ii) Explain if the normal distribution is a suitable model for the waiting times for customers to get pastries from this bakery. [1]

7 A deck of 12 cards consists of 2 gold cards and 10 silver cards. The cards are identical other than their colours. In a game, three cards are drawn from the deck of cards at random without replacement. The number of gold cards drawn is denoted by G.

(i) Explain the significance of stating that the cards are identical other than their colours. [1]

(ii) Show that $\mathrm{P}(G=2)=\frac{1}{22}$ and find the probabilities of all other possible combinations of the cards drawn. [4]

A gold card is worth 4 points while a silver card is worth 1 point. The score X is the sum of the points of each of the three drawn cards.

(iii) Using your answers in part **(ii)**, find the probability distribution of X. [2]

(iv) If the player of the game wins \$10x for a score of x points, what is the expected winnings of the player? [1]

8 For events A and B, it is given that $\mathrm{P}(A)=\frac{11}{20}$, $\mathrm{P}(A\cup B)=\frac{4}{5}$ and $\mathrm{P}(B|A)=\frac{4}{11}$.

(i) Find

(a) $\mathrm{P}(B)$, [3]

(b) $\mathrm{P}(A\cap B')$. [1]

(ii) Determine if the events A and B' are mutually exclusive. [1]

A third event C is such that $\mathrm{P}(C)=0.6$ and B and C are independent.

(iii) Find $\mathrm{P}(B\cap C)$. [1]

(iv) Find the range of values of $\mathrm{P}(A\cap B'\cap C)$. [2]

9 In a particular stage of a computer game, a player is given a first set of 20 tasks,

- If he completes fewer than 13 tasks, he has to retry the stage.
- If he completes more than 16 tasks, he progresses directly to the next stage.
- Otherwise, he is given a second set of 10 tasks.
  - If he completes more than 7 of these 10 tasks, he progresses to the next stage.
  - Otherwise, he has to retry the stage.

It is known that the player's performances in the two sets of tasks are independent.

Let X and Y be the number of tasks completed in the first and second sets of tasks respectively.

(i) State two assumptions needed for X to be well-modelled by a binomial distribution. [2]

Assume now that X has the distribution B(20, 0.7) and Y has the distribution B(10, 0.8),

(ii) Find the probability that the player is given a second set of tasks. [2]

(iii) Using a tree diagram or otherwise, find the probability that the player

(a) progresses to the next stage, [3]

(b) completes exactly 15 tasks in the first set of tasks given he does not progress to the next stage. [2]

10 **In this question you should state clearly all the distributions that you use, together with the values of the appropriate parameters.**

In a fruit shop, the masses, in kg, of a certain type of apple and pear are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean	Standard deviation
Apple	0.2	0.05
Pear	0.3	0.08

(i) An apple and a pear are chosen at random.

(a) Find the probability that the mass of the apple is more than 0.25 kg and the mass of the pear is less than 0.25 kg. [1]

(b) Find the probability that the mass of the apple is within 0.075 kg of the mass of the pear. [4]

Apples are sold at \$5 per kg and pears at \$7 per kg.

(ii) Find the probability that the total price of 10 randomly chosen apples and 5 randomly chosen pears exceeds \$22. [4]

11 Cardiovascular fitness is commonly measured using VO_2 max, the maximum amount of oxygen the body can use during intense exercise. The higher the value of VO_2 max, the fitter the person is. In a study on cardiovascular fitness, the resting heart rate x, measured in beats/min and the VO_2 max y, measured in ml/kg/min, of 8 randomly selected people are given in the table below.

x	78	75	65	64	62	56	53	48
y	36	41	47	48	48	56	58	63

(i) Draw a scatter diagram for these values, labelling the axes. [1]

(ii) Find the product moment correlation coefficient between y and x and comment on its value in the context of the data. [2]

(iii) Use an appropriate regression line to estimate the VO_2 max of a person with a resting heart rate of 60 beats/min, leaving your answer to the nearest integer. Comment on the reliability of your estimate. [3]

(iv) Use a specific example to explain why extrapolating using your regression line in part **(iii)** to predict the VO_2 max of people with very high or very low resting heart rate is not advisable. [1]

Due to an instrumentation issue, all the resting heart rate values are to be increased by v beats/min.

(v) Find the value of the product moment correlation coefficient, and the gradient and y-intercept of the regression line of y on x after these adjustments, leaving your answer in terms of v where applicable. [3]

12 Supernova School is a school which specialises in sports. The mean reaction time of the students in this school is 250 milliseconds. To reduce the reaction times of students, the teachers implemented a reflex training programme. After its completion, 50 students were randomly selected from the school, and their reaction times x (in milliseconds) were recorded as follows:

$$\sum(x-200)=2159.62 \qquad \sum(x-200)^2=112081.09$$

(i) Calculate the unbiased estimates of the population mean and variance of the reaction time of students. [2]

(ii) Test, at 1% level of significance, whether the programme was effective. You should state your hypotheses and define any symbols you use. [5]

(iii) Explain why the test in part **(ii)** may not be appropriate if only a sample of 10 students was taken. [1]

In another school, Galaxy School, the reaction times of its students follow a normal distribution with mean μ_0 milliseconds and variance 153 milliseconds^2. The school also implemented the reflex training programme with its students.

(iv) A random sample of 40 students is taken and the mean reaction time of this sample is computed to be 241 milliseconds. A test carried out at 5% level of significance indicated that the reflex training programme was not successful. Find the range of values of μ_0. [4]