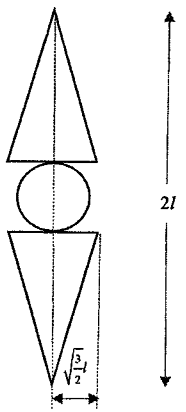
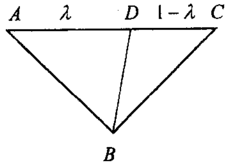
Q1	Suggested Answers
	Since C has an oblique asymptote with gradient 1.5, $p=1.5$ C contains the point (3, 7): $7=\frac{1.5(3)^2+q(3)+r}{3-1}$ $r=0.5-3q$ $\therefore y=\frac{1.5x^2+qx+0.5-3q}{x-1}$ Turning point at (3, 7): $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(3x+q)(x-1)-(1.5x^2+qx+0.5-3q)(1)}{(x-1)^2}$ $0=\frac{(3(3)+q)(3-1)-(1.5(3)^2+q(3)+0.5-3q)}{(3-1)^2}$ $(9+q)(2)-14=0$ $\therefore q=-2$ $\therefore r=0.5-3(-2)=6.5$

Q2	Suggested Answers
	Method 1: Let the radius of the sphere be r and the height of the cone be h. $2h+2r=2l \Rightarrow h=l-r$ Total volume, $V=2\left(\frac{1}{3}\pi h\times\frac{3}{2}l^2\right)+\frac{4}{3}\pi r^3$ $=\pi l^2(l-r)+\frac{4}{3}\pi r^3$ $\frac{\mathrm{d}V}{\mathrm{d}r}=\pi l^2(-1)+4\pi r^2$ $\frac{\mathrm{d}V}{\mathrm{d}r}=0 \Rightarrow 4\pi r^2=\pi l^2$ Since $r>0$, $r=\frac{l}{2}$ $\frac{\mathrm{d}^2V}{\mathrm{d}r^2}=8\pi r>0$ for $r=\frac{l}{2}$, ie minimum volume when $r=\frac{l}{2}$. 
	Method 2: Let the radius of the sphere be r and the height of the cone be h. $2h+2r=2l \Rightarrow h=l-r$ Total volume, $V=2\left(\frac{1}{3}\pi h\times\frac{3}{2}l^2\right)+\frac{4}{3}\pi r^3=\pi l^2h+\frac{4}{3}\pi(l-h)^3$ $\frac{\mathrm{d}V}{\mathrm{d}h}=\pi l^2-4\pi(l-h)^2=0$ $l^2=4(l-h)^2$ $l=2(l-h)$ or $l=2(h-l)$ $l=2l-2h$ or $3l=2h$ $h=\frac{l}{2}$ or $h=\frac{3l}{2}$ (reject $\because l>h$) $\frac{\mathrm{d}^2V}{\mathrm{d}h^2}=8\pi(l-h)$ When $h=\frac{l}{2}$, $\frac{\mathrm{d}^2V}{\mathrm{d}h^2}=4\pi l>0$, ie minimum volume when $r=\frac{l}{2}$.

Q3	Suggested Answers
(a)	**Method 1** $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC} \Rightarrow \left\|\overrightarrow{AB}+\overrightarrow{BC}\right\|^2=\left\|\overrightarrow{AC}\right\|^2$ $\Rightarrow \left(\overrightarrow{AB}+\overrightarrow{BC}\right)\cdot\left(\overrightarrow{AB}+\overrightarrow{BC}\right)=\overrightarrow{AC}\cdot\overrightarrow{AC} \quad (\because \|\mathbf{v}\|^2=\mathbf{v}\cdot\mathbf{v})$ $\Rightarrow \overrightarrow{AB}\cdot\overrightarrow{AB}+2\left(\overrightarrow{AB}\cdot\overrightarrow{BC}\right)+\overrightarrow{BC}\cdot\overrightarrow{BC}=\overrightarrow{AC}\cdot\overrightarrow{AC}$ $\Rightarrow AB^2+BC^2+2\left(\overrightarrow{AB}\cdot\overrightarrow{BC}\right)=AC^2$ $\Rightarrow AC^2+2\left(\overrightarrow{AB}\cdot\overrightarrow{BC}\right)=AC^2$ since $AB^2+BC^2=AC^2$ $\Rightarrow \overrightarrow{AB}\cdot\overrightarrow{BC}=0$ So AB is perpendicular to BC and hence $\angle ABC=90°$.
	Method 2 $AB^2+BC^2=AC^2 \Rightarrow \left\|\overrightarrow{AB}\right\|^2+\left\|\overrightarrow{BC}\right\|^2=\left\|\overrightarrow{AC}\right\|^2$ $\Rightarrow \left\|\overrightarrow{AB}\right\|^2+\left\|\overrightarrow{BC}\right\|^2=\left\|\overrightarrow{AB}+\overrightarrow{BC}\right\|^2$ $\Rightarrow \left\|\overrightarrow{AB}\right\|^2+\left\|\overrightarrow{BC}\right\|^2=\left(\overrightarrow{AB}+\overrightarrow{BC}\right)\cdot\left(\overrightarrow{AB}+\overrightarrow{BC}\right)$ $\Rightarrow \left\|\overrightarrow{AB}\right\|^2+\left\|\overrightarrow{BC}\right\|^2=\overrightarrow{AB}\cdot\overrightarrow{AB}+\overrightarrow{BC}\cdot\overrightarrow{BC}+2\left(\overrightarrow{AB}\cdot\overrightarrow{BC}\right)$ $\Rightarrow \left\|\overrightarrow{AB}\right\|^2+\left\|\overrightarrow{BC}\right\|^2=\left\|\overrightarrow{AB}\right\|^2+\left\|\overrightarrow{BC}\right\|^2+2\left(\overrightarrow{AB}\cdot\overrightarrow{BC}\right)$ $\Rightarrow \overrightarrow{AB}\cdot\overrightarrow{BC}=0$ So AB is perpendicular to BC and hence $\angle ABC=90°$.
(b)	**Method 1** By ratio theorem, $\mathbf{d}=\overrightarrow{OD}=(1-\lambda)\overrightarrow{OA}+\lambda\overrightarrow{OC}=(1-\lambda)\mathbf{a}+\lambda\mathbf{c}$.  Area of triangle ABD $=\frac{1}{2}\left\|\overrightarrow{AB}\times\overrightarrow{BD}\right\|$ $=\frac{1}{2}\left\|(\mathbf{b}-\mathbf{a})\times(\mathbf{d}-\mathbf{b})\right\|$ $=\frac{1}{2}\left\|(\mathbf{b}-\mathbf{a})\times((1-\lambda)\mathbf{a}+\lambda\mathbf{c}-\mathbf{b})\right\|$ $=\frac{1}{2}\left\|(1-\lambda)(\mathbf{b}\times\mathbf{a})+\lambda(\mathbf{b}\times\mathbf{c})-\lambda(\mathbf{a}\times\mathbf{c})+\mathbf{a}\times\mathbf{b}\right\|$ $=\frac{1}{2}\left\|(\lambda-1)(\mathbf{a}\times\mathbf{b})+\lambda(\mathbf{b}\times\mathbf{c})-\lambda(\mathbf{a}\times\mathbf{c})+\mathbf{a}\times\mathbf{b}\right\|$ since $\mathbf{b}\times\mathbf{a}=-\mathbf{a}\times\mathbf{b}$. $=\frac{1}{2}\left\|\lambda(\mathbf{a}\times\mathbf{b})+\lambda(\mathbf{b}\times\mathbf{c})-\lambda(\mathbf{a}\times\mathbf{c})\right\|$ $=\frac{\lambda}{2}\left\|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}\right\|$ since $0<\lambda<1$. So $k=\frac{\lambda}{2}$.

	Method 2 Since triangles ABD and ABC share the same height, Area of triangle ABD $=\frac{\lambda}{\lambda+(1-\lambda)}(\text{Area of triangle } ABC)$ $=\lambda\left(\frac{1}{2}\left\|\overrightarrow{AB}\times\overrightarrow{AC}\right\|\right)$ $=\frac{\lambda}{2}\left\|(\mathbf{b}-\mathbf{a})\times(\mathbf{c}-\mathbf{a})\right\|$ $=\frac{\lambda}{2}\left\|\mathbf{b}\times\mathbf{c}-\mathbf{b}\times\mathbf{a}-\mathbf{a}\times\mathbf{c}\right\|$ $=\frac{\lambda}{2}\left\|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}\right\|$ So $k=\frac{\lambda}{2}$.

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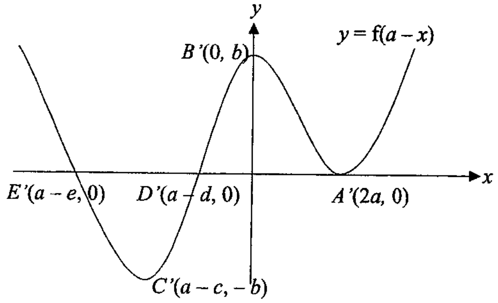
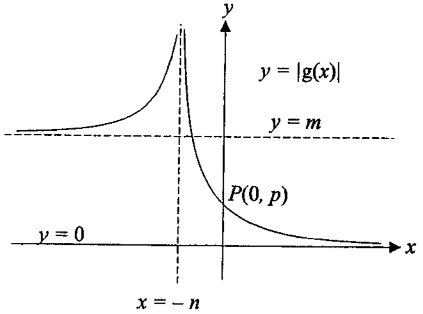
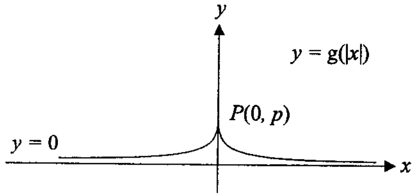
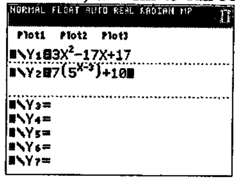
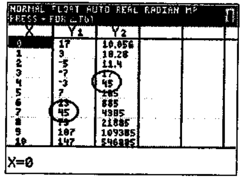
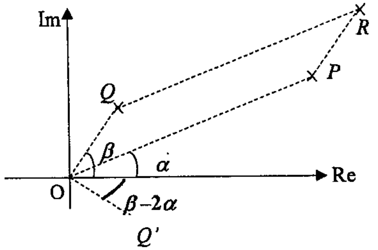
Q4	Suggested Answers
(a)	
(b)(i)	
(b)(ii)	

Q5	Suggested Answers
(a)	$\sum_{r=m+1}^{n}\left(3r^2-17r+17\right)$ $=3\sum_{r=m+1}^{n} r^2+\sum_{r=m+1}^{n}(-17r+17)$ $=3\left(\sum_{r=1}^{n} r^2-\sum_{r=1}^{m} r^2\right)+17\sum_{r=m+1}^{n}(-r+1)$ $=3\left[\frac{n}{6}(n+1)(2n+1)-\frac{m}{6}(m+1)(2m+1)\right]$ $+17\left[\frac{n-m}{2}(-m+(-n+1))\right]$ $=\frac{n}{2}(n+1)(2n+1)-\frac{m}{2}(m+1)(2m+1)$ $+17\left[\frac{n-m}{2}(1-m-n)\right]$
(b)	From GC, the term 45 can be found in both series.  NORMAL FLOAT AUTO REAL RADIAN MP Plot1 Plot2 Plot3 Y1=3X²−17X+17 Y2=7(5^(X−3))+10 Y3= Y4= Y5= Y6= Y7=  NORMAL FLOAT AUTO REAL RADIAN MP PRESS + FOR ΔTbl X · Y1 · Y2 0 · 17 · 10.056 1 · 3 · 10.28 2 · -5 · 11.4 3 · -7 · 17 4 · -3 · 45 5 · 7 · 185 6 · 23 · 885 7 · 45 · 4385 8 · 73 · 21885 9 · 107 · 109385 10 · 147 · 546885 X=0
(c)	The *n*th term of J, $j_n=\frac{b}{2}(3^n-1)-\frac{b}{2}(3^{n-1}-1)$ $=\frac{b}{2}(3^n-3^{n-1})$ $=\frac{b}{2}(3^{n-1})(3-1)$ $=b3^{n-1}$ Given b is a positive odd integer, then j_n is a positive odd integer since 3^{n-1} is odd. Since the arithmetic progression is 1, 3, 5, …, the set of **all** positive odd integers, each of j_n must be a part of the arithmetic progression.

Q6	Suggested Answers
(a)	If $\|p\|=\|q\|$, $OPRQ$ forms a rhombus. $\arg(r)=\frac{1}{2}(\beta-\alpha)+\alpha=\frac{1}{2}(\alpha+\beta)$
(b)	 $\angle POQ'=\beta-\alpha$
(b)(i)	**Method 1:** $\arg(q')=-(\beta-2\alpha)$ since $\beta>2\alpha$ **Method 2:** $\arg(q')=\alpha-(\beta-\alpha)=2\alpha-\beta$
(b)(ii)	$\|q'\|=\|q\|$ Let F be the foot of perpendicular from Q' to the real axis. Real part of $q'=\|q\|\cos(-\beta+2\alpha)$ (Adjacent side of $\Delta OQ'F$) Imaginary part of $q'=[\|q\|\sin(-\beta+2\alpha)]$ (Opposite side of $\Delta OQ'F$ but negative) $\therefore q'=\|q\|\cos(-\beta+2\alpha)+\mathrm{i}\|q\|\sin(-\beta+2\alpha)$ Alternative: $q'=\|q\|\cos(\beta-2\alpha)-\mathrm{i}\|q\|\sin(\beta-2\alpha)$

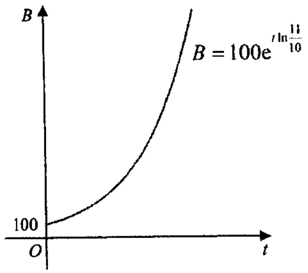
Q7	Suggested Answers
(a)	$x=\sin^2\theta$ $\frac{\mathrm{d}x}{\mathrm{d}\theta}=2\sin\theta\cos\theta$ When $x=0,\theta=0$. When $x=\frac{1}{2},\theta=\frac{\pi}{4}$ $\int_0^{\frac{1}{2}}\sqrt{\frac{16x}{1-x}}\ \mathrm{d}x=\int_0^{\frac{\pi}{4}}\sqrt{\frac{16\sin^2\theta}{1-\sin^2\theta}}(2\sin\theta\cos\theta)\ \mathrm{d}\theta$ $=\int_0^{\frac{\pi}{4}}\sqrt{\frac{16\sin^2\theta}{\cos^2\theta}}(2\sin\theta\cos\theta)\ \mathrm{d}\theta$ $=\int_0^{\frac{\pi}{4}}\frac{4\sin\theta}{\cos\theta}(2\sin\theta\cos\theta)\ \mathrm{d}\theta$ $\left(\because 0\le\theta\le\frac{\pi}{2},\sqrt{\cos^2\theta}=\cos\theta\right)$ $=\int_0^{\frac{\pi}{4}}8\sin^2\theta\ \mathrm{d}\theta$ $=4\int_0^{\frac{\pi}{4}}1-\cos 2\theta\ \mathrm{d}\theta$ $=[4\theta-2\sin 2\theta]_0^{\frac{\pi}{4}}$ $=\pi-2$
(b)	$\int_a^b\frac{\|x-1\|}{\sqrt{\frac{1}{2}x^2-x+1}}\ \mathrm{d}x$ $=\int_a^1\frac{-(x-1)}{\sqrt{\frac{1}{2}x^2-x+1}}\ \mathrm{d}x+\int_1^b\frac{(x-1)}{\sqrt{\frac{1}{2}x^2-x+1}}\ \mathrm{d}x$ $=-\left[\frac{\left(\frac{1}{2}x^2-x+1\right)^{\frac{1}{2}}}{\frac{1}{2}}\right]_a^1+\left[\frac{\left(\frac{1}{2}x^2-x+1\right)^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^b$ $=-\left[2\left(\frac{1}{2}x^2-x+1\right)^{\frac{1}{2}}\right]_a^1+\left[2\left(\frac{1}{2}x^2-x+1\right)^{\frac{1}{2}}\right]_1^b$ $=-\left[2\left(\frac{1}{2}\right)^{\frac{1}{2}}-2\left(\frac{1}{2}a^2-a+1\right)^{\frac{1}{2}}\right]+\left[2\left(\frac{1}{2}b^2-b+1\right)^{\frac{1}{2}}-2\left(\frac{1}{2}\right)^{\frac{1}{2}}\right]$ $=-2\sqrt{2}+2\left(\frac{1}{2}a^2-a+1\right)^{\frac{1}{2}}+2\left(\frac{1}{2}b^2-b+1\right)^{\frac{1}{2}}$

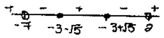
Q8	Suggested Answers
(a)	Note that the sequence is an arithmetic progression with common difference 1.5 $u_{50} = u_1 + 49(1.5)$ $99u_1 = u_1 + 73.5$ $u_1 = 0.75$
(b)	**Method 1** $\sum_{k=1}^{n} u_{2k} = u_2 + u_4 + \ldots + u_{2n}$ is a sum of n terms of an arithmetic sequence with first term $u_2 = 0.75 + 1.5 = 2.25$ and common difference $2(1.5) = 3$ $u_2 + u_4 + \ldots + u_{2n} = \frac{n}{2}\left(2(2.25) + (n-1)(3)\right)$ $= \frac{n}{2}\left(1.5 + 3n\right)$ From GC, for $\frac{n}{2}\left(1.5 + 3n\right) > 2025$, least $n = 37$
	Method 2 $\sum_{k=1}^{n} u_{2k} = \sum_{k=1}^{n}\left(2.25 + (2k-1)3\right) > 2025$ Using GC, least $n = 37$
(c)(i)	$\frac{u_k}{u_{25}} = \frac{u_5}{u_k}$ $u_k^2 = u_5 u_{25}$ $\left[0.75 + (k-1)(1.5)\right]^2 = \left[0.75 + (5-1)(1.5)\right]\left[0.75 + (25-1)(1.5)\right]$ $0.75 + (k-1)(1.5) = \sqrt{6.75 \times 36.75} = 15.75$ $k = 11$
(c)(ii)	**Method 1** Common ratio $= \frac{15.75}{36.75} = \frac{3}{7}$ $S_{13} = \frac{36.75\left(1 - (\frac{3}{7})^{13}\right)}{1 - \frac{3}{7}}$ $= 64.31144$ $= 64.311$
	Method 2 Using GC, $\sum_{r=1}^{13}\left(36.75\left(\frac{3}{7}\right)^{r-1}\right) = 64.311$

Q9	Suggested Answers
(a)	$k=\frac{3}{2}$ When $x=\frac{3}{2}$, the function will be undefined.
(b)	$\mathrm{D_f}=\mathbb{R}\setminus\left\{\frac{3}{2}\right\}$ $\mathrm{R_f}=\mathbb{R}\setminus\left\{\frac{1}{2}\right\}$ Since $\mathrm{R_f}\not\subset\mathrm{D_f}$, f^2 does not exist.
(c)	$y=\frac{x+1}{2x-3}$ $(2x-3)y=x+1$ $2xy-3y=x+1$ $2xy-x=3y+1$ $x=\frac{3y+1}{2y-1}$ $\mathrm{f}^{-1}(x)=\frac{3x+1}{2x-1}$
(d)	**Method 1:** $\mathrm{fh}(x)=\frac{3x-7}{2x-1}$ $\mathrm{f}^{-1}\mathrm{fh}(x)=\mathrm{f}^{-1}\left(\frac{3x-7}{2x-1}\right)$ $\mathrm{h}(x)=\mathrm{f}^{-1}\left(\frac{3x-7}{2x-1}\right)$ $=\frac{3\left(\frac{3x-7}{2x-1}\right)+1}{2\left(\frac{3x-7}{2x-1}\right)-1}$ $=\frac{9x-21+2x-1}{6x-14-2x+1}$ $=\frac{11x-22}{4x-13}$
	Method 2: $\mathrm{fh}(x)=\frac{3x-7}{2x-1}$ $\mathrm{fh}(x)=\frac{\mathrm{h}(x)+1}{2\mathrm{h}(x)-3}=\frac{3x-7}{2x-1}$ $(2x-1)(\mathrm{h}(x)+1)=(3x-7)(2\mathrm{h}(x)-3)$ $2x\mathrm{h}(x)+2x-\mathrm{h}(x)-1=6x\mathrm{h}(x)-9x-14\mathrm{h}(x)+21$ $-4x\mathrm{h}(x)+13\mathrm{h}(x)=-11x+22$ $\mathrm{h}(x)=\frac{11x-22}{4x-13}$

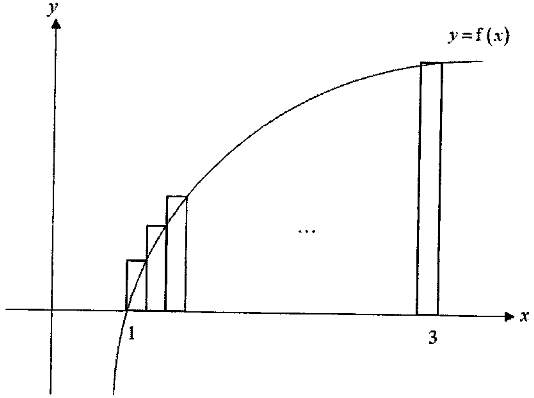
Q10	Suggested Answers
(a)	$\cos(t^3)=1-\frac{(t^3)^2}{2!}+\frac{(t^3)^4}{4!}+\cdots=1-\frac{t^6}{2}+\frac{t^{12}}{24}+\cdots$ $\int_0^x t^2\cos(t^3)\mathrm{d}t$ $\approx\int_0^x t^2(1-\frac{t^6}{2}+\frac{t^{12}}{24})\mathrm{d}t$ $=\int_0^x t^2-\frac{t^8}{2}+\frac{t^{14}}{24}\mathrm{d}t$ $=[\frac{t^3}{3}-\frac{t^9}{18}+\frac{t^{15}}{360}]_0^x$ $=\frac{x^3}{3}-\frac{x^9}{18}+\frac{x^{15}}{360}$
(b)	$\int_0^{0.1}t^2\cos(t^3)\,\mathrm{d}t\approx\frac{0.1^3}{3}-\frac{0.1^9}{18}+\frac{0.1^{15}}{360}\approx0.00033(5\text{d.p.})$
(c)	$\int_0^x t^2\cos(t^3)\mathrm{d}t=\frac{1}{3}[\sin(t^3)]_0^x=\frac{1}{3}\sin(x^3)$ $\int_0^{0.1}t^2\cos(t^3)\,\mathrm{d}t=\frac{1}{3}\sin(0.1^3)=0.00033\ (5\text{ d.p.})$
(d)	The approximation was accurate up to 5 d.p. Approximation was good since $x=0.1$ is close to zero.

Q11	Suggested Answers
(a)	$x^2-y^2=16\Rightarrow y=\sqrt{x^2-16}\left(\because y\geq0\right)$ Area required $=\int_5^8\sqrt{x^2-16}-\left(-\frac{1}{2}x+\frac{11}{2}\right)\mathrm{d}x$ $=8.4723$ $=8.47\text{units}^2$
(b)	Volume required $=\pi\int_4^5 x^2-16\ \mathrm{d}x+\frac{1}{3}\pi(3)^2(6)$ $=\pi\left[\frac{x^3}{3}-16x\right]_4^5+18\pi$ $=\pi\left[-\frac{115}{3}-\left(-\frac{128}{3}\right)\right]+18\pi$ $=\frac{67}{3}\pi\text{units}^3$
(c)	Volume required $=\pi(5)^2(3)-\pi\int_0^3 y^2+16\ \mathrm{d}y$ $=75\pi-\pi\left[\frac{y^3}{3}+16y\right]_0^3$ $=75\pi-57\pi$ $=18\pi\text{ units}^3$

Q12	Suggested Answers
(a)	$\frac{\mathrm{d}B}{\mathrm{d}t} = kB$, where $k \in \mathbb{R}$.
(b)	$\frac{1}{B}\frac{\mathrm{d}B}{\mathrm{d}t} = k$ Integrating both sides w.r.t. t, $\int \frac{1}{B}\,\mathrm{d}B = \int k\,\mathrm{d}t$ $\ln B = kt + C$, where C is an arbitrary constant (note that $B \geq 0$ since it refers to mass) $B = \mathrm{e}^{kt+C} = D\mathrm{e}^{kt}$ Given $B = 100$ when $t = 0$: $100 = D$ Given $B = 110$ when $t = 1$: $110 = 100\mathrm{e}^{k}$ $k = \ln\frac{11}{10}$. $\therefore B = 100\mathrm{e}^{t\ln\frac{11}{10}}$ or $100\left(\frac{11}{10}\right)^{t}$
(c)	 The biomass would grow indefinitely if it was left on its own.
(d)	$\frac{\mathrm{d}B}{\mathrm{d}t} = mB\frac{1}{3+2t^2}$, where $m \in \mathbb{R}$. $\frac{1}{B}\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{m}{3+2t^2}$ Integrating both sides w.r.t.t: $\int \frac{1}{B}\,\mathrm{d}B = \int \frac{m}{3+2t^2}\,\mathrm{d}t$ $= \frac{m}{2}\int \frac{1}{\frac{3}{2}+t^2}\,\mathrm{d}t$ $\ln B = \frac{m}{2}\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{2}{3}}t\right) + C$ $B = \mathrm{e}^{\frac{m}{\sqrt{6}}\tan^{-1}\left(\sqrt{\frac{2}{3}}t\right)+C}$ $= D\mathrm{e}^{\frac{m}{\sqrt{6}}\tan^{-1}\left(\sqrt{\frac{2}{3}}t\right)}$ Given $B = 100$ when $t = 0$: $100 = D$ Given $B = 101$ when $t = 1$: $101 = 100\mathrm{e}^{\frac{m}{\sqrt{6}}\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)}$ $m = \left(\ln\frac{101}{100}\right)\frac{\sqrt{6}}{\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)} = 0.03559595$ $B = 100\mathrm{e}^{\frac{0.03559595}{\sqrt{6}}\tan^{-1}\left(\sqrt{\frac{2}{3}}t\right)}$ $= 100\mathrm{e}^{0.014532\tan^{-1}\left(\sqrt{\frac{2}{3}}t\right)}$ $= 100\mathrm{e}^{0.0145\tan^{-1}\left(\sqrt{\frac{2}{3}}t\right)}$
(e)	As $t \to \infty$, $\tan^{-1}\left(\sqrt{\frac{2}{3}}t\right) \to \frac{\pi}{2}$ $\therefore B \to 100\mathrm{e}^{0.014532\left(\frac{\pi}{2}\right)} = 102.30893$ grams

Q1	Suggested Answers
(a)	**Method 1:** From GC, set required is $\{x \in \mathbb{R} : 2 \le x \le 8\}$.
	Method 2: $3\lvert x-3\rvert \le \lvert 1-2x\rvert$ $9(x-3)^2 \le (1-2x)^2$ $9x^2-54x+81 \le 1-4x+4x^2$ $5x^2-50x+80 \le 0$ $x^2-10x+16 \le 0$ $(x-2)(x-8) \le 0$ The set required is $\{x \in \mathbb{R} : 2 \le x \le 8\}$.
(b)	$\dfrac{x+18}{x^2+5x-14} \ge -1$ $\dfrac{x+18+(x^2+5x-14)}{(x+7)(x-2)} \ge 0 \quad x \ne -7 \text{ or } 2$ $\dfrac{x^2+6x+4}{(x+7)(x-2)} \ge 0$ $\dfrac{(x+3)^2-5}{(x+7)(x-2)} \ge 0$ $\dfrac{(x+3-\sqrt{5})(x+3+\sqrt{5})}{(x+7)(x-2)} \ge 0$  $\therefore x < -7 \text{ or } -3-\sqrt{5} \le x \le -3+\sqrt{5} \text{ or } x > 2$

Q2	Suggested Answers
(a)	$-\mathrm{i}x^3+5\mathrm{i}x^2+ax+b=0$ Replacing $a=\lambda\mathrm{i}$ and $b=\mu\mathrm{i}$ where $\lambda, \mu \in \mathbb{R}$: $-\mathrm{i}x^3+5\mathrm{i}x^2+\lambda\mathrm{i}x+\mu\mathrm{i}=0$ $x^3-5x^2-\lambda x-\mu=0$ Since the coefficients of the equation are all real, the complex roots occur in conjugate pair.
(b)	$\mathrm{i}x^3+5\mathrm{i}x^2+a^*x+b=0$ Since a is purely imaginary, $a^*=-a$: $-\mathrm{i}(-x)^3+5\mathrm{i}(-x)^2+a(-x)+b=0$ $-x=2-\mathrm{i},\ 2+\mathrm{i}$ and 1 $\therefore x=-2+\mathrm{i},\ -2-\mathrm{i}$ and -1

Q3	Suggested Answers
(a)	 The width of each of the n rectangles is $\dfrac{2}{n}$. The height of the first rectangle is $\mathrm{f}\left(1+\dfrac{2}{n}\right)$, second rectangle is $\mathrm{f}\left(1+\dfrac{4}{n}\right)$, …and the n rectangle is $\mathrm{f}\left(1+\dfrac{2n}{n}\right)$. The area of the n rectangles is $\dfrac{2}{n}\left\{\mathrm{f}\left(1+\dfrac{2}{n}\right)+\mathrm{f}\left(1+\dfrac{4}{n}\right)+\ldots+\mathrm{f}\left(1+\dfrac{2n}{n}\right)\right\}$. When $n \to \infty$, the area of the n rectangles tends towards the area under the curve of $y=\mathrm{f}(x)$ from $x=1$ to $x=3$ which is $\int_1^3 \mathrm{f}(x)\ \mathrm{d}x$.
(b)	$\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\ln\left(1+\dfrac{2k}{n}\right)\dfrac{2}{n}$ $=\int_1^3 \ln x\ \mathrm{d}x$ $=[x\ln x]_1^3-\int_1^3 \dfrac{1}{x}(x)\ \mathrm{d}x$ $=[x\ln x]_1^3-\int_1^3 1\ \mathrm{d}x$ $=[x\ln x]_1^3-[x]_1^3$ $=3\ln 3-(3-1)$ $=3\ln 3-2$

Q4	Suggested Answers
(a)	Differentiating wrt x : $3x^2+x\frac{\mathrm{d}y}{\mathrm{d}x}+y+6y^2\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-y-3x^2}{x+6y^2}$
(b)	If tangent is parallel to y-axis, $x+6y^2=0$, ie, $x=-6y^2$ Point of contact of tangent with C is given by $\left(-6y^2\right)^3+y\left(-6y^2\right)+2y^3=k$ ie, $-216y^6-6y^3+2y^3=k\Rightarrow 216y^6+4y^3+k=0$ (shown) For y to be real, discriminant ≥ 0, ie, $4^2-4(216)(k)\geq 0$ $4\geq 216k\Rightarrow k\leq\frac{4}{216}$, ie, $k\leq\frac{1}{54}$
(c)	When $x=-6$, $-6y^2=-6\Rightarrow y=\pm 1$ Therefore, $k=-220$ or -212

Q5	Suggested Answers
(a)	Let θ be the acute angle between p_1 and p_2. Then $\cos\theta=\frac{\begin{pmatrix}4\\1\\1\end{pmatrix}\bullet\begin{pmatrix}4\\3\\-1\end{pmatrix}}{\left\|\begin{pmatrix}4\\1\\1\end{pmatrix}\right\|\left\|\begin{pmatrix}4\\3\\-1\end{pmatrix}\right\|}=\frac{18}{\sqrt{18}\cdot\sqrt{26}}=\frac{3}{\sqrt{13}}$. Therefore $\sin\theta=\frac{2}{\sqrt{13}}$.
(b)	Since $4(1)+0+4=8$ and $4(1)+3(0)-4=0$, the point $A(1,0,4)$ lies on p_1 and p_2. $\begin{pmatrix}4\\1\\1\end{pmatrix}\times\begin{pmatrix}4\\3\\-1\end{pmatrix}=\begin{pmatrix}-4\\8\\8\end{pmatrix}=4\begin{pmatrix}-1\\2\\2\end{pmatrix}$. The vector equation of the line of intersection of p_1 and p_2 is $\mathbf{r}=\begin{pmatrix}1\\0\\4\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\2\end{pmatrix}$, $\lambda\in\mathbb{R}$.

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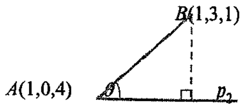
<table>
<tr><td>(c)</td><td>
$\overrightarrow{AB}\cdot\begin{pmatrix}-1\\2\\2\end{pmatrix}=\begin{pmatrix}0\\3\\-3\end{pmatrix}\cdot\begin{pmatrix}-1\\2\\2\end{pmatrix}=6-6=0$
So AB is perpendicular to l.
$AB=\left|\begin{pmatrix}0\\3\\-3\end{pmatrix}\right|=\sqrt{3^2+(-3)^2}=3\sqrt{2}$.
By (a), perpendicular distance from B to p_2
$=AB\sin\theta$
$=3\sqrt{2}\left(\frac{2}{\sqrt{13}}\right)$
$=\frac{12}{\sqrt{26}}$


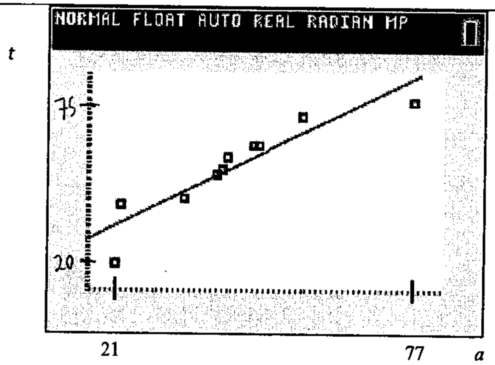
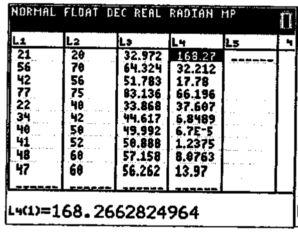
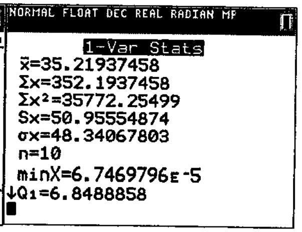
</td></tr>
<tr><td>(d)</td><td>
Since p_3 is perpendicular to both p_1 and p_2, the normal vectors of p_1 and p_2 are both parallel to p_3.
From (b), we infer that a normal vector of p_3 is $\begin{pmatrix}-1\\2\\2\end{pmatrix}$.
Equation of p_3 is therefore
$\mathbf{r}\cdot\begin{pmatrix}-1\\2\\2\end{pmatrix}=\begin{pmatrix}1\\3\\1\end{pmatrix}\cdot\begin{pmatrix}-1\\2\\2\end{pmatrix}=-1+6+2=7$
which in cartesian form is $-x+2y+2z=7$.
</td></tr>
</table>

<table>
<tr><th>Q6</th><th>Suggested Answers</th></tr>
<tr><td>(a)(i)</td><td>
If two scores are equal (Event C), suppose x and x, sum of two scores will be $2x$ which is even (which is thus not odd). A and C cannot occur concurrently.
So Event A and C are mutually exclusive
</td></tr>
<tr><td>(a)(ii)</td><td>
A and B are independent.
P(A) = P(1 odd and 1 even) = $2(\frac{1}{2}(\frac{1}{2}))=\frac{1}{2}$
P(B) = P(at least 1 of scores >4)
= P(1st die =5,6) +P(2nd die = 5,6) – P(1st and 2nd die =5 or 6)
$=\frac{1}{3}+\frac{1}{3}-(\frac{1}{3})^2=\frac{5}{9}$
$P(A\cap B)=2P(5,2\text{ or }4\text{ or }6)+2P(6,1\text{ or}3)=\frac{10}{36}=\frac{5}{18}$
Alternatively, by using table of outcomes:
<table>
<tr><th></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th></tr>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>⑦</td></tr>
<tr><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>⑦</td><td>8</td></tr>
<tr><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>⑨</td></tr>
<tr><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>⑨</td><td>10</td></tr>
<tr><td>5</td><td>6</td><td>⑦</td><td>8</td><td>⑨</td><td>10</td><td>⑪</td></tr>
<tr><td>6</td><td>⑦</td><td>8</td><td>⑨</td><td>10</td><td>⑪</td><td>12</td></tr>
</table>
$P(B)=\frac{20}{36}=\frac{5}{9}$
$P(A\cap B)=\frac{10}{36}=\frac{5}{18}$ (Look for number of ◯)
$P(A)P(B)=\frac{1}{2}(\frac{5}{9})=\frac{5}{18}=P(A\cap B)$
Thus A and B are independent.
</td></tr>
<tr><td>(b)</td><td>
$P(C\,|\,B)$
$=\frac{P(C\cap B)}{P(B)}$
$=\frac{P(5,5)+P(6,6)}{P(B)}$
$=\frac{\frac{2}{36}}{\frac{5}{9}}$
$=\frac{1}{10}$
</td></tr>
</table>

<table>
<tr><th>Q7</th><th>Suggested Answers</th></tr>
<tr><td>(a)</td><td>s =0,1,2,3,4,5 ;
f= 5,4,3,2,1,0
d = 5,3,1
D can take values 1, 3 or 5</td></tr>
<tr><td>(b)</td><td>$S \sim \mathrm{B}\left(5, \frac{1}{3}\right)$
$\mathrm{P}(D=1)$
$= \mathrm{P}(S=3 \text{ or } S=2)$
$= \frac{40}{81}$</td></tr>
<tr><td>(c)</td><td><table><tr><td>d</td><td>1</td><td>3</td><td>5</td></tr><tr><td>P(D=d)</td><td>40/81</td><td>10/27</td><td>11/81</td></tr></table>
$\mathrm{E}(D^2) = \sum d^2 \mathrm{P}(D=d)$ = 65/9.</td></tr>
<tr><td>(d)</td><td>$S \sim \mathrm{B}\left(5, \frac{1}{3}\right)$
$\mathrm{E}(S) = 5\times\frac{1}{3} = \frac{5}{3}$, $\mathrm{Var}(S) = 5\times\frac{1}{3}\times\frac{2}{3} = \frac{10}{9}$
$\mathrm{E}\left(S^2\right) = \mathrm{Var}(S) + (\mathrm{E}(S))^2 = \frac{10}{9} + (\frac{5}{3})^2 = \frac{35}{9}$</td></tr>
<tr><td>(e)</td><td>$D^2 = \lvert S-(5-S)\rvert^2 = 4S^2 - 20S + 25$
$\mathrm{E}(D^2) = \mathrm{E}(4S^2 - 20S + 25)$
$= 4\mathrm{E}(S^2) - 20\mathrm{E}(S) + 25$
$= 4(\frac{35}{9}) - 20\left(\frac{5}{3}\right) + 25$
$= \frac{65}{9}$ (verified)</td></tr>
</table>

Q8	Suggested Answers
(a)	$\bar{x} = \frac{\sum x}{n} = \frac{80.1}{45} = 1.78$ $s^2 = \frac{1}{n-1}\sum(x-\bar{x})^2$ $= \frac{1}{44}(24.29)$ $= 0.55205$ (5sf) $= 0.552$ (3sf)
(b)	Let μ be the population mean mass of adult trout. To test $\mathrm{H}_0 : \mu = w$ $\mathrm{H}_1 : \mu \neq w$ at 10% level of significance Under H_0, since $n = 45$ is large, by the Central Limit Theorem, $\bar{X} \sim \mathrm{N}\left(w, \frac{s^2}{45}\right)$ approximately $Z = \frac{\bar{X} - w}{s/\sqrt{45}} \sim \mathrm{N}(0,1)$ approximately Reject H_0 if $z_{\text{calc}} \le -1.6448$ or $z_{\text{calc}} \ge 1.6448$ (5 s.f.) $z_{\text{calc}} = \frac{1.78 - w}{\frac{s}{\sqrt{45}}}$ Since H_0 is not rejected, $-1.6448 < z_{\text{calc}} < 1.6448$ $-1.6448 < \frac{1.78-w}{\frac{s}{\sqrt{45}}} < 1.6448$ $-1.6448 < \frac{1.78-w}{\frac{\sqrt{0.55205}}{\sqrt{45}}} < 1.6448$ $-0.18218 < 1.78 - w < 0.18218$ $1.5978 < w < 1.9622$ $1.60 < w < 1.96$ $\{w \in \mathbb{R} : 1.60 < w < 1.96\}$
(c)	Since the <u>sample size (n=45) is large</u>, we can approximate the <u>sample mean mass</u> of adult trout with a normal distribution using Central limit theorem.

Q9	Suggested Answers
(a)	The probability that a plate being faulty is constant. The event that a plate is faulty is independent of the other plates (being faulty).
(b)	Let X be the number of faulty plates out of 100 plates. $X \sim \mathrm{B}(100, p)$ $\mathrm{P}(X=3)=\binom{100}{3}p^3(1-p)^{100-3}$ $=161700p^3(1-p)^{97}.$
(c)	Since $X=3$ is the mode $\mathrm{P}(X=3)>\mathrm{P}(X=2)$ $161700p^3(1-p)^{97}>4950p^2(1-p)^{98}$ $\frac{98}{3}p>1-p$ $\frac{101}{3}p>1$ and $p>\frac{3}{101}$ $\mathrm{P}(X=3)>\mathrm{P}(X=4)$ $161700p^3(1-p)^{97}>3921225p^4(1-p)^{96}$ $\frac{97}{4}p<1-p$ $\frac{101}{4}p<1$ $p<\frac{4}{101}$ Hence $\frac{3}{101}<p<\frac{4}{101}$
(d)	P(no faulty items)+P(1 faulty plate) +P(1 faulty bowl) = 0.88 $(0.99)^2(1-q)^2+2(0.99)(0.01)(1-q)^2+(0.99)^2(2q)(1-q)=0.88$ $0.9999(1-q)^2+(1.9602)q(1-q)=0.88$ Using GC, since $0<q<1$, $q=0.33333\approx 0.333$

Q10	Suggested Answers
(a)	By GC, $\bar{a}=42.8$ and $\bar{t}=\frac{485+k}{10}$. Since the point $(\bar{a},\bar{t})$ lies on the line $t=0.8957914a+14.16013$, $\frac{485+k}{10}=0.8957914(42.8)+14.16013$ $k=40$
(b)	
(c)	Time spent by a 60-year old $0.8957914(60)+14.16013$ $=67.9$ ≈ 68 minutes.
(d)	By GC, the sum of residuals of the least squares regression line of t on a $=352.1937458$ Since the sum of residuals of the least squares regression line of t on a is the least amongst all the lines $t=ma+c$ which approximate the same set of data, $S=\sum_{i=1}^{10}\left[t_i-(ma_i+c)\right]^2\geq 353$. 

(d)

```
NORMAL FLOAT DEC REAL RADIAN MP
L1    L2    L3      L4      L5     4
21    20    32.972  168.27  ------
56    70    64.324  32.212
42    56    51.783  17.78
77    75    83.136  66.196
22    40    33.868  37.607
34    42    44.617  6.8489
40    50    49.992  6.7E-5
41    52    50.888  1.2375
48    60    57.158  8.0763
47    60    56.262  13.97
L4(1)=168.2662824964
```

```
NORMAL FLOAT DEC REAL RADIAN MP
        1-Var Stats
 x̄=35.21937458
 Σx=352.1937458
 Σx²=35772.25499
 Sx=50.95554874
 σx=48.34067803
 n=10
 minX=6.7469796ᴇ-5
↓Q1=6.8488858
```

In the GC screen on the left, L4 gives the values of $\left[t_i-(ma_i+c)\right]^2$.

On the right, sum of squares of residuals for the least squares regression line of t on a is $\sum x = 352.1937458$.

(e) Model (A): $t = p + \frac{q}{a} = p + q\left(\frac{1}{a}\right)$ so we consider the regression line of t on $\frac{1}{a}$.

Model (B): $t = pe^{qa} \Rightarrow \ln t = qa + \ln p$ so we consider the regression line of $\ln t$ on a.

Model	Product moment correlation coefficient, r
A	-0.929
B	0.833

Since the $|r|_{\text{Model A}} = 0.929 > |r|_{\text{Model B}} = 0.833$, Model A is more appropriate as its magnitude is closer to 1 than Model B.

Q11	Suggested Answers
(a)	Let X be the mass of a randomly chosen apple. $X \sim \mathrm{N}(\mu, \sigma^2)$ $\mathrm{P}(X > 120) = 0.15$ $\mathrm{P}(Z > \frac{120-\mu}{\sigma}) = 0.15$ $\frac{120-\mu}{\sigma} = 1.0364$ $120 - \mu = 1.0364\sigma$..........(1) $\mathrm{P}(X < 80) = 0.10$ $\mathrm{P}(Z < \frac{80-\mu}{\sigma}) = 0.1$ $\frac{80-\mu}{\sigma} = -1.2816$ $80 - \mu = -1.2816\sigma$............(2) Using GC, $\mu = 102.12 \approx 102, \quad \sigma = 17.256 \approx 17.3$(shown)
(b)	$X_1 - X_2 \sim \mathrm{N}(0,\ 17.256^2 + 17.256^2)$ i.e. $X_1 - X_2 \sim \mathrm{N}(0,\ 595.54)$ $\mathrm{P}(\lvert X_1 - X_2 \rvert \geq 20)$ $= \mathrm{P}(X_1 - X_2 \geq 20) + \mathrm{P}(X_1 - X_2 \leq -20)$ $= 2 \times \mathrm{P}(X_1 - X_2 \geq 20)$ $= 0.41247 \approx 0.412$
(c)	$X \sim \mathrm{N}(102.12,\ 17.256^2)$ $\mathrm{P}(110 < X < 140) = 0.30988$ Expected number of premium apples = 20(0.30988) = 6.1976 ≈ 6.20
(d)	**Method 1:** Let T be the total mass of 20 apples in a box $T = X_1 + X_2 + \ldots + X_{20}$ $\overline{T} = \frac{X_1 + X_2 + \ldots + X_{20}}{20} \sim \mathrm{N}\left(102.12,\ \frac{17.256^2}{20}\right)$ i.e. $\overline{T} \sim \mathrm{N}\ (102.12,\ 14.888)$ $\mathrm{P}(\overline{T} \leq 105) = 0.77229 \approx 0.772$
	Method 2: $T = X_1 + X_2 + \ldots + X_{20} \sim \mathrm{N}(20 \times 102.12,\ 20 \times 17.256^2)$ $T = X_1 + X_2 + \ldots + X_{20} \sim \mathrm{N}\ (2042.4,\ 5955.4)$ $\mathrm{P}(T \leq 105 \times 20) = 0.77228 \approx 0.772$
(e)	**Method 1:** $C = 0.0029(T_1 + T_2 + T_3)$ $C \sim \mathrm{N}\left(0.0029(3 \times 20 \times 102.12),\ 0.0029^2(3 \times 20 \times 17.256^2)\right)$ i.e. $C \sim \mathrm{N}(17.769,\ 0.15025)$ $\mathrm{P}(C > 18) = 0.27560 \approx 0.276$
	Method 2: $T_1 + T_2 + T_3 \sim \mathrm{N}\left(3 \times 20 \times 102.12,\ 3 \times 20 \times 17.256^2\right)$ i.e. $T_1 + T_2 + T_3 \sim \mathrm{N}(6127.2,\ 17866.17)$ $\mathrm{P}\left(T_1 + T_2 + T_3 > \frac{18 \times 1000}{2.9}\right) = 0.27551 \approx 0.276$