BP - 445

NANYANG JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

Higher 2

CANDIDATE NAME			
CT CLASS	2 4	Centre Number/ Index Number	/

MATHEMATICS

9758/01

Paper 1

2 September 2025

3 hours

Additional Materials: Printed Answer Booklet

List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **100**.

This document consists of **6** printed pages and **2** blank pages.

NANYANG JUNIOR COLLEGE
Internal Examinations

1 Curve C has equation $y=\dfrac{px^2+qx+r}{x-1}$, where p, q and r are real numbers. Given that C has a turning point at $(3,\ 7)$ and an oblique asymptote parallel to the line $y=\dfrac{3}{2}x+\sqrt{5}$, find p, q and r. [4]

2 An ornament consists of two identical solid right circular cones whose flat bases are separated by a solid sphere, such that the sphere is in contact with the two bases. The axis passes through the vertex of each cone and the centre of the sphere. It is given that the distance between the vertices of the cones is $2l$, and the radii of the bases of the cones are $\sqrt{\dfrac{3}{2}}\,l$, where l is a constant. Find the radius of the sphere in terms of l if the total volume of the ornament is to be a minimum.

[The volume of a sphere with radius r is given by $V=\dfrac{4}{3}\pi r^3$.] [6]

3 **(a)** A triangle ABC is such that $AB^2+BC^2=AC^2$.

By considering $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$ and using the fact that $|\mathbf{v}|^2=\mathbf{v}\cdot\mathbf{v}$ for any vector $\mathbf{v}$, prove that $\angle ABC$ is a right-angle. [4]

(b) In a triangle ABC, the point D divides AC in the ratio $\lambda:1-\lambda$, where $0<\lambda<1$.

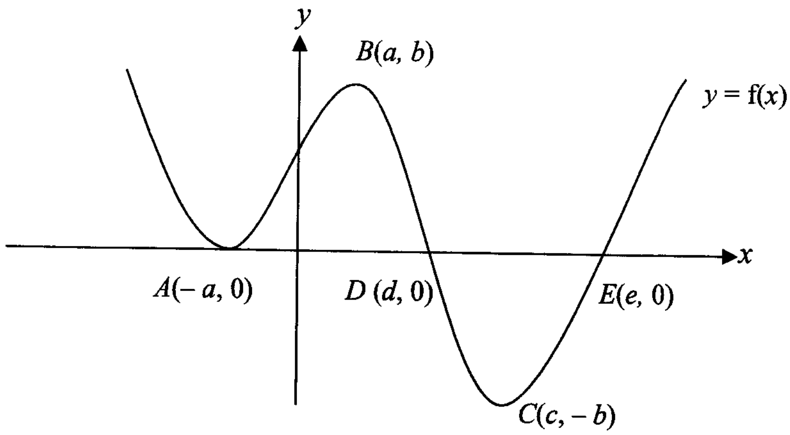
Let the position vectors of A, B, C and D be denoted by $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ respectively.

Show that the area of triangle ABD is given by

$$k\left|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}\right|$$

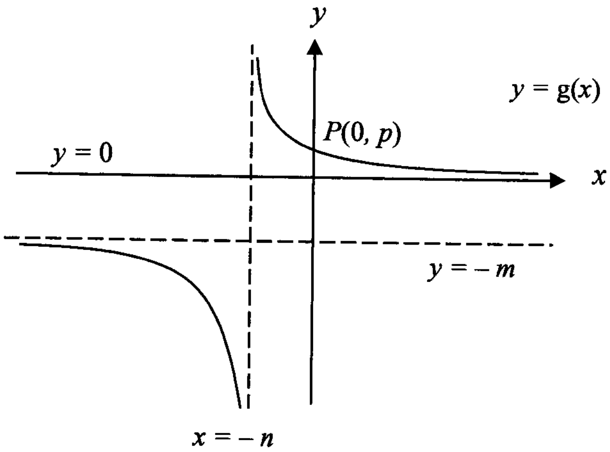
where k is to be determined in terms of λ. [4]

4 **(a)** The graph of $y=\mathrm{f}(x)$ has turning points at $A(-a,0)$, $B(a,b)$ and $C(c,-b)$ and intersects the x-axis at the points $D(d,0)$ and $E(e,0)$. The graph of $y=\mathrm{f}(x)$ is as shown in the diagram below.



Sketch the graph of $y=\mathrm{f}(a-x)$, labelling the coordinates of the corresponding points of A, B, C, D, and E clearly. [3]

(b) The graph of $y=\mathrm{g}(x)$ has asymptotes $y=0$, $y=-m$ and $x=-n$, and intersects the y-axis at $P(0,p)$, where $p<m$. The graph of $y=\mathrm{g}(x)$ is as shown in the diagram below.



Sketch on separate diagrams the graphs of

(i) $y=|\mathrm{g}(x)|$ and [3]

(ii) $y=\mathrm{g}(|x|)$, [2]

labelling clearly the asymptote(s) and coordinates of P on each graph.

5 The nth term of a series G is given by $g_n = 3n^2 - 17n + 17$, where $n \geq 1$.

(a) Given that $\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$, find $\sum_{r=m+1}^{n} g_r$. (You need not simplify your answer.) [4]

The nth term of a series H is given by $h_n = 7(5^{n-3}) + 10$, where $n \geq 1$.

(b) State the smallest number that can be found in both series H and G. [1]

The sum of the first n terms of a series J is given by $\frac{b}{2}(3^n - 1)$, where b is a positive odd integer.

(c) By finding the nth term of series J, explain why each of the terms in series J is a term of the arithmetic progression with first term 1 and common difference 2. [3]

6 The points P, Q and R representing the complex numbers p, q and r on an Argand diagram are such that $\arg(p) = \alpha$ and $\arg(q) = \beta$, where $0 < \alpha < \beta < \frac{\pi}{2}$, $\beta > 2\alpha$ and $r = p + q$.

(a) If $|p| = |q|$, describe the shape of the quadrilateral of $OPRQ$. Hence find $\arg(r)$ in terms of α and β. [3]

(b) The point Q', representing the complex number q', is the reflection of the point Q in OP. State the angle POQ'. [1]

By leaving your answers in terms of α, β and $|q|$ where applicable, hence, or otherwise,

(i) find the argument of the complex number q', [1]

(ii) find the real and imaginary parts of q' and write down q' in $a + ib$ form. [3]

7 **(a)** Use the substitution $x = \sin^2\theta$, where $0 \leq \theta \leq \frac{\pi}{2}$, to find $\int_0^{\frac{1}{2}} \sqrt{\frac{16x}{1-x}}\, dx$ exactly. [4]

(b) Find $\int_a^b \frac{|x-1|}{\sqrt{\frac{1}{2}x^2 - x + 1}}\, dx$ in terms of a and b, where $a < 1 < b$. [4]

8 A sequence is such that $u_{n+1} = u_n + 1.5$, for $n = 1, 2, 3, \ldots$.

(a) Given that $u_{50} = 99u_1$, show that $u_1 = 0.75$. [2]

(b) Find the least value of n such that $u_2 + u_4 + u_6 + \ldots + u_{2n}$ is more than 2025. [2]

(c) It is given that u_{25}, u_k and u_5 are the first three terms of a geometric progression.

(i) Find k. [2]

(ii) Find the sum of the first 13 terms of the geometric series, giving your answer to 3 decimal places. [2]

9 The function f is defined by

$$f(x)=\frac{x+1}{2x-3} \quad \text{for } x\in\mathbb{R},\ x\neq k.$$

(a) State the value of k and explain why this value has to be excluded from the domain of f. [2]

(b) Determine, with a reason, if f^2 exists. [2]

(c) Find $f^{-1}(x)$. [2]

(d) Hence find $h(x)$ for which $fh(x)=\frac{3x-7}{2x-1}, x\neq\frac{1}{2}$. [3]

10 In the question you may use expansions from the List of Formulae (MF27).

(a) Find the Maclaurin expansion of $\cos(t^3)$ in ascending powers of t, up to and including the term in t^{12}.

Hence, find the Maclaurin series of $\int_0^x t^2\cos(t^3)\,dt$ up to and including the term in x^{15} given that x is small. [4]

(b) Use your expansion from part **(a)** to find an approximate value for $\int_0^{0.1} t^2\cos(t^3)\,dt$, correct to 5 decimal places. [1]

(c) Find $\int_0^x t^2\cos(t^3)\,dt$ in terms of x. Hence evaluate $\int_0^{0.1} t^2\cos(t^3)\,dt$, correct to 5 decimal places. [3]

(d) Comparing your answers to parts **(b)** and **(c)**, and with reference to the value of x, comment on the accuracy of your approximations. [1]

11 The curve C has equation $x^2-y^2=16$, where $y\geq 0$. The line L has equation $y=-\frac{1}{2}x+\frac{11}{2}$.

(a) Find the area enclosed by C, L and the line $x=8$. [3]

(b) For $x>0$, the region bounded by C, L and the x-axis is rotated about the x-axis through 2π radians. Find the exact volume generated. [4]

(c) The region bounded by C, the line $x=5$ and the x-axis is rotated about the y-axis through 2π radians. Find the exact volume generated. [4]

12 Scientists are studying the growth of a newly discovered biomass that entered Earth's atmosphere together with an asteroid. A sample of 100 grams of the biomass was collected initially. It was found that on the next day, the sample grew to 110 grams. The scientists observed that the rate of growth of biomass was proportional to the amount of biomass present. The amount of biomass at time t days is denoted by B grams.

(a) Write down a differential equation relating B and t. [1]

(b) Solve the differential equation obtained in part **(a)**, expressing B in terms of t. [4]

(c) Sketch the graph of B against t, and explain what would happen if the biomass was left on its own. [2]

To limit the growth of the biomass, the scientist collected a second sample of 100 grams of the biomass and introduced a nutrient depletion serum to the sample the moment it was collected. The rate of growth of the biomass for this second sample is modelled by the differential equation

$$\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{mB}{3+2t^2}$$

where m is a constant.

After 1 day, the second sample grew to 101 grams.

(d) For this second sample, find B in terms of t. [5]

(e) Determine the amount of the second biomass sample after a long time, leaving your answer to 5 decimal places. [1]

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NANYANG JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

Higher 2

CANDIDATE NAME

CT CLASS 2 4

Centre Number/ Index Number /

MATHEMATICS

9758/02

Paper 2

17 September 2025

3 hours

Additional Materials: Printed Answer Booklet

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Internal Examinations

Section A: Pure Mathematics [40 marks]

1 **(a)** Find the set of values of x for which $3|x-3| \leq |1-2x|$. [2]

(b) Without using a calculator, solve $\dfrac{x+18}{x^2+5x-14} \geq -1$. [4]

2 **(a)** It is given that the roots to the equation $-ix^3+5ix^2+ax+b=0$, where a and b are purely imaginary, are $2+i$, $2-i$ and 1. Explain why the complex roots occur in conjugate pairs. [2]

(b) By using **(a)** and an appropriate substitution, find the roots of the equation $ix^3+5ix^2+a^*x+b=0$, where the complex conjugate of a is denoted by a^*. [4]

3 **(a)** Given that f is a continuous function, explain, with the aid of a sketch, why the value of

$$\lim_{n\to\infty} \frac{2}{n}\left\{f\left(1+\frac{2}{n}\right)+f\left(1+\frac{4}{n}\right)+\ldots+f\left(1+\frac{2n}{n}\right)\right\}$$

is $\int_1^3 f(x)\,dx$. [3]

(b) Hence evaluate $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n} \ln\left(1+\frac{2k}{n}\right)\frac{2}{n}$ exactly. [4]

4 The equation of a curve C is $x^3+xy+2y^3=k$, where k is a constant.

(a) Find $\dfrac{dy}{dx}$ in terms of x and y. [2]

It is given that C has a tangent which is parallel to the y-axis.

(b) Show that the y-coordinate of the points of contact of the tangent with C must satisfy $216y^6+4y^3+k=0$. Hence show that $k \leq \dfrac{1}{54}$. [6]

(c) Find the possible values of k in the case where the line $x=-6$ is a tangent to C. [2]

5 Two planes p_1 and p_2 have respective cartesian equations given by

$$4x+y+z=8 \text{ and } 4x+3y-z=0.$$

(a) Find the sine of the acute angle between p_1 and p_2 in the form $\frac{m}{\sqrt{n}}$ where m and n are positive integers to be determined. [3]

(b) Verify that the point A with coordinates (1,0,4) lies on p_1 and p_2. Hence, without the use of a calculator, find the vector equation of the line l formed by the intersection of p_1 and p_2. [3]

(c) It is given that B is a point on p_1 with coordinates (1,3,1). Show that AB is perpendicular to l and hence use **(a)** to deduce exactly the shortest distance from B to p_2. [3]

(d) Find the cartesian equation of the plane p_3 which contains B and is perpendicular to both p_1 and p_2. [2]

Section B: Probability and Statistics [60 marks]

6 Two fair six-sided dice are thrown. Events A, B and C are defined as follows.

A: sum of the two scores is odd

B: at least one of the two scores is greater than 4

C: the two scores are equal

(a) Find, giving your reasons clearly in each case, which pair of the events are

(i) mutually exclusive, [1]

(ii) independent. [3]

(b) Find P(C | B). [2]

7 The random variable S is the number of successes in 5 independent trials of an experiment in which the probability of success in any trial is $\frac{1}{3}$. The random variable D is the difference between the number of successes and the number of failures in 5 such trials.

(a) State the values that D can take. [1]

(b) Show that $\mathrm{P}(D=1)=\frac{40}{81}$. [1]

(c) By finding the probability distribution of D, find the exact value of $\mathrm{E}(D^2)$. [3]

(d) Find the exact values of $\mathrm{E}(S)$ and $\mathrm{E}(S^2)$. [2]

(e) Hence, by showing that $D^2=4S^2-20S+25$, verify the correctness of the value of $\mathrm{E}(D^2)$ found in part **(c)**. [2]

8 Helen is a conservationist who monitors the health of fish in a river. She is investigating whether a new water-treatment plant has affected the mass of adult trout in the river. Previously, the mean mass of adult trout in the river has been w kg. Helen carries out a test, at the 10% level of significance to find out whether there has been a change in the mean mass of adult trout in the river.

Helen catches 45 adult trout and measures the mass, X, kilograms, of each fish. Her summarised data are as follows.

$$n = 45 \qquad \sum x = 80.1 \qquad \sum (x - \bar{x})^2 = 24.29$$

(a) Calculate unbiased estimates of the population mean and variance of the mass of adult trout. [2]

(b) Use an algebraic method to calculate the set of values of w for which there is insufficient evidence to conclude that there has been a change in the mean mass of adult trout in the river. You should state your hypotheses and define any symbols you use. [6]

(c) Explain why there is no need for Helen to know anything about the population distribution of the mass of adult trout. [2]

9 A small ceramic workshop produces 100 plates each working day. Some of the plates turn out to be faulty.

(a) State, in the context of the question, two assumptions needed for the number of faulty plates made in a day to be well modelled by a binomial distribution. [2]

Assume now that the number of faulty plates produced each working day has the distribution $\mathrm{B}(100, p)$.

(b) Show that the probability that exactly 3 faulty plates are produced on a randomly chosen working day is $161700p^3(1-p)^{97}$. [1]

(c) Given that the most likely number of faulty plates produced on a working day is 3, find the possible range of values of p, leaving your answer in exact form. [4]

The workshop also produces bowls on each working day. The number of faulty bowls also follows a binomial distribution. The probability that a bowl is faulty is q. Faults on plates are independent of faults on bowls. The plates and bowls are sold in sets of 2 randomly chosen bowls and 2 randomly chosen plates. In the case where $p = 0.01$, the probability that a set contains at most 1 faulty item is 0.88.

(d) Write down an equation satisfied by q. Hence find the value of q. [4]

10 The director of an art gallery wishes to find out the relationship between the time spent, t minutes, in the art gallery and the age, a (in years), of the visitors. Ten visitors were randomly selected and the data are summarised as follows:

t (minutes)	20	70	56	75	k	42	50	52	60	60
a (years)	21	56	42	77	22	34	40	41	48	47

(a) A linear model is proposed for the relationship between t and a. Given that the least squares regression line of t on a is

$$t = 0.8957914a + 14.16013,$$

show that $k = 40$. [2]

(b) Sketch a scatter diagram of t against a for the data given in the table and draw the line given in **(a)** on the same diagram. [2]

(c) Use the least squares regression line of t on a in **(a)** to estimate the time spent in the art gallery by a 60-year old visitor. [1]

(d) A regression line $t = ma + c$ where m and c are constants is used to model the above set of data. The residual for a data point (a_i, t_i) where $i = 1, 2, \ldots, 10$ which is plotted on a scatter diagram is defined to be the number

$$t_i - (ma_i + c).$$

The sum of the squares of these residuals, denoted by S, is given by

$$S = \sum_{i=1}^{10} \left[t_i - (ma_i + c)\right]^2.$$

Find an inequality that is satisfied by S. [2]

(e) The director wishes to explore the following two models which represent the relationship between t and a, where p and q are constants.

(A) $t = p + \dfrac{q}{a}$, (B) $t = pe^{qa}$.

For each model, find the product moment correlation coefficient and hence explain with a reason which model is more appropriate. [3]

11 In this question you should state the parameters of any normal distributions you use.

A machine grades apples according to their mass. Apples with mass exceeding 120g are labelled as "large" and apples with mass less than 80g are labelled as "small". A large batch of apples is graded and it is found that 15% are "large" and 10% are "small". It is known that the masses of the apples are normally distributed.

(a) Find the mean mass of a randomly chosen apple from the batch and show that the standard deviation is 17.3 g, correct to 3 significance figures. [3]

(b) Two apples are chosen in random. Find the probability that the difference in their masses is at least 20g. [3]

The apples are labelled and packed into boxes. Each box contains 20 apples.

(c) Apples with mass between 110g and 140g are labelled as "premium". Find the expected number of "premium" apples in a randomly chosen box. [2]

(d) Find the probability that the mean mass of apples in a randomly chosen box is at most 105g. [3]

(e) The boxes of apples are sold by weight at a price of $2.90 per kilogram. Find the probability that the total price of 3 randomly chosen boxes of apples exceeds $18. [3]

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