RAFFLES INSTITUTION
2025 YEAR 6 PRELIMINARY EXAMINATION
Higher 2

MATHEMATICS

9758/01

Paper 1

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **8** printed pages.
RAFFLES INSTITUTION
Mathematics Department

1 A curve has equation $y = ax + b + \frac{c}{x^2 - 1}$, where a, b and c are real constants. It is given that the curve crosses the x-axis at $x = 2$. The normal to the curve at the point $(0, 3)$ meets the x-axis at $x = 7.5$. Find the values of a, b and c. [4]

2 The point P travels along the curve C with equation $y = x\sin^{-1} x$, $-1 < x < 1$. Let the gradient of the curve C at the point P be m.

If the x-coordinate of P is increasing at the rate of 9 units per second when $x = \frac{1}{2}$, find the exact value of the rate at which m is changing at this instant. [4]

3 Relative to the origin O, the points A and C have position vectors $\mathbf{a}$ and $2\mathbf{b}$ such that

$$\mathbf{a} = 5p\mathbf{i} - 2p\mathbf{j} + 4p\mathbf{k} \text{ and } \mathbf{b} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k},$$

where p is a positive constant. It is given that $|\mathbf{a}| = 2|\mathbf{b}|$.

(a) Find the exact value of p. [2]

(b) Evaluate $(\mathbf{a} + 2\mathbf{b}) \cdot (\mathbf{a} - 2\mathbf{b})$. [1]

(c) Evaluate $\mathbf{a} \times \mathbf{b}$ and hence find the area of triangle OAC. [3]

(d) Use a geometrical reason to explain why $|\mathbf{a} \times \mathbf{b}| = \frac{1}{4}(|\mathbf{a} + 2\mathbf{b}|)(|\mathbf{a} - 2\mathbf{b}|)$. [2]

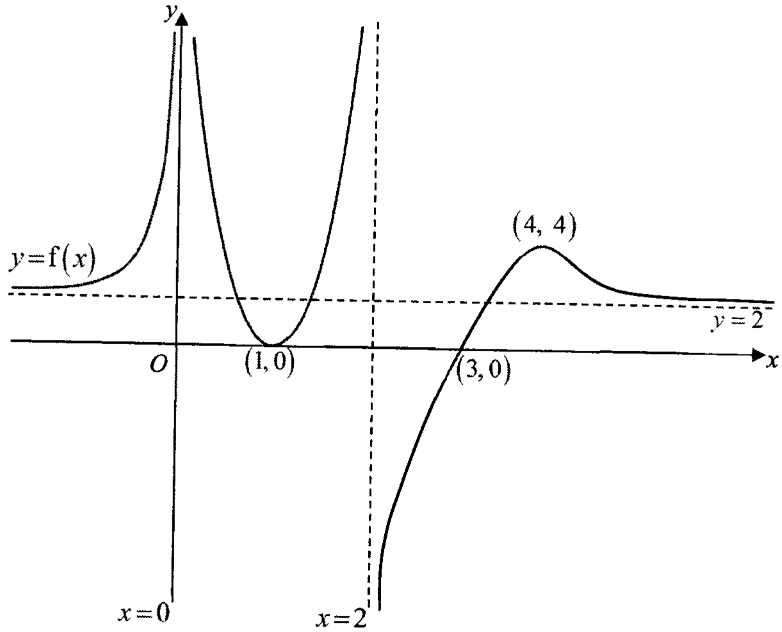
4 **(a)** Find $\int \frac{9x}{(2x-1)(x+1)^2} \, dx$. [4]

(b) **(i)** Differentiate $\frac{1}{x^2 + 1}$ with respect to x. [1]

(ii) Differentiate $\ln\sqrt{x^2 + 1}$ with respect to x. [1]

(iii) Hence find $\int \frac{x \ln\sqrt{x^2 + 1}}{(x^2 + 1)^2} \, dx$. [4]

5 **(a)** The diagram below shows a sketch of the graph of $y = f(x)$. The curve passes through the points with coordinates $(1, 0)$ and $(3, 0)$, and has turning points at $(1, 0)$ and $(4, 4)$. The asymptotes are $x = 0$, $x = 2$ and $y = 2$.



Sketch on separate diagrams, the graphs of

(i) $y = f'(x)$, [3]

(ii) $y = \dfrac{1}{f(x)}$, [3]

showing clearly the main features of the graphs.

(b) Describe a sequence of transformations which transform the graph of $y = \dfrac{x^2+1}{x}$ to the graph of $y = \dfrac{2x^2-5x+4}{x-2}$. [3]

6 **(a)** A geometric series has first term 2 and common ratio 0.8. Find algebraically, the least value of m for the sum of the first $4m$ terms of the series to be greater than 99% of the sum to infinity. [3]

(b) A finite arithmetic progression A has n terms, first term a and common difference d.

In another progression B, the kth term is obtained by adding the kth positive odd integer to the corresponding term of A. That is, the first term of B is obtained by adding 1 to the first term of A, the second term of B is obtained by adding 3 to the second term of A and so on.

It is given that the twelfth term of A is 25, the sum of all the terms of A is 676 and the sum of all the terms of B is twice the sum of all the terms of A.

(i) Find the values of n, a and d. [4]

(ii) Obtain the sum of the first ten terms of B. [2]

7 The function f is defined by $\mathrm{f}: x \mapsto 1+\dfrac{3}{\mathrm{e}^{x}-2}$ for $x \in \mathbb{R}$, $x > \ln 2$.

(a) Find $\mathrm{f}^{-1}(x)$ and state its domain. [3]

Another function g is defined by

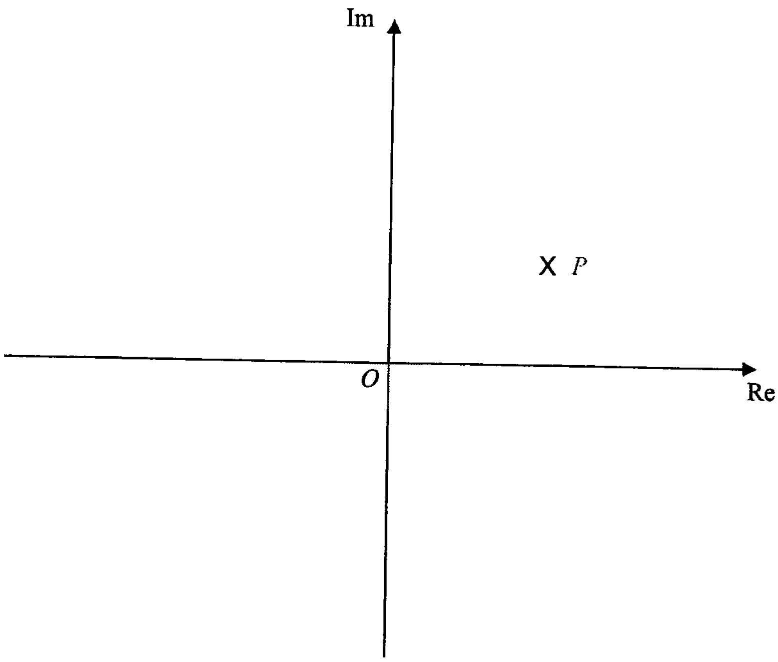
$$\mathrm{g}: x \mapsto \begin{cases} 2(x-1)^2+2 & \text{for } x \le 0, \\ 2+|2-x| & \text{for } 0 < x \le 3. \end{cases}$$

(b) Sketch the graph of $y=\mathrm{g}(x)$ and explain why the composite function $\mathrm{f}^{-1}\mathrm{g}$ exists. [4]

(c) Hence find the exact value of k such that $\mathrm{f}^{-1}\mathrm{g}(k)=\ln\dfrac{5}{2}$. [3]

8 **Do not use a calculator in answering this question.**

(a)



The point P on the Argand diagram represents the complex number w given by $w = u + \text{i}v$, where u and v are real and positive.

(i) Explain algebraically why $\arg(kw) = \arg(w)$ for any real constant $k > 1$. [1]

Points Q and R represent the complex numbers kw and $\text{i}kw$, where k is a real constant and $k > 1$.

(ii) On the same Argand diagram in the Printed Answer Booklet, plot the points Q and R. Show clearly the geometrical relationship between the points P, Q and R. [2]

(b) In another Argand diagram, the points A, B and C represent the complex numbers z, $\mathrm{f}(z)$ and $\mathrm{f}(\mathrm{f}(z))$ respectively, where $\mathrm{f}(z)=z^2-2z$.

(i) Show that $\mathrm{f}(\mathrm{f}(z))-\mathrm{f}(z)=z(z-2)(z-3)(z+1)$. [2]

It is given that ABC is a right-angled triangle, described in an anticlockwise sense, with a right angle at B, and $BC=mBA$, where m is a positive real constant.

(ii) By considering $\dfrac{\mathrm{f}(\mathrm{f}(z))-\mathrm{f}(z)}{\mathrm{f}(z)-z}$, or otherwise, show that

$$(z-2)(z+1)=m\mathrm{i}.$$ [2]

(iii) In the case where $z=x+2\mathrm{i}$, where x is a positive real number, find x and m. Hence obtain the complex number represented by the point B. [5]

9 Decontamination is a water treatment method to remove a certain undesirable chemical from contaminated water. As the liquid agent is continuously added to the contaminated water in a large tank, the undesirable chemical in the water is removed gradually.

The rate of decrease of the concentration of the undesirable chemical is proportional to the product of its current concentration and the rate at which the liquid agent flows into the tank. At time t minutes after the start of the treatment process, the concentration of the undesirable chemical is C mg/L and the liquid agent flows into the tank at a rate of $\dfrac{1}{1+t^2}$ L/min. It is known that the initial concentration of the undesirable chemical in the tank is 50 mg/L and after 10 minutes, its concentration is measured to be 20 mg/L.

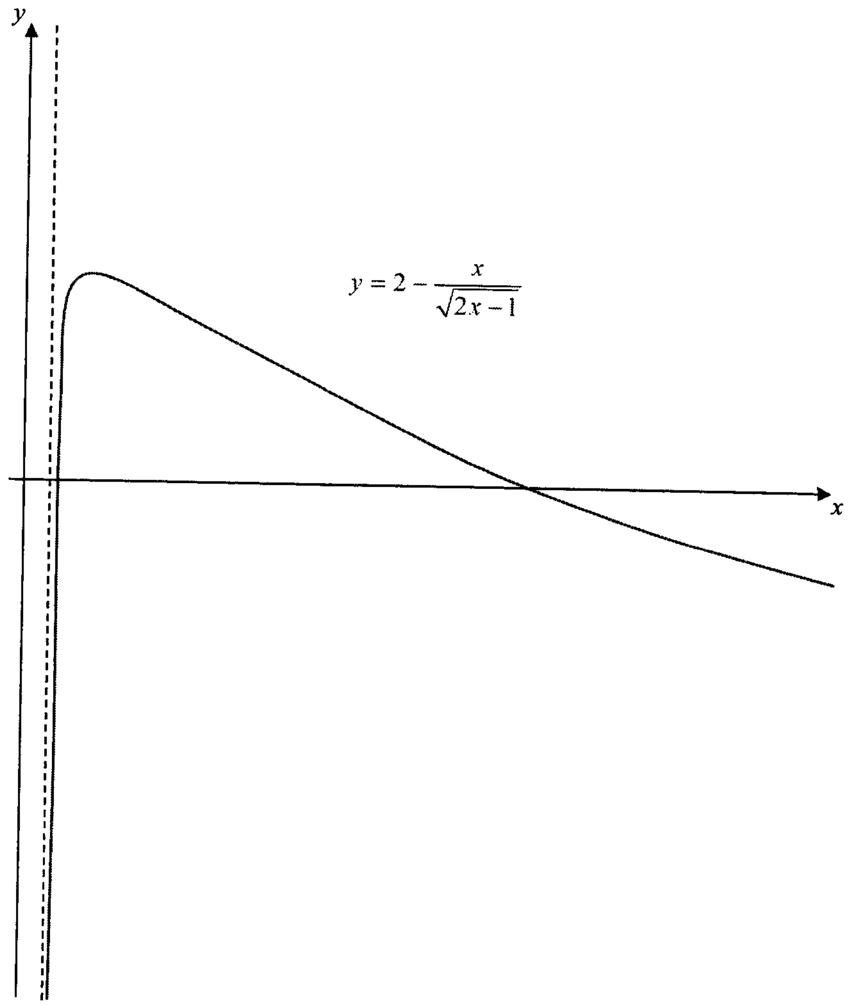
(a) By setting up and solving a differential equation relating C and t, show that $C=50\mathrm{e}^{q\tan^{-1}t}$, giving the value of q correct to 5 decimal places. [7]

(b) Determine the value of C when $t=50$. [1]

(c) Sketch the graph of C against t and state what happens to C in the tank for large values of t. [3]

10 **(a)** Using the substitution $u=\sqrt{2x-1}$, show that $\int \frac{x}{\sqrt{2x-1}}\,\mathrm{d}x = \int \frac{u^2+1}{2}\,\mathrm{d}u$. [2]

(b) **(i)** The diagram below shows the graph of $y=2-\frac{x}{\sqrt{2x-1}}$. Indicate on the same diagram in the Printed Answer Booklet, the equation of the asymptote of the curve and the coordinates of its turning point and points of intersection with the x-axis. [2]



(ii) The region R is bounded by the curve $y=2-\frac{x}{\sqrt{2x-1}}$ and the lines $x=1$, $x=3$ and $y=\frac{1}{2}$. Find the exact area of R. [5]

(c) The region S is bounded by the curves $y=2-\frac{x}{\sqrt{2x-1}}$, $x=2(y-1)^2+1$, the lines $x=1$, $x=4$ and the x-axis.

(i) Express $x=2(y-1)^2+1$ in the form $y=\mathrm{f}(x)$. [1]

(ii) On the same diagram as in part **(b)(i)**, sketch the graph of $x=2(y-1)^2+1$. [2]

(iii) Find the volume of the solid generated when S is rotated through 2π radians about the x-axis. Give your answer correct to 3 decimal places. [3]

11 A curve C has parametric equations

$$x=2\cos\theta+\cos 2\theta, \qquad y=2\sin\theta-\sin 2\theta,$$

where $0<\theta<2\pi$ and $\theta\neq\frac{2\pi}{3}, \pi, \frac{4\pi}{3}$.

The point P with parameter p lies on C, and the lines M_p and N_p are the tangent and normal to C at P respectively.

(a) It is given that

$$\sin A+\sin B=2\sin\frac{A+B}{2}\cos\frac{A-B}{2} \quad \text{and}$$
$$\cos A-\cos B=-2\sin\frac{A+B}{2}\sin\frac{A-B}{2}.$$

Using these two results, show that the gradient of M_p is $-\tan\left(\frac{p}{2}\right)$. [2]

(b) For $p=\frac{\pi}{3}$, M_p and N_p cut the x-axis at points Q and R respectively. Find the area of ΔPQR. [4]

(c) The point S with parameter s lies on C, and the line N_s is the normal to C at S. It is given that M_p and N_s are parallel.

Given $p\neq\frac{\pi}{3}, \frac{5\pi}{3}$ and $p<s$, by using the result $\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$, find an equation relating p and s. [2]

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RAFFLES INSTITUTION
Mathematics Department

Section A: Pure Mathematics [40 marks]

1 **(a)** Without using a calculator, solve the inequality $\dfrac{x^2-5x+6}{x^2-4}<\dfrac{2x-3}{x+2}$. [4]

(b) **(i)** Sketch on the same diagram the graphs of $y=\ln x$ and $y=x-5$, giving the equations of any asymptotes and the x-coordinates of the points of intersection between the two graphs. [2]

(ii) Hence solve the inequality $\ln|x|<|x|-5$. [2]

2 **(a)** In a triangle ABC, $AB=2$, angle $CAB=x$ radians and angle $CBA=\dfrac{\pi}{6}$ radians.

(i) Show that $AC=\dfrac{2}{\cos x+\sqrt{3}\sin x}$. [2]

(ii) Given that x is a sufficiently small angle, show that

$$AC\approx a+bx+cx^2,$$

where a, b and c are constants to be determined. [3]

(b) It is given that $2xy+\ln y=\ln 3$. Show that

$$(2xy^2+y)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+4y^2\frac{\mathrm{d}y}{\mathrm{d}x}-\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=0.$$

Hence find the Maclaurin series for y, up to and including the term in x^2. [5]

3 The terms of the sequence U are given by

$$u_1 = k \text{ and } u_{n+1} = \frac{8u_n - 14}{u_n - 1}, \quad n \geq 1.$$

(a) For the following values of k, describe the behaviour of the sequence U.

(i) $k = 3$ [1]

(ii) $k = 10$ [1]

(b) Find the possible value(s) of k if the sequence U is a constant sequence. [2]

The nth term of the sequence V is given by $v_n = \frac{a^n}{b} + \frac{b}{a(1-a^n)} - \frac{a}{n+1}$, where a and b are non-zero real constants and $a \neq \pm 1$.

(c) For some values of a, $v_n \to L$ as $n \to \infty$. Find, with justification, the range of values of a for L to exist, and state the value of L in terms of a and b. [3]

The nth term of the sequence W is given by

$$w_n = \begin{cases} u_n & \text{when } n \text{ is even,} \\ v_n & \text{when } n \text{ is odd.} \end{cases}$$

It is given that the sequence W converges when the sequences U and V converge to the same limit. The sequence W diverges otherwise.

(d) For $k = 10$, by using part **(a)(ii)** and part **(c)**, find the range of values of b for the sequence W to converge. Hence explain whether $\sum_{r=1}^{\infty} w_r$ is a convergent series.

[3]

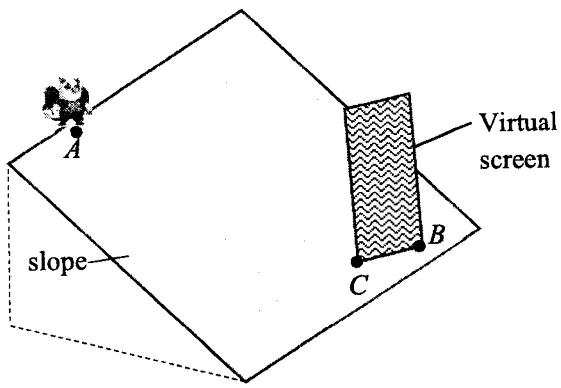
4 In a computer game, a slope can be modelled as a plane p containing three points, $A(1,\ 0,\ -3)$, $B(1,\ 4,\ -15)$ and $C(2,\ 3,\ -14)$.

(a) Show that p has equation $\mathbf{r}.\begin{pmatrix}2\\3\\1\end{pmatrix}=-1$. [2]

In Stage 1 of the game, Griffles travels on the slope from A along a path with equation $\mathbf{r}=\begin{pmatrix}1\\0\\-3\end{pmatrix}+\lambda\begin{pmatrix}1\\m\\n\end{pmatrix}$, where m and n are constants and λ is a real parameter.

(b) Find an equation relating m and n. [1]

In the rest of the question, it is given that $m=\frac{1}{3}$, $n=-3$ and Griffles is modelled as a point travelling on the slope. An obstacle in the form of a rectangular virtual screen stands on the slope with its base modelled by the **line segment** BC (see diagram).



(c) By considering the **line segment** BC, show that Griffles is able to successfully navigate this obstacle without colliding into it. [3]

A laser gun with a sensor is mounted at the point $E(5,\ 2,\ 1)$ above the slope. The sensor is activated when an object is within a range of 5 units.

(d) Show that Griffles does not activate the sensor. [3]

After completing Stage 1, Griffles is teleported to another slope. This slope can be modelled as a plane q parallel to p such that E is equidistant from p and q.

(e) Find the cartesian equation of q. [3]

Section B: Probability and Statistics [60 marks]

5 A discrete random variable X has the following probability distribution:

$$P(X = x) = k\left(x^2 + x\right) \text{ for } x = 1, 2, 3, 4, 5, \text{ where } k \text{ is a constant.}$$

(a) Show that $k = \frac{1}{70}$. [1]

(b) Find the exact values of $E(X)$ and $Var(X)$. [3]

(c) Two independent observations X_1 and X_2 are taken of X. Find the probability that the difference between the two observations is at least 3. [3]

6 Kitty has 5 small, 3 medium and 2 large spherical charms and each of the 10 charms is uniquely designed.

(a) Kitty arranges all the charms in a circle on a corkboard with charms of the same sizes next to each other. How many ways are there for her to do so? [2]

(b) On another occasion, Kitty arranges all the charms in a line at the base of a photo frame with none of the small charms next to each other. How many ways are there for her to do so? [2]

The diameters of all the small, medium and large charms are 0.5 cm, 1 cm and 2 cm respectively. Each of the spherical charms has a hole through its centre that allows it to be threaded through a chain.

(c) Kitty makes a keychain that includes a 6 cm chain with one end attached to a keyring, as shown in **Fig. 1**. It is given that the 6 cm chain is fully threaded with charms, with no gaps between them, and is stretched taut in a straight line. For example, **Fig. 2** shows the 6 cm chain threaded with 2 large and 2 medium charms.

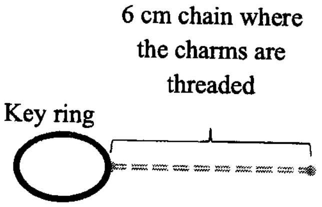


Fig. 1

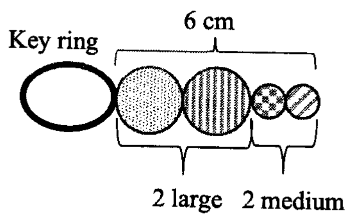


Fig. 2

How many different ways can she make a keychain with at least one charm of each of the three sizes? [4]

7 **In this question you should state the parameters of any distributions you use.**

A snack company produces two types of potato chips, Rays and Luffles. The mass, in grams, of a regular packet of Rays follows the distribution $\mathrm{N}(100,\ \sigma^2)$ and the mass, in grams, of a regular packet of Luffles follows the distribution $\mathrm{N}(120,\ 16)$. The masses of all regular packets of potato chips are independent of one another.

(a) If the probability of the mass of a randomly chosen packet of Rays not exceeding $(100-\sigma^2)$ grams is less than 0.2, find the possible range of values of σ. [2]

It is given that $\sigma = 3$ for the rest of this question.

(b) Find the probability that the total mass of 2 randomly chosen regular packets of Rays and 3 randomly chosen packets of Luffles is greater than 0.55 kg. [3]

(c) The snack company decides to launch a new product called Mega Jumbo Pack, which consists of $(24-n)$ regular packets of Rays and n regular packets of Luffles. It is given that the probability of the mass of a Mega Jumbo Pack exceeding 20 times the mass of a regular packet of Luffles by more than 500 g is at least 0.1. Find the minimum value of n. [4]

8 A small cafe sells one type of coffee. The selling price of a cup of coffee is reviewed and adjusted at the beginning of every year depending on market conditions. Based on sales figures collected over 7 years, the cafe owner, Mr Tay, studied the effect of the selling price of a cup of coffee, \$$x$, on the average number of cups, y cups, sold per day within the year. The data is shown in the table below.

x	2.0	2.2	2.5	3.0	4.0	4.4	4.5
y	280	250	190	150	90	100	70

(a) On the grid in the Printed Answer Booklet, draw a scatter diagram of the data. [2]

Mr Tay realised that one of the values of y has been wrongly stated in the data table.

(b) Indicate the corresponding point on your diagram by labelling it P. Explain why the scatter diagram for the remaining points may be consistent with a model of the form $y = a + \frac{b}{x}$, where a and b are constants. [2]

(c) Omitting P, calculate the product moment correlation coefficient and the least squares estimates of a and b for the model $y = a + \frac{b}{x}$. [2]

(d) Use the model $y = a + \frac{b}{x}$ with the values of a and b found in part **(c)** to estimate the value of y that has been wrongly stated in the data table. Give two reasons why you would expect this estimate to be reliable. [3]

9 Based on observations over a long period, the mean time taken by male students in Griffles Junior College (GJC) to complete a 2.4 km run is known to be 11.3 minutes with a standard deviation of 2.2 minutes. As part of a review of the current physical training programme, the Physical Education (PE) department in GJC decided to test, at the 2.5% level of significance, whether the mean time taken by male students in GJC to complete a 2.4 km run has changed. The time taken, x minutes, by a random sample of 8 Year 5 male students from GJC to complete a 2.4 km run, are as follows.

11	11.5	10.8	11.2	11.4	11	11.8	12.5

(a) Write down the null and alternative hypotheses for this test, defining any symbols you use. [2]

(b) Stating a necessary assumption, find the critical region for this test. Hence state the conclusion of the test in the context of the question. [6]

The PE department in GJC also conducted a second test, at the 3% level of significance, to determine whether the mean time taken by female students in GJC to complete a 2.4 km run is less than 14.5 minutes. A random sample of n female students from GJC is taken, where n is large. The mean and standard deviation of the time taken by this sample to complete a 2.4 km run are found to be 14.2 minutes and 1.5 minutes respectively.

(c) Given that the PE department concludes that the mean time taken by female students to complete a 2.4 km run is less than 14.5 minutes, find the range of values that n can take. [5]

10 A bag contains 4 identical red balls and 6 identical blue balls.

(a) In the first game, the balls are randomly picked by a player from the bag, one at a time, without replacement, until there are no balls left in the bag.

(i) Find the probability that the last ball picked is red. [1]

(ii) Find the probability that in the first five picks, exactly 2 blue balls are picked given that at least 3 red balls are picked. [3]

(b) In the second game, each ball picked by a player will be placed back into the bag before the next ball is picked. The player makes a total of 20 random picks from the bag and the colour of each pick is recorded.

(i) Find the probability that the colour red is recorded exactly 4 times. [2]

(ii) Find the probability that the colour red is recorded more than 4 times but not more than 8 times. [2]

(iii) Find the probability that the 8^{th} pick is the 6^{th} time the colour blue is recorded. [2]

(iv) The second game is played by 50 randomly chosen players. Estimate the probability that the average number of times the colour red is recorded is more than 8.5. [4]

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