2025 JC2 H2 Maths Prelim Paper 1 Solutions

No	Solution	
1	$f(x)=ax^3+bx^2+cx+d$ $f(2)=8a+4b+2c+d=10 \quad -(1)$ $f(-2)=-8a+4b-2c+d=-2 \quad -(2)$ $f'(x)=3ax^2+2bx+c$ $f'(-1)=3a-2b+c=0 \quad -(3)$ $\int_{-2}^{2} f(x)\ dx=16 \Rightarrow \int_{-2}^{2}(ax^3+bx^2+cx+d)\ dx=16$ $\left[\frac{1}{4}ax^4+\frac{1}{3}bx^3+\frac{1}{2}cx^2+dx\right]_{-2}^{2}=16$ $\frac{16}{3}b+4d=16 \quad -(4)$ Using GC to solve (1), (2), (3), (4) $a=3, b=0, c=-9, d=4$	
2	Let r and h be the radius and the height of the cylinder respectively. External Surface Area $p=2\pi r^2+2\pi rh$ $\Rightarrow h=\frac{p-2\pi r^2}{2\pi r}=\frac{p}{2\pi r}-r$ Volume $V=\pi r^2 h$ $=\pi r^2\left(\frac{p}{2\pi r}-r\right)$ $=\frac{pr}{2}-\pi r^3$ $\frac{dV}{dr}=\frac{p}{2}-3\pi r^2$ For stationary values, $\frac{dV}{dr}=\frac{p}{2}-3\pi r^2=0 \quad \Rightarrow \quad r=\left(\frac{p}{6\pi}\right)^{\frac{1}{2}}$ $\frac{d^2V}{dr^2}=-6\pi r$ When $r=\left(\frac{p}{6\pi}\right)^{\frac{1}{2}}$, $\frac{d^2V}{dr^2}=-6\pi\left(\frac{p}{6\pi}\right)^{\frac{1}{2}}<0$ $\therefore$ V is a maximum when $r=\left(\frac{p}{6\pi}\right)^{\frac{1}{2}}$ cm.	

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3(i)

$l_1: \mathbf{r}=\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}-2\\1\\2\end{pmatrix}$, $l_2: \mathbf{r}=\begin{pmatrix}0\\5\\-8\end{pmatrix}+\mu\begin{pmatrix}2\\-2\\3\end{pmatrix}$, $\lambda \,\&\, \mu \in \mathbb{R}$

$\begin{pmatrix}-2\\1\\2\end{pmatrix}\neq k\begin{pmatrix}2\\-2\\3\end{pmatrix}$, where k is a scalar

l_1 is not parallel to l_2.

Equate the two lines:

$$\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}-2\\1\\2\end{pmatrix}=\begin{pmatrix}0\\5\\-8\end{pmatrix}+\mu\begin{pmatrix}2\\-2\\3\end{pmatrix}$$

$2\lambda+2\mu=1$ (1)

$-\lambda-2\mu=-3$ (2)

$-2\lambda+3\mu=8$ (3)

No solution from GC

OR

Solution for equations (1) and (2) $\lambda=-2$, $\mu=\frac{5}{2}$.

check that it satisfies (3).

$\text{LHS}=-2(2)+3\left(\frac{5}{2}\right)=\frac{7}{2}$

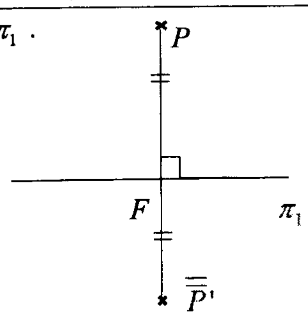
RHS=8

Hence (3) is not satisfied.

Since l_1 and l_2 are non-parallel and non-intersecting, they are skew lines.

(ii)

Let F be foot of perpendicular from P to π_1 .



Equation of line through P and F:

$$\mathbf{r}=\begin{pmatrix}1\\5\\0\end{pmatrix}+\alpha\begin{pmatrix}0\\1\\-1\end{pmatrix}=\begin{pmatrix}1\\5+\alpha\\-\alpha\end{pmatrix},\ \alpha\in\mathbb{R}$$

Substitute into equation of plane π_1:

$$\begin{pmatrix}1\\5+\lambda\\-\lambda\end{pmatrix}\bullet\begin{pmatrix}0\\1\\-1\end{pmatrix}=2$$

	$5+2\alpha=2 \Rightarrow \alpha=-\frac{3}{2} \quad \therefore \overrightarrow{OF}=\begin{pmatrix}1\\ \frac{7}{2}\\ \frac{3}{2}\end{pmatrix}$ $\overrightarrow{OP'}=2\overrightarrow{OF}-\overrightarrow{OP}=\begin{pmatrix}2\\7\\3\end{pmatrix}-\begin{pmatrix}1\\5\\0\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}$ Coordinates of P' are $(1,2,3)$. **Alternative method to find foot of perpendicular:** $Q(0,2,0)$ is a point on plane π $\overrightarrow{PF}=\left[\overrightarrow{PQ}\bullet\frac{1}{\sqrt{1+1}}\begin{pmatrix}0\\1\\-1\end{pmatrix}\right]\frac{1}{\sqrt{1+1}}\begin{pmatrix}0\\1\\-1\end{pmatrix}$ $=\frac{1}{2}\left[\begin{pmatrix}-1\\-3\\0\end{pmatrix}\bullet\begin{pmatrix}0\\1\\-1\end{pmatrix}\right]\begin{pmatrix}0\\1\\-1\end{pmatrix}=\frac{-3}{2}\begin{pmatrix}0\\1\\-1\end{pmatrix}$ $\overrightarrow{OF}=\frac{-3}{2}\begin{pmatrix}0\\1\\-1\end{pmatrix}+\begin{pmatrix}1\\5\\-0\end{pmatrix}=\begin{pmatrix}1\\ \frac{7}{2}\\ \frac{3}{2}\end{pmatrix}$ $\overrightarrow{OP'}=2\overrightarrow{OF}-\overrightarrow{OP}=\begin{pmatrix}2\\7\\3\end{pmatrix}-\begin{pmatrix}1\\5\\0\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}$ Coordinates of P' are $(1,2,3)$.	
(iii)	Let $A(1,2,0)$ that lies on l_1. $\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}=\begin{pmatrix}1\\5\\0\end{pmatrix}-\begin{pmatrix}1\\2\\0\end{pmatrix}=\begin{pmatrix}0\\3\\0\end{pmatrix}$ Normal of plane π_2:	

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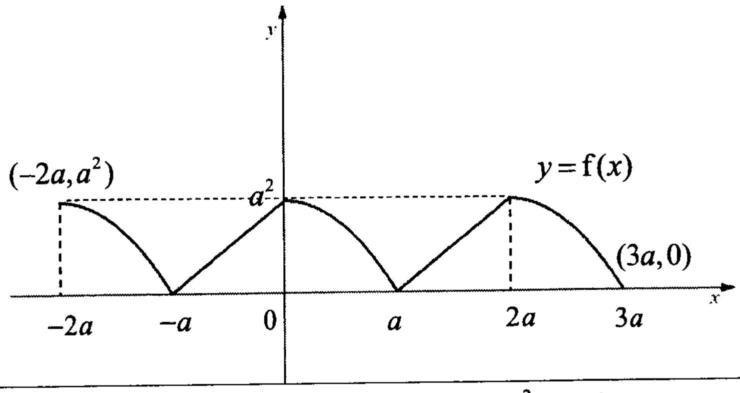
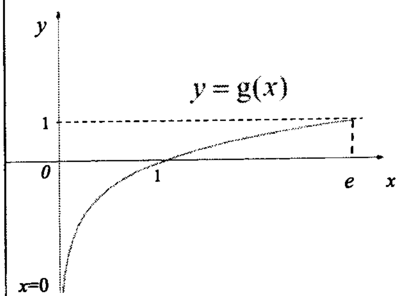
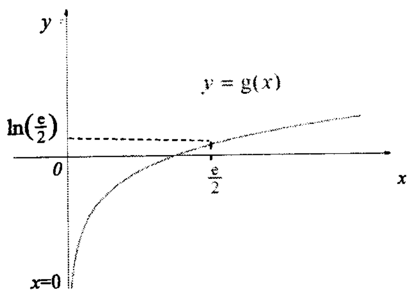
	$\underset{\sim}{n} = \begin{pmatrix}-2\\1\\2\end{pmatrix} \times \begin{pmatrix}0\\3\\0\end{pmatrix} = \begin{pmatrix}-6\\0\\-6\end{pmatrix} = -6\begin{pmatrix}1\\0\\1\end{pmatrix}$ Equation of plane containing P and l_1 : $\underset{\sim}{r}.\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}1\\0\\1\end{pmatrix}.\begin{pmatrix}1\\2\\0\end{pmatrix} = 1$ Cartesian equation of π_2 is $x + z = 1$	
4(a) (i)	 	
(ii)	$\mathrm{f}\left(\frac{101a}{2}\right) = \mathrm{f}\left(50a + \frac{a}{2}\right) = \mathrm{f}\left(\frac{a}{2}\right) = a^2 - \left(\frac{a}{2}\right)^2 = \frac{3a^2}{4}$	
(b)(i)	 For hg to exist, $R_g \subseteq D_h$ $R_g = (-\infty, 1]$ $D_h = (0, 4)$ Since $R_g \not\subset D_h$, hg does not exist.	
(ii)	Since h is a quadratic function with maximum point at $x = 1$, by symmetry, when $\mathrm{h}(x) = \frac{e}{2}$, $x = 0$ and $x = 2$. The other value of x is 2. Since h^{-1} exists, h is one to one and $R_h = D_{h^{-1}} = \left(0, \frac{e}{2}\right]$ (given) From the graph of h, restricted $D_h = [2, 4)$	

	Alternative Method 1 to find x: Since h is a quadratic function, its roots are at -2 and 4. Let $h(x)=a(x+2)(x-4)$ $h(0)=\frac{e}{2}$ $\frac{e}{2}=-8a \Rightarrow a=-\frac{e}{16}$ $h(x)=-\frac{e}{16}(x+2)(x-4)$ Solving $h(x)=\frac{e}{2}$, $x=0$ or 2 The other value of x is 2. **Alternative Method 2 to find x:** Since maximum point at $x=1$, Let $h(x)=a(x-1)^2+b$ $h(4)=0 \Rightarrow 9a+b=0$ ---(1) $h(0)=\frac{e}{2} \Rightarrow a+b=\frac{e}{2}$ ---(2) $(1)-(2)$: $8a=-\frac{e}{2} \Rightarrow a=-\frac{e}{16}$ $(2) \Rightarrow b=\frac{e}{2}+\frac{e}{16}=\frac{9e}{16}$ $h(x)=-\frac{e}{16}(x-1)^2+\frac{9e}{16}$ Solving $h(x)=\frac{e}{2}$, $x=0$ or 2 The other value of x is 2. **Alternative Method 3 to find x:** Let $h(x)=ax^2+bx+c$ $h(0)=\frac{e}{2}$, $c=\frac{e}{2}$ $h(4)=0$, $16a+4b+\frac{e}{2}=0$ ----- (1) $h'(1)=0$, $h'(x)=2ax+b$ $2a+b=0$ ----- (2) Solving (1) & (2), (by GC or manually) $a=-\frac{e}{16}, b=\frac{e}{8}$ Or $a=-0.1698926143$, $b=0.3397852286$	

	$\therefore \mathrm{h}(x) = -\frac{\mathrm{e}}{16}x^2 + \frac{\mathrm{e}}{8}x + \frac{\mathrm{e}}{2}$ Solving $\mathrm{h}(x) = \frac{\mathrm{e}}{2}$, $x = 0$ or 2 The other value of x is 2.	
(iii)	To find range of gh: $\mathrm{D_h} \xrightarrow{\mathrm{h}} \mathrm{R_h} \xrightarrow{\mathrm{g}} \mathrm{R_{gh}}$ $[2,4) \xrightarrow{\mathrm{h}} \left(0, \frac{\mathrm{e}}{2}\right] \xrightarrow{\mathrm{g}} (-\infty, 1-\ln 2]$  $\therefore\ \mathrm{R_{gh}} = \left(-\infty, \ln\frac{\mathrm{e}}{2}\right]$ or $(-\infty, 1-\ln 2]$ or $(-\infty, 0.307]$	
5(a)	$\frac{a+8d}{a+2d} = \frac{a+10d}{a+8d}$ $(a+8d)^2 = (a+2d)(a+10d)$ $a^2 + 16ad + 64d^2 = a^2 + 12ad + 20d^2$ $4ad + 44d^2 = 0$ $4d(a+11d) = 0$ $d = 0$ or $a + 11d = 0$ (reject since terms of AP, GP are distinct) $d = -\frac{1}{11}a$	

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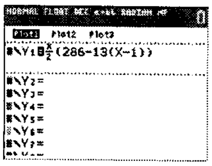
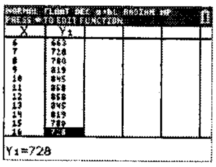
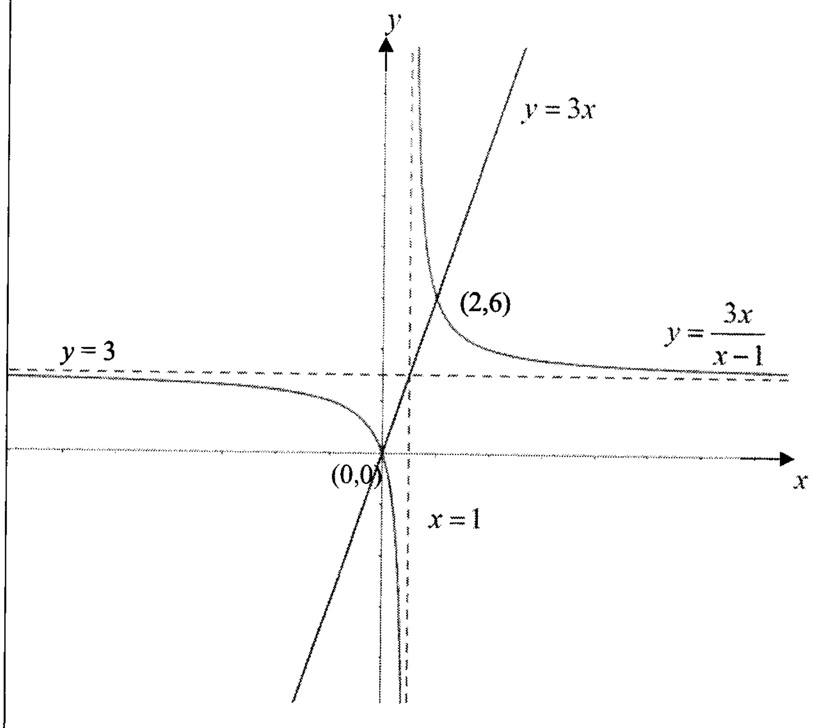
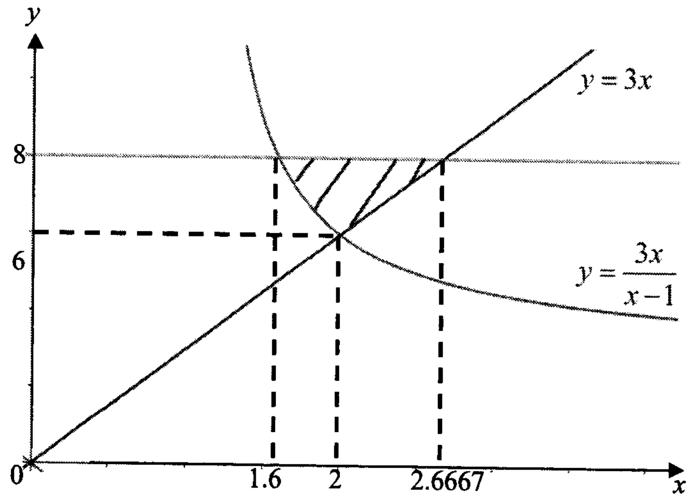
(b)	$S_1 = a = 143$ $d = -\frac{1}{11}(143) = -13$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ $= \frac{n}{2}\left[286 - 13(n-1)\right]$ From GC, largest $S_n = 858$ 	
(c)	$r = \frac{a+8d}{a+2d} = \frac{-11d+8d}{-11d+2d} = \frac{-3d}{-9d} = \frac{1}{3}$ Since $\lvert r \rvert = \frac{1}{3} < 1$,the geometric series is convergent.	
(d)	Let u_n be the nth term of the geometric progression. $u_{n+1} + u_{n+2} + \ldots$ $= S_\infty - S_n$ $= \frac{b}{1-r} - \frac{b(1-r^n)}{1-r}$ $= \frac{br^n}{1-r}$ $= \frac{b\left(\frac{1}{3}\right)^n}{1-\frac{1}{3}}$ $= \frac{3}{2}\left(\frac{1}{3}\right)^n b$ $= \frac{b}{2}\left(\frac{1}{3}\right)^{n-1} \leq \frac{1}{2}b \quad \text{since } \left(\frac{1}{3}\right)^{n-1} \leq 1$ Alternative:	

	$u_{n+1}+u_{n+2}+u_{n+3}+\ldots$ $=b\left(\frac{1}{3}\right)^{n}+b\left(\frac{1}{3}\right)^{n+1}+b\left(\frac{1}{3}\right)^{n+2}+\ldots$ $=\frac{b\left(\frac{1}{3}\right)^{n}}{1-\frac{1}{3}}$ $=\frac{3}{2}b\left(\frac{1}{3}\right)^{n}$ $=\frac{b}{2}\left(\frac{1}{3}\right)^{n-1}\leq\frac{b}{2}$ since $\left(\frac{1}{3}\right)^{n-1}\leq 1$	
6 (i)	1. Reflection of the graph in the x-axis followed by 2. Scaling of the resulting graph by a factor $\frac{1}{3}$ parallel to the x-axis.	
(ii)	From (i), $g(x)=-f(3x)$ $\therefore\ a=-1, b=3, c=0, d=0.$	
(iii)	$g'(x)=-3f'(3x)$ $g'(2)=-3f'(6)$ Since $f'(6)=4$ $g'(2)=-3f'(6)=-3(4)=-12$	

7(a)(i)		
(a)(ii)	 Area of region bounded by both curves and $y = 8$ $= \int_6^8 \left(\frac{y}{3} - \frac{y}{y-3} \right) dy$ $= 1.13 \text{ units}^2$ (3 s.f.)	

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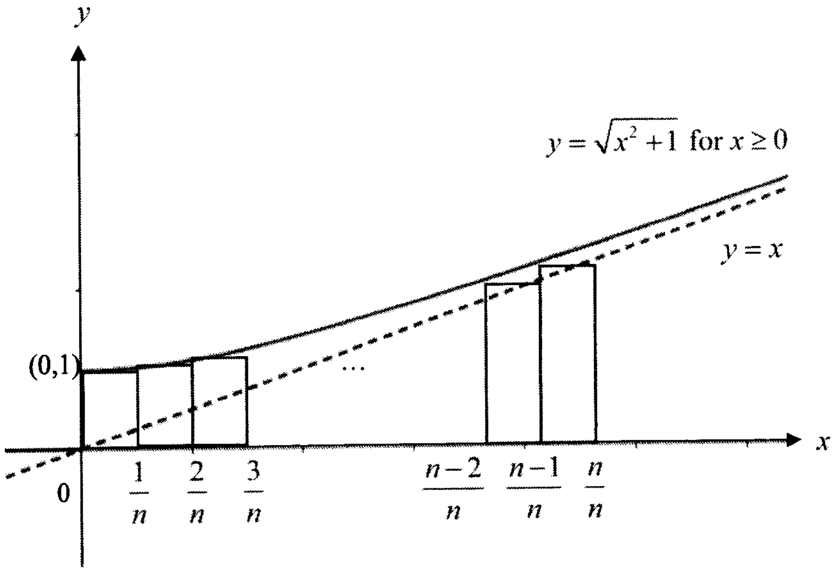
	Alternative Method: Area of region bounded by both curves and $y = 8$ $= (2.6667 - 1.6)(8) - \int_{1.6}^{2} \frac{3x}{x-1}\, dx - \int_{2}^{2.6667} 3x\, dx$ $= 8.5336 - 2.7325 - 4.6669$ $= 1.13 \text{ units}^2$ (3 s.f.)	
7(b)(i)	 If we split the area of the region enclosed by the curve $y = f(x) = \sqrt{x^2+1}$, the x-axis and the lines $x = 0$ and $x = 1$ into n rectangles, each of width $\frac{1}{n}$ as shown, the sum of the area of the n rectangles is given by $A = \frac{1}{n} \times f(0) + \frac{1}{n} \times f\left(\frac{1}{n}\right) + \ldots + \frac{1}{n} \times f\left(\frac{n-1}{n}\right)$ $= \frac{1}{n}\left\{f(0) + f\left(\frac{1}{n}\right) + \ldots + f\left(\frac{n-1}{n}\right)\right\}$ $= \frac{1}{n}\left\{1 + \sqrt{\frac{1}{n^2}+1} + \sqrt{\frac{4}{n^2}+1} + \ldots + \sqrt{\frac{(n-1)^2}{n^2}+1}\right\}$ As the number of rectangles increases, i.e. $n \to \infty$, the width of each rectangle decreases, then the sum of the area of the n	

	rectangle approaches the area of the region enclosed by curve $y=\sqrt{x^2+1}$, the x-axis and the lines $x = 0$ and $x = 1$ which is given by $\int_0^1 \sqrt{x^2+1}\ \mathrm{d}x$. Hence $S=\lim_{n\to\infty} A$ $=\lim_{n\to\infty}\frac{1}{n}\left\{1+\sqrt{\frac{1}{n^2}+1}+\sqrt{\frac{4}{n^2}+1}+\ldots+\sqrt{\frac{(n-1)^2}{n^2}+1}\right\}$ $=\int_0^1 \mathrm{f}(x)\ \mathrm{d}x$ $=\int_0^1 \sqrt{1+x^2}\ \mathrm{d}x$	
(b)(ii)	Actual area under the graph $y=\sqrt{x^2+1}$ from $x=0$ to $x=1$ $\int_0^1 \sqrt{x^2+1}\ \mathrm{d}x=1.1478=1.15$ (3 s.f.)	
8(i)	$(x-y)^2=2\mathrm{e}^{xy}$ Differentiate with respect to x $2(x-y)\left(1-\frac{\mathrm{d}y}{\mathrm{d}x}\right)=2\mathrm{e}^{xy}\left(x\frac{\mathrm{d}y}{\mathrm{d}x}+y\right)$ $(x-y)-(x-y)\frac{\mathrm{d}y}{\mathrm{d}x}=x\mathrm{e}^{xy}\frac{\mathrm{d}y}{\mathrm{d}x}+y\mathrm{e}^{xy}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x-y-y\mathrm{e}^{xy}}{x-y+x\mathrm{e}^{xy}}$ (shown)	
(ii)	When $x = 0$, $(0-y)^2=2\mathrm{e}^{(0)y}$ $y=\pm\sqrt{2}$ $y=\sqrt{2}\ (\because y>0)$ At $(0, \sqrt{2})$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{0-\sqrt{2}-\sqrt{2}\mathrm{e}^{0(\sqrt{2})}}{0-\sqrt{2}+(0)\mathrm{e}^{0(\sqrt{2})}}$ $=\frac{-2\sqrt{2}}{-\sqrt{2}}$ $=2$	

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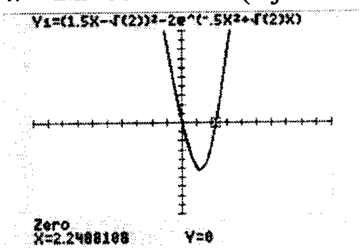
	Gradient of normal $=-\frac{1}{2}$ Equation of the normal to the curve at (0, $\sqrt{2}$) is $y-\sqrt{2}=-\frac{1}{2}(x-0)$ $y=-\frac{1}{2}x+\sqrt{2}$	
(iii)	Substitute $y=-\frac{1}{2}x+\sqrt{2}$ into $(x-y)^2=2e^{xy}$, $\left(x+\frac{1}{2}x-\sqrt{2}\right)^2=2e^{x\left(-\frac{1}{2}x+\sqrt{2}\right)}$ $\left(\frac{3}{2}x-\sqrt{2}\right)^2=2e^{-\frac{1}{2}x^2+\sqrt{2}x}$ Using G.C., $x=2.2488$ or $x=0$ (reject $\because$ it's point A)  when $x=2.2488$, $y=-\frac{1}{2}(2.2488)+\sqrt{2}=0.28980$ (5 s.f.) $\therefore B(2.25,0.290)$	

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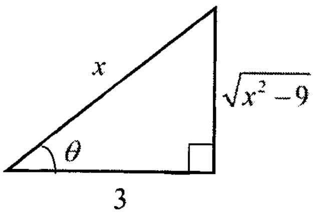
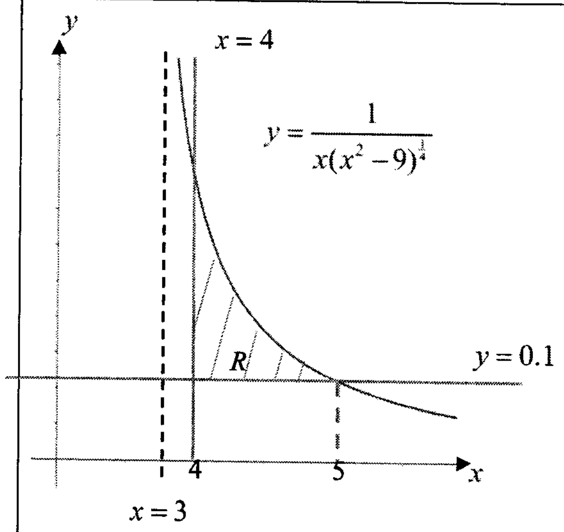
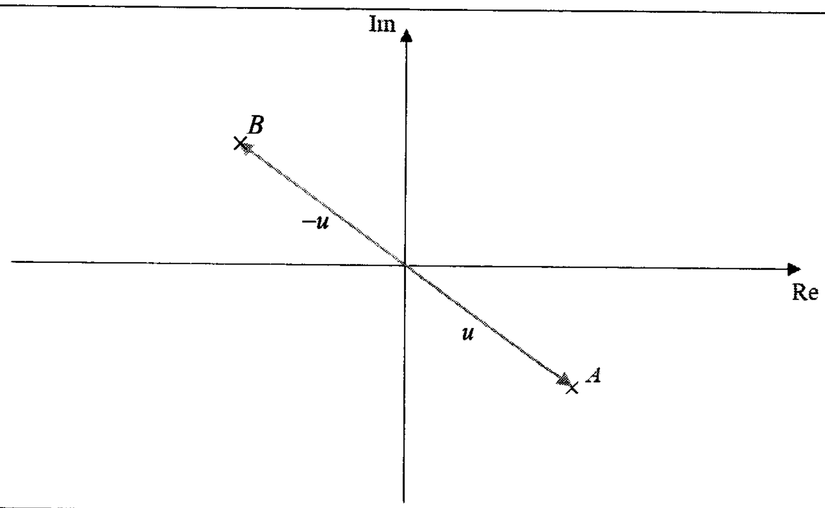
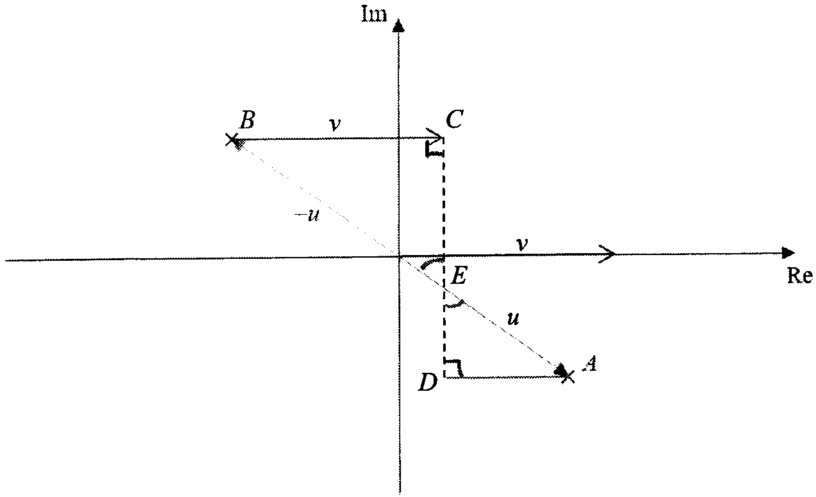
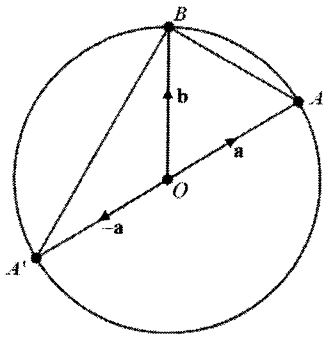
9(a)	$\int \frac{1}{x^2\sqrt{x^2-9}}\,\mathrm{d}x$ let $x = 3\sec\theta$ $= \int \frac{1}{9\sec^2\theta\sqrt{9\sec^2\theta-9}}\frac{\mathrm{d}x}{\mathrm{d}\theta}\,\mathrm{d}\theta$ $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 3\sec\theta\tan\theta$ $= \int \frac{1}{9\sec^2\theta\sqrt{9\sec^2\theta-9}}(3\sec\theta\tan\theta)\,\mathrm{d}\theta$ $= \int \frac{1}{9\sec\theta\sqrt{\sec^2\theta-1}}(\tan\theta)\,\mathrm{d}\theta$ $= \int \frac{1}{9\sec\theta\sqrt{\tan^2\theta}}(\tan\theta)\,\mathrm{d}\theta$  $= \int \frac{1}{9\sec\theta}\,\mathrm{d}\theta$ $= \frac{1}{9}\int \cos\theta\,\mathrm{d}\theta$ $= \frac{1}{9}\sin\theta + C$ $x = 3\sec\theta$ => $\cos\theta = \frac{3}{x}$ $= \frac{\sqrt{x^2-9}}{9x} + C$ where C is an arbitrary constant	
9(b)		

	Volume of solid $=\pi\int_4^5 y^2 \text{ d}x-\pi(0.1)^2(1)$ $=\pi\int_4^5 \frac{1}{x^2\sqrt{x^2-9}}\text{d}x-\pi(0.1)^2(1)$ $=\pi\left[\frac{\sqrt{x^2-9}}{9x}\right]_4^5-\frac{\pi}{100}$ $=\pi\left(\frac{\sqrt{5^2-9}}{9\times5}-\frac{\sqrt{4^2-9}}{9\times4}\right)-\frac{\pi}{100}$ $=\pi\left(\frac{4}{45}-\frac{\sqrt{7}}{36}\right)-\frac{\pi}{100}$ $=\pi\left(\frac{71}{900}-\frac{\sqrt{7}}{36}\right)$ units^3	
10(a)	$\lvert z\rvert+5w=0$ (1) $\text{i}z-4w=-4+7\text{i}$ (2) From (1): $w=-\frac{1}{5}\lvert z\rvert$ (3) Substitute (3) into (2): $\text{i}z+\frac{4}{5}\lvert z\rvert=-4+7\text{i}$ Let $z=x+y\text{i}$, $\text{i}(x+y\text{i})+\frac{4}{5}\sqrt{x^2+y^2}=-4+7\text{i}$ $\left(-y+\frac{4}{5}\sqrt{x^2+y^2}\right)+x\text{i}=-4+7\text{i}$ Comparing imaginary parts: $x=7$ Comparing real parts:	

	$-y+\frac{4}{5}\sqrt{7^2+y^2}=-4$ $5(y-4)=4\sqrt{49+y^2}$ (*) $25(16-8y+y^2)=16(49+y^2)$ $9y^2-200y-384=0$ $(y-24)(9y+16)=0$ $y=24$ or $y=-\frac{16}{9}$ Note that $y=-\frac{16}{9}$ does not satisfy (*) and hence $y=24$ $\therefore\ z=7+24\text{i}$ Substitute into (3): $w=-\frac{1}{5}\sqrt{7^2+24^2}$ $=-\frac{1}{5}\sqrt{625}$ $\therefore\ w=-5$	
10(b) (i)		
10(b) (ii)	As points C and D represent complex numbers that form a complex conjugate pair, therefore line segment CD must be **vertical**, and points C and D must be equidistant from the real axis. Given that $\angle CDA=90^\circ$, and CD is vertical, thus we require line segment DA to be **horizontal**.	

	Since A and B are also equidistant from the real axis, DA being horizontal implies that BC must also be horizontal. Since C represents the complex number $v - u$, the vector BC must represent the complex number v. Since BC is horizontal, v must be a real number. Hence, $\text{Im}(v) = 0$.  Alternatively, $u = a + ib$ $v = c + id$ $v - u = (c-a) + (d-b)i$ $(v-u)^* = (c-a) + (b-d)i$ $A(a,b)$ $C(c-a, d-b)$ $D(c-a, b-d)$ C and D share the same x coordinate. Therefore CD is vertical. $\angle CDA = 90^\circ$ implies that DA must be horizontal, hence A and D must share the same y-coordinate. $b - d = b \Rightarrow d = 0 \Rightarrow \text{Im}(v) = 0$	
	Let $\overrightarrow{OA} = \begin{pmatrix} a \\ b \end{pmatrix}$ where $u = a + bi$, where $a, b \in \mathbb{R}$. Let $\overrightarrow{OV} = \begin{pmatrix} c \\ d \end{pmatrix}$ where $v = c + di$, where $c, d \in \mathbb{R}$.	

	Then $\overrightarrow{OC}=\overrightarrow{OV}-\overrightarrow{OA}=\begin{pmatrix}c\\d\end{pmatrix}-\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}c-a\\d-b\end{pmatrix}$. $\overrightarrow{OD}=\overrightarrow{OV^*}-\overrightarrow{OA^*}=\begin{pmatrix}c-a\\-(d-b)\end{pmatrix}=\begin{pmatrix}c-a\\b-d\end{pmatrix}$ Since $\angle CDA=90^\circ$, then $\overrightarrow{CD}\bullet\overrightarrow{AD}=0$ $\overrightarrow{CD}=\overrightarrow{OD}-\overrightarrow{OC}=\begin{pmatrix}c-a\\b-d\end{pmatrix}-\begin{pmatrix}c-a\\d-b\end{pmatrix}=\begin{pmatrix}0\\2b-2d\end{pmatrix}$ $\overrightarrow{DA}=\overrightarrow{OA}-\overrightarrow{OD}=\begin{pmatrix}a\\b\end{pmatrix}-\begin{pmatrix}c-a\\b-d\end{pmatrix}=\begin{pmatrix}2a-c\\d\end{pmatrix}$ $\overrightarrow{CD}\bullet\overrightarrow{DA}=0$ $\begin{pmatrix}0\\2b-2d\end{pmatrix}\bullet\begin{pmatrix}2a-c\\d\end{pmatrix}=0$ $2d(b-d)=0$ $d=0$ or $d=b$ If $d=b$, then $\overrightarrow{OC}=\begin{pmatrix}c-a\\0\end{pmatrix}$ and $\overrightarrow{OD}=\begin{pmatrix}c-a\\0\end{pmatrix}$ then C and D are the same point. Then $\angle CDA\neq 90^\circ$. Hence $d=0\Rightarrow \text{Im}(v)=0$ (shown)	
11 (i)	 Since AA' is a straight line passing through the origin O, $\therefore \overrightarrow{OA'}=-\mathbf{a}$ $\overrightarrow{BA}=\overrightarrow{OA}-\overrightarrow{OB}$ $=\mathbf{a}-\mathbf{b}$ $\overrightarrow{BA'}=\overrightarrow{OA'}-\overrightarrow{OB}$ $=-\mathbf{a}-\mathbf{b}$	

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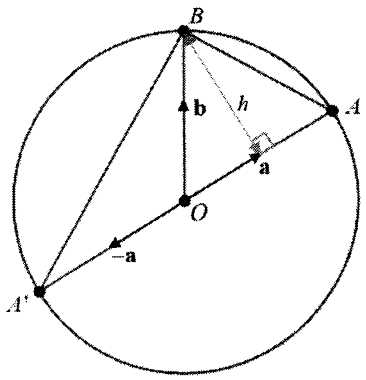
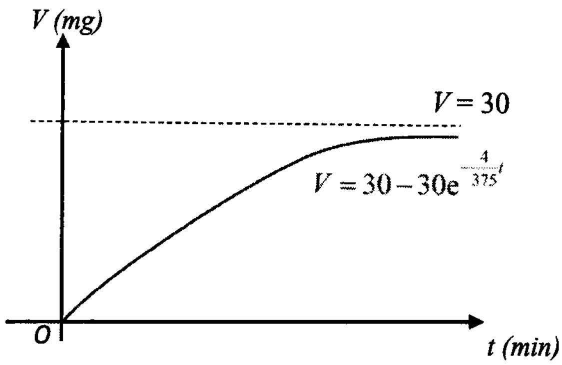
	$\overrightarrow{BA}\cdot\overrightarrow{BA'}=(\mathbf{a}-\mathbf{b})\cdot(-\mathbf{a}-\mathbf{b})$ $=-\mathbf{a}\cdot\mathbf{a}-\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{a}+\mathbf{b}\cdot\mathbf{b}$ $=-\lvert\mathbf{a}\rvert^2+\lvert\mathbf{b}\rvert^2$ $(\lvert\mathbf{a}\rvert=\lvert\mathbf{b}\rvert$ $\because$ radius of circle$)$ $=0$ $\therefore \overrightarrow{BA}\perp\overrightarrow{BA'}$, hence $\measuredangle ABA'=90^\circ$ (shown).	
(ii)	By Vector Product, $\Delta OAB=\frac{1}{2}\lvert\mathbf{a}\times\mathbf{b}\rvert$  From the above diagram, $\frac{1}{2}\times\lvert\overrightarrow{OA}\rvert\times h=\text{area of }\Delta OAB=\frac{1}{2}\lvert\mathbf{a}\times\mathbf{b}\rvert$ (1) Area of $\Delta ABA'=\frac{1}{2}\times AA'\times h$ $=\frac{1}{2}\times 2\lvert\overrightarrow{OA}\rvert\times h$ $=\left[\frac{1}{2}\times\lvert\overrightarrow{OA}\rvert\times h\right]\times 2$, substituting (1) $=\frac{1}{2}\lvert\mathbf{a}\times\mathbf{b}\rvert\times 2$ $=\lvert\mathbf{a}\times\mathbf{b}\rvert$ where $k=1$	
12 (i)	$\frac{dV}{dt}=\frac{dV_{in}}{dt}-\frac{dV_{out}}{dt}$ $=0.32-kV$	

	When $\frac{\mathrm{d}V}{\mathrm{d}t}=0.12$ and $V=18.75$ mg, $0.12=0.32-k(18.75)$ $k=\frac{0.2}{18.75}=\frac{4}{375}$ $\frac{\mathrm{d}V}{\mathrm{d}t}=0.32-\frac{4}{375}V=\frac{1}{375}(120-4V)=\frac{4}{375}(30-V)$ (shown)	
(ii)	$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{4}{375}(30-V)$ $\int\frac{1}{30-V}\,\mathrm{d}V=\int\frac{4}{375}\,\mathrm{d}t$ $-\ln\lvert 30-V\rvert=\frac{4}{375}t+C$ $\ln\lvert 30-V\rvert=-\frac{4}{375}t-C$ $\lvert 30-V\rvert=\mathrm{e}^{-\frac{4}{375}t-C}$ $30-V=\pm\mathrm{e}^{-C}\mathrm{e}^{-\frac{4}{375}t}$ $30-V=A\mathrm{e}^{-\frac{4}{375}t}$ where $A=\pm\mathrm{e}^{-C}$ $V=30-A\mathrm{e}^{-\frac{4}{375}t}$ When $t=0, V=0$ $A=30$ $V=30-30\mathrm{e}^{-\frac{4}{375}t}$	
(iii)		

(iv)	As the maximum mass of particulates is 30 mg, the extraction device is effective enough to meet the recommendation.	
(v)	$\frac{dV}{dt}=0.32-\frac{2}{375}V=\frac{1}{375}(120-2V)=\frac{2}{375}(60-V)$	
(vi)	At steady state, $\frac{dV}{dt}=0$ $0=60-V$ $V=60$	

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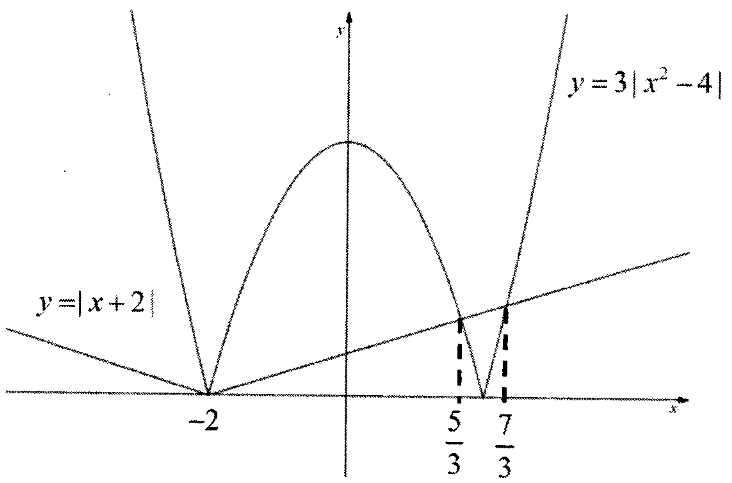
No	Solution	
1(i)	**Method 1: Algebraic Method** $3\lvert x^2-4\rvert \le \lvert x+2\rvert$ $3\lvert x^2-4\rvert \le \lvert x+2\rvert$ $9(x^2-4)^2 \le (x+2)^2$ $9(x-2)^2(x+2)^2 \le (x+2)^2$ $(x+2)^2\left[9(x-2)^2-1\right] \le 0$ $(x+2)^2(3x-5)(3x-7) \le 0$ + + − + -2 $\frac{5}{3}$ $\frac{7}{3}$ $\frac{5}{3} \le x \le \frac{7}{3}$ or $x=-2$	
	Method 2: Graphical Method  Using GC to find x-coordinates of the points of intersection between $y=\lvert x+2\rvert$ and $y=3\lvert x^2-4\rvert$: $x=-2$ or $\frac{5}{3}$ or $x=\frac{7}{3}$	

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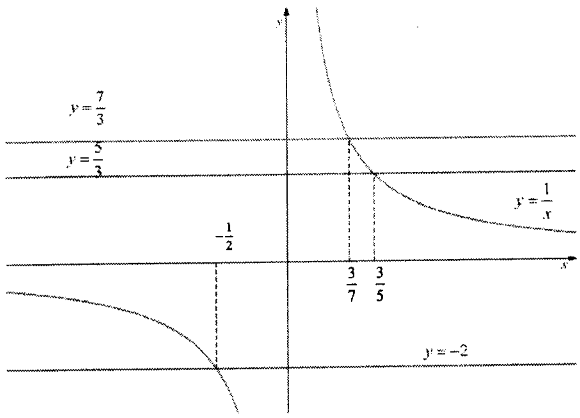
	From the graph, $\frac{5}{3} \le x \le \frac{7}{3}$ or $x = -2$	
(ii)	$\frac{3\lvert 1-4x^2\rvert}{x^2} \le \left\lvert \frac{1+2x}{x} \right\rvert$ $3\left\lvert \frac{1-4x^2}{x^2} \right\rvert \le \left\lvert \frac{1}{x}+2 \right\rvert$ $3\left\lvert \frac{1}{x^2}-4 \right\rvert \le \left\lvert \frac{1}{x}+2 \right\rvert$ Replace x with $\frac{1}{x}$, $\frac{5}{3} \le \frac{1}{x} \le \frac{7}{3}$ or $\frac{1}{x} = -2$  From the graph, $\frac{3}{7} \le x \le \frac{3}{5}$ or $x = -\frac{1}{2}$	
2(a) (i)	Since $x = \alpha$ is a root, $f(\alpha) = 0$ $\Rightarrow p\alpha^6 + q\alpha^4 + r = 0$ $f(-\alpha) = p(-\alpha)^6 + q(-\alpha)^4 + r$ $= p\alpha^6 + q\alpha^4 + r$ $= 0$ $\Rightarrow x = -\alpha$ is also a root.	
2 (a) (ii)	$x = 5$ and $x = \beta$ are roots	

	$\Rightarrow x=-5$ and $x=-\beta$ are also roots. Since coefficients of f are all real, $x=\beta^*$ and $x=-\beta^*$ are also roots. Therefore, the remaining roots are -5, $\beta^*, -\beta,$ and $-\beta^*$.	
(b)	**Method 1:** Since $1+i$ is a root, $z^3+(3-a)z^2-(2+6\mathrm{i})z-6=(z-1-\mathrm{i})(z^2+Bz+C)$ Comparing coefficients, Constant: $-6=C(-1-\mathrm{i})$ $C=3-3\mathrm{i}$ ---(1) z^2: $3-a=B-1-\mathrm{i} \Rightarrow a=4-B+\mathrm{i}$ ------(2) z: $-2-6\mathrm{i}=3-3\mathrm{i}+B(-1-\mathrm{i})$ $(1+\mathrm{i})B=5+3\mathrm{i}$ $B=\dfrac{5+3\mathrm{i}}{1+\mathrm{i}}$ $B=4-\mathrm{i}$ $a=4-B+\mathrm{i}=4-(4-\mathrm{i})+\mathrm{i}=2\mathrm{i}$ $z^3+(3-a)z^2-(2+6\mathrm{i})z-6=0$ $\Rightarrow (z-1-\mathrm{i})(z^2+(4-\mathrm{i})z+3-3\mathrm{i})=0$ $z=1+\mathrm{i}$ or $z^2+(4-\mathrm{i})z+3-3\mathrm{i}=0$ $(z+3)(z+1-\mathrm{i})=0$ $z=-3$ or $z=-1+\mathrm{i}$ OR $z^2+(4-\mathrm{i})z+3-3\mathrm{i}=0$ $z=\dfrac{-(4-\mathrm{i})\pm\sqrt{(4-\mathrm{i})^2-4(1)(3-3\mathrm{i})}}{2}$ $z=\dfrac{-4+\mathrm{i}\pm(2+\mathrm{i})}{2}$ $z=-3$ or $z=-1+\mathrm{i}$ Therefore, the other roots are -3 and $-1+\mathrm{i}$.	
	Method 2: Since $1+\mathrm{i}$ is a root,	

	$(1+\text{i})^3+(3-a)(1+\text{i})^2-(2+6\text{i})(1+\text{i})-6=0$ $-2+2\text{i}+(3-a)(2\text{i})-(-4+8\text{i})-6=0$ $(3-a)2\text{i}=4+6\text{i}$ $3-a=\dfrac{4+6\text{i}}{2\text{i}}$ $3-a=3-2\text{i}$ $a=2\text{i}$ $\therefore z^3+(3-2\text{i})z^2-(2+6\text{i})z-6=(z-1-\text{i})(z^2+Bz+3-3\text{i})$ Comparing coefficients, $z^2:\ 3-2\text{i}=B-1-\text{i}$ $B=4-\text{i}$ $z^3+(3-a)z^2-(2+6\text{i})z-6=0$ $\Rightarrow (z-1-\text{i})(z^2+(4-\text{i})z+3-3\text{i})=0$ $z=1+\text{i}$ or $z^2+(4-\text{i})z+3-3\text{i}=0$ $(z+3)(z+1-\text{i})=0$ $z=-3$ or $z=-1+\text{i}$ OR $z^2+(4-\text{i})z+3-3\text{i}=0$ $z=\dfrac{-(4-\text{i})\pm\sqrt{(4-\text{i})^2-4(1)(3-3\text{i})}}{2}$ $z=\dfrac{-4+\text{i}\pm(2+\text{i})}{2}$ $z=-3$ or $z=-1+\text{i}$ Therefore, the other roots are -3 and $-1+\text{i}$.	
3(a)	$BC=2$ E is the midpoint of BC. $\therefore BE=EC=1$ By Pythagoras' theorem, $AE=\sqrt{2^2-1^2}=\sqrt{3}$ In ΔAED, $\tan\left(\dfrac{\pi}{4}+x\right)=\dfrac{\sqrt{3}}{ED}$ $ED=\dfrac{\sqrt{3}}{\tan\left(\dfrac{\pi}{4}+x\right)}$	

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	$\therefore BD = BE + ED = 1 + \dfrac{\sqrt{3}}{\tan\left(\dfrac{\pi}{4}+x\right)}$ (shown)	
(b)	$BD = 1 + \dfrac{\sqrt{3}}{\tan\left(\dfrac{\pi}{4}+x\right)}$ $= 1 + \dfrac{\sqrt{3}}{\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}}$ $= 1 + \dfrac{\sqrt{3}}{\dfrac{1+\tan x}{1-\tan x}}$ Since $\tan x \approx x$, $BD \approx 1 + \sqrt{3}(1-x)(1+x)^{-1}$ $= 1 + \sqrt{3}(1-x)(1-x+x^2)$ $= 1 + \sqrt{3}(1-x+x^2-x+x^2)$ $= 1 + \sqrt{3} - 2\sqrt{3}x + 2\sqrt{3}x^2$ where $a = 1+\sqrt{3}$, $b = -2\sqrt{3}$, $c = 2\sqrt{3}$	
4(i)	Note that from $23u_{r+1} = 19u_r + 16$ $\therefore u_{r+1} = \dfrac{19}{23}u_r + \dfrac{16}{23}$ From GC, $u_{10} = 3.6417$	
(ii)	As $r \to \infty$, $u_r \to l$ and $u_{r+1} \to l$: $23l = 19l + 16$ $4l = 16$ $l = 4$	
(iii)	When u_r exceeds 99.9% of l: $u_r > \dfrac{99.9}{100}l$ $u_r > \dfrac{99.9}{100}(4)$ $u_r > 3.996$	

	$\therefore\ u_{r+1}=\frac{19}{23}u_r+\frac{16}{23}$ By GC, <table><tr><td>r</td><td>u_r</td></tr><tr><td>33</td><td>$3.9956<3.996$</td></tr><tr><td>34</td><td>$3.9963>3.996$</td></tr><tr><td>35</td><td>$3.9970>3.996$</td></tr></table> $\therefore$ Smallest $r=34$	
(iv)	From $S_n=\sum_{r=1}^{n}(u_r-c)$ where $u_r=4-2\left(\frac{19}{23}\right)^{n-1}$, $S_n=\sum_{k=1}^{n}\left[4-2\left(\frac{19}{23}\right)^{k-1}-c\right]$ $=\sum_{k=1}^{n}(4-c)-2\sum_{k=1}^{n}\left(\frac{19}{23}\right)^{k-1}$ $=(4-c)n-2\left[1+\frac{19}{23}+\left(\frac{19}{23}\right)^2+\ldots+\left(\frac{19}{23}\right)^{n-1}\right]$ $=(4-c)n-2\times\frac{1\left[1-\left(\frac{19}{23}\right)^n\right]}{1-\frac{19}{23}}$ $=(4-c)n-\frac{23}{2}\left[1-\left(\frac{19}{23}\right)^n\right]$ Hence, $A=4-c$ and $B=-\frac{23}{2}$	
(v)	$S_n=(4-c)n-\frac{23}{2}\left[1-\left(\frac{19}{23}\right)^n\right]$ As $n\to\infty$, $\left(\frac{19}{23}\right)^n\to 0$ and $(4-c)n\to\infty$ if $c\neq 4$. Hence if S_n diverges if $c\neq 4$	

5(i)

$$x = \tan\theta - \cot\theta,\ y = \tan\theta + \cot\theta,\ \ 0 < \theta < \frac{\pi}{2}$$

$$\frac{dx}{d\theta} = \sec^2\theta + \cos ec^2\theta \qquad \frac{dy}{d\theta} = \sec^2\theta - \cos ec^2\theta$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$$

$$= \frac{\sec^2\theta - \cos ec^2\theta}{\sec^2\theta + \cos ec^2\theta}$$

$$= \frac{\frac{1}{\cos^2\theta} - \frac{1}{\sin^2\theta}}{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}$$

$$= \frac{\sin^2\theta - \cos^2\theta}{\sin^2\theta + \cos^2\theta}$$

$$= \sin^2\theta - \cos^2\theta$$

$$= -\cos 2\theta \text{ (Shown)}$$

Alternative method

$$x = \tan\theta - \cot\theta,\ y = \tan\theta + \cot\theta,\ \ 0 < \theta < \frac{\pi}{2}$$

$$\frac{dx}{d\theta} = \sec^2\theta + \cos ec^2\theta \qquad \frac{dy}{d\theta} = \sec^2\theta - \cos ec^2\theta$$

$$\frac{dy}{dx} = \frac{\sec^2\theta - \cos ec^2\theta}{\sec^2\theta + \cos ec^2\theta}$$

$$= \frac{\sec^2\theta\left(1 - \frac{\cos ec^2\theta}{\sec^2\theta}\right)}{\sec^2\theta\left(1 - \frac{\cos ec^2\theta}{\sec^2\theta}\right)}$$

$$= \frac{\left(1 - \cot^2\theta\right)}{1 + \cot^2\theta}$$

$$= \frac{1 - \cot^2\theta}{\cos ec^2\theta}$$

$$= \frac{1}{\cos ec^2\theta} - \cos^2\theta$$

$$= \sin^2\theta - \cos^2\theta$$

$$= -\left(\cos^2\theta - \sin^2\theta\right)$$

$$= -\cos 2\theta \text{ (Shown)}$$

(ii)	At $\theta = p$, Equation of $L: y-(\tan p+\cot p) = -\cos 2p(x-(\tan p-\cot p))$ $y+(\cos 2p)x = \tan p+\cot p+\cos 2p\tan p-\cos 2p\cot p$ $y+(\cos 2p)x = \tan p(1+\cos 2p)+\cot p(1-\cos 2p)$ $y+(\cos 2p)x = \tan p(2\cos^2 p)+\cot p(2\sin^2 p)$ $y+(\cos 2p)x = \frac{\sin p}{\cos p}(2\cos^2 p)+\frac{\cos p}{\sin p}(2\sin^2 p)$ $y+(\cos 2p)x = 2\sin p\cos p+2\sin p\cos p$ $y+(\cos 2p)x = 2\sin 2p$ (shown)	
(iii)	Given $\frac{dp}{dt} = 0.2$ When $y=0$, $(\cos 2p)x = 2\sin 2p$ $x = 2\tan 2p$ Note: Since $0<\theta<\frac{\pi}{4} \Rightarrow 0<p<\frac{\pi}{4} \Rightarrow 0<2p<\frac{\pi}{2}$ $\tan 2p>0$ $Q(2\tan 2p, 0)$ When $x=0$, $y=2\sin 2p$ Note: Since $0<\theta<\frac{\pi}{4} \Rightarrow 0<p<\frac{\pi}{4} \Rightarrow 0<2p<\frac{\pi}{2}$ $\sin 2p>0$ $R(0, 2\sin 2p)$ Area of OQR, $A = \frac{1}{2}(2\sin 2p)(2\tan 2p) = 2\tan 2p\sin 2p$ $\frac{dA}{dp} = 2\left(\tan 2p(2\cos 2p)+(\sin 2p)2\sec^2 2p\right)$ $= 4\left(\sin 2p+\tan 2p\sec 2p\right)$ At $p=\frac{\pi}{6}$, $\frac{dA}{dp} = 4\left(\sin\left(\frac{\pi}{3}\right)+\tan\left(\frac{\pi}{3}\right)\sec\left(\frac{\pi}{3}\right)\right) = 4\left(\frac{\sqrt{3}}{2}+2\sqrt{3}\right) = 4\left(\frac{5}{2}\right)\sqrt{3} = 10\sqrt{3}$ $\frac{dA}{dt} = \left(\frac{dA}{dp}\right)\left(\frac{dp}{dt}\right) = 10\sqrt{3}\times 0.2 = 2\sqrt{3}$ units^2/s	

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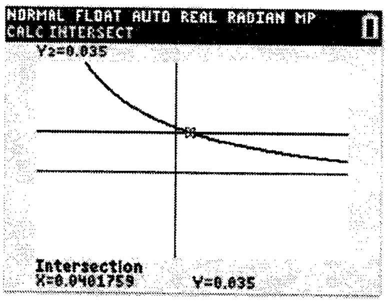
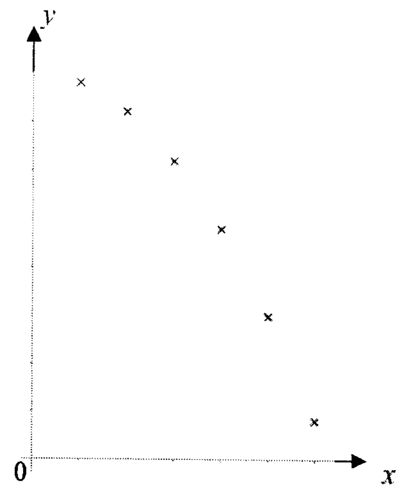
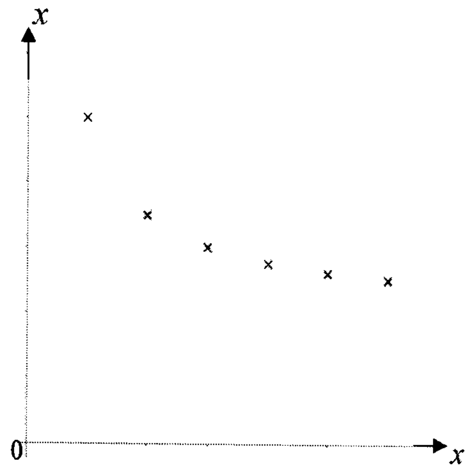
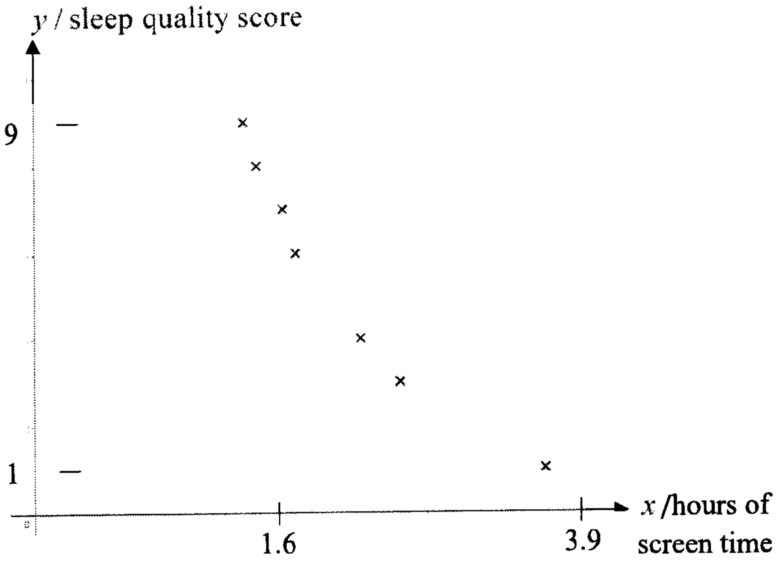
6(a) (i)	Case 1: Two married couples are in the same taxi Number of ways = 2 Case 2: Two married couples are in different taxis Number of ways = ${}^2C_1 \times {}^4C_2 \times {}^2C_2 = 12$ Total number of ways = 2 + 12 = 14	
(ii)	The two married couples must sit at the back in different taxis. Number of ways to sit the two married couples = 2 × (2 × 2) × (2 × 2) = 32 Number of ways to sit the 4 singles = $4!$ or ${}^4C_2 \times 2! \times 2! = 24$ Required number of ways = 2 × (2 × 2) × (2 × 2) × 4! = 768	
7 (a)	If A and B are independent, $P(A \cap B) = P(A)P(B)$ Since $B \subset A$, $P(A \cap B) = P(B)$ and since $A \subset S$, $P(A) \neq 1$ So $P(A \cap B) = P(B) \neq P(A)P(B)$. Hence A and B are not independent. OR A and B are not independent since the occurrence of B implies the occurrence of A. (or the non-occurrence of A implies the non-occurrence of B) OR From the Venn Diagram, $B \subset A \subset S$. Hence $P(A\|B) = 1 \neq P(A)$ as seen from the Venn Diagram. OR From the Venn Diagram, $B \subset A \subset S$. Hence $P(B\|A) \neq P(B)$ as the reduced sample space $A \neq S$ as seen from the Venn Diagram. $P(A\|B) = 1$	
7(b)(i)	$P(X = 2)$ $= P(\text{'2' appears once and '0' appears twice or '1' appears twice and '0' appears once})$ $= \frac{1}{2} \times \frac{1}{6} \times \frac{1}{6} \times \frac{3!}{2!} + \frac{1}{3} \times \frac{1}{3} \times \frac{1}{6} \times \frac{3!}{2!}$ $= \frac{7}{72}$	
(ii)	$P(X = 6)$ $= P(\text{'2' appears 3 times})$ $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ $= \frac{1}{8}$	

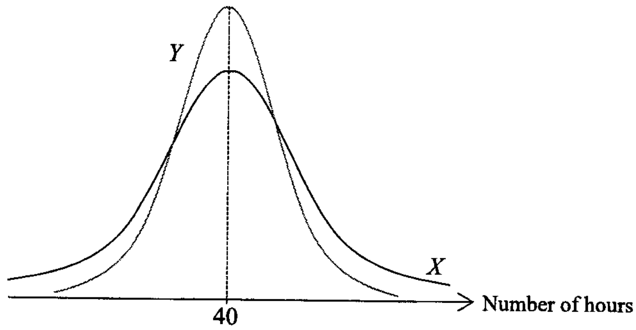
	$P(X=5)$ $=P('2'\text{appears twice and }'1'\text{ appears once})$ $=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{3}\times\frac{3!}{2!}$ $=\frac{1}{4}$ $P(X=3)$ $=P('2','1','0'\text{ all appear once or }'1'\text{ appear 3 times})$ $=\frac{1}{2}\times\frac{1}{3}\times\frac{1}{6}\times3!+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}$ $=\frac{11}{54}$ Or Complement Method $P(X=3)=1-\frac{1}{216}-\frac{1}{36}-\frac{7}{72}-\frac{7}{24}-\frac{1}{4}-\frac{1}{8}=\frac{11}{54}$	
(iii)	Required Probability $=\mathrm{P}(X=0)\times[\mathrm{P}(X>2)]^2\times[\mathrm{P}(X=1\text{ or }2)]^2\times\frac{5!}{2!2!}$ $=\frac{1}{216}\times\left(1-\frac{1}{216}-\frac{1}{36}-\frac{7}{72}\right)^2\times\left(\frac{1}{36}+\frac{7}{72}\right)^2\times\frac{5!}{2!2!}$ $=0.00164$ (3 s.f.)	
8(i)	Assumptions: Whether a toy is faulty or not is independent of other toys. The probability of a toy being faulty is constant at 0.02 for each toy.	
(ii)	Let X be the random variable denoting the number of faulty toys in 15 blind boxes. $X\sim\mathrm{B}(15,0.02)$ $\mathrm{P}(X\geq3)=1-\mathrm{P}(X\leq2)=0.00304=0.00304$ (3 s.f.)	
(iii)	**Method 1:** Let Y be the random variable denoting the number of faulty toys in 13 blind boxes. $Y\sim\mathrm{B}(13, 0.02)$ Required probability $=\mathrm{P}(Y=1)\times(0.02)$ $=0.204026\times(0.02)$ $=0.00408$ (3 s.f.)	

	Method 2: Required probability $= {}^{13}C_1(0.02)(0.98)^{12}\times(0.02) = 0.00408\,(3\text{ s.f.})$	
(iv)	Let K be the random variable denoting the number of faulty keychains out of 3. $K \sim \mathrm{B}(3, p)$ $\mathrm{P}(K=1)$ $= {}^{3}C_1 p^1(1-p)^2$ $= 3p(1-p)^2$	
(v)	Let A be the number of faulty toys in 2 blind boxes. $A \sim \mathrm{B}(2, 0.02)$ $K \sim \mathrm{B}(3, p)$ P(1 faulty toy and no faulty keychains | at most 1 faulty toy in the pack) = 0.035 $\dfrac{\mathrm{P}(A=1)\cdot\mathrm{P}(K=0)}{\mathrm{P}(A=0)\cdot\mathrm{P}(K=0)+\mathrm{P}(A=1)\cdot\mathrm{P}(K=0)+\mathrm{P}(A=0)\cdot\mathrm{P}(K=1)} = 0.035$ $\dfrac{0.0392\cdot(1-p)^3}{0.9604\cdot(1-p)^3+0.0392\cdot(1-p)^3+0.9604\cdot{}^{3}C_1 p(1-p)^2} = 0.035$ **Method 1: Using GC** From GC,  Since $0 < p \le 1, p = 0.0401759 = 0.0401$ (3 s. f.) **Method 2: Algebraic approach**	

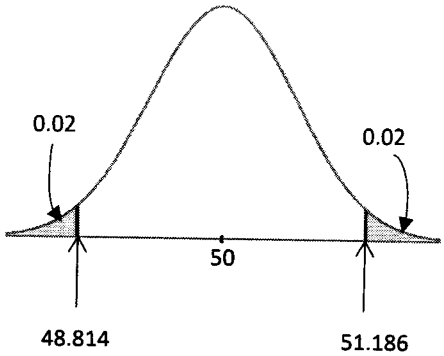
	$\dfrac{0.0392(1-p)^3}{0.9604(1-p)^3+0.0392(1-p)^3+0.9604\times{}^3C_1p(1-p)^2}=0.035$ $\dfrac{0.0392(1-p)}{0.9604(1-p)+0.0392(1-p)+0.9604(3p)}=0.035$ $0.0392(1-p)=0.033614(1-p)+0.001372(1-p)+0.100842p$ $0.004214-0.105056p=0$ $p=\dfrac{43}{1072}=0.0401\ (3\text{ s.f.})$	

9(i)	(A): $y = a + bx^2$, where $a > 0$ and $b < 0$  (B): $y = c + \dfrac{d}{x}$ where $c > 0$ and $d > 0$  	

(ii)		
(iii)	From the scatter diagram, as x increases, y decreases at a decreasing rate. Case (B) is the most appropriate model. $r = 0.99868 = 0.999$ (3 s.f.)	
(iv)	Case (B): $y = -4.6636 + \frac{21.716}{x}$ $y = -4.66 + \frac{21.7}{x}$ (3 s.f.) When $x=3$, $y = -4.6636 + \frac{21.716}{3} = 2.5751$ Sleep quality score =3 (nearest integer) Yes, the estimate is reliable as the $r = 0.999$ is close to 1 and $x=3$ is within the data range.	
(v)	Since $t = 60x$, $y = -4.6636 + \frac{21.716(60)}{60x}$ $y = -4.6636 + \frac{1303.0}{t}$ $y = -4.66 + \frac{1300}{t}$ (3 s.f.)	

	No, the r value <u>does not differ/ stays the same</u> as it is <u>unaffected by change of units/ scaling</u>.	
(vi)	I disagree with Alvin because the outcome of the study determines how sleep quality is linearly correlated to the amount of screen time and does not imply that the increased amount of the screen time causes poorer sleep quality.	
10 (a)(i)		
(ii)	$\frac{t-40}{\alpha}=1.8$ $t=40+1.8\alpha$	
(iii)	required probability $=\left[\mathrm{P}(Z>1.8)\right]^2=(0.035930)^2=0.00129$	
(iv)	$Y \sim N(40,\beta^2)$ $\mathrm{P}(Y>42)\geq 0.25$ $\mathrm{P}(\frac{Y-40}{\beta}>\frac{42-40}{\beta})\geq 0.25$ $\mathrm{P}(Z>\frac{2}{\beta})\geq 0.25$ $\frac{2}{\beta}\leq 0.67449$ $\beta\geq 2.9652=2.97$	
(b) (i)	Let S be the random variable denoting the mass, in kg, of a small bag of rice. $S \sim \mathrm{N}(1, 0.16^2)$ $\mathrm{P}(S>1.25)=0.059085$ Let W be the number of small bags of rice with a mass greater than 1.25 kg out of 300 bags. $W \sim B(300, 0.059085)$ $E(W)=300\times 0.059085=17.7$ Expected number of small bags of rice with a mass greater than 1.25 kg is 17.7	

(b)(ii)	Let L be the random variable denoting the mass, in kg, of a big bag of rice. Since $n = 30$ is large enough, by Central Limit Theorem, $\bar{L} \sim \mathrm{N}\left(5, \frac{0.5^2}{30}\right)$ approximately. Let $W = S_1 + S_2 + \cdots + S_5 - \bar{L}$. $\mathrm{E}(W) = 5\mathrm{E}(S) - \mathrm{E}\left(\bar{L}\right)$ $= 5(1) - 5$ $= 0$ $\mathrm{Var}(W) = 5\mathrm{Var}(S) + \mathrm{Var}\left(\bar{L}\right)$ $= 5(0.16^2) + \frac{0.5^2}{30}$ $= \frac{409}{3000}$ or 0.13633 (5sf) $\therefore W \sim \mathrm{N}\left(0, \frac{409}{3000}\right)$ approximately Required Prob. $= \mathrm{P}(W > 0.25)$ $= 0.249$ (3sf)	
11 (i)	Let X mg be the Vitamin C content in a randomly chosen bottle of orange juice and μ mg be the population mean vitamin C content in the bottles of orange juice. $\mathrm{H}_0 : \mu = 50$ $\mathrm{H}_1 : \mu \neq 50$ Test at 4 % level of significance. Under H_0, $\bar{X} \sim \mathrm{N}\left(50, \frac{2^2}{12}\right)$.	

	Critical Region:  Reject H_0 if $\bar{x} \le 48.814$ or $\bar{x} \ge 51.186$ i.e. $\bar{x} \le 48.8$ or $\bar{x} \ge 51.2$ (3sf)	
(ii)	In the earlier test, the distribution of the Vitamin C content in the bottles of juice was known to be normally distributed with a known variance. Therefore, regardless of the sample size, the sample mean Vitamin C content in the bottles of juice will still be normally distributed. However, the distribution of the new "Vitamin Boost" Vitamin C content is unknown. Hence, a larger sample size of at least 30 needs to be taken so that Central Limit Theorem can be applied to approximate the distribution of the sample mean Vitamin C content of the new juice line to a normal distribution.	
(iii)	Unbiased estimate of population mean, $\bar{y} = \frac{1865}{30} = 62.167\text{ (5sf)} = 62.2\text{ (3sf)}$ Unbiased estimate of population variance, $s^2 = \frac{1}{29}\left[116275 - \frac{1865^2}{30}\right] = 11.523\text{ (5sf)} = 11.5\text{ (3sf)}$	
(iv)	Let Y mg be the Vitamin C content in a randomly chosen bottle of "Vitamin Boost" Orange Juice, and let μ_Y mg be the population mean Vitamin C content in the "Vitamin Boost" Orange Juice bottles. $H_0: \mu_Y = 60$ $H_1: \mu_Y > 60$ Test at 5% level of significance. Under H_0, since $n = 30$ is large enough, by the Central Limit Theorem, $\bar{Y} \sim N\left(60, \frac{11.523}{30}\right)$ approximately.	

Method 1: p-value Method

$p\text{-value} = \mathrm{P}\left(\overline{Y} \geq 62.167\right)$

$= 0.000236$ or 2.36×10^{-4} (3sf)

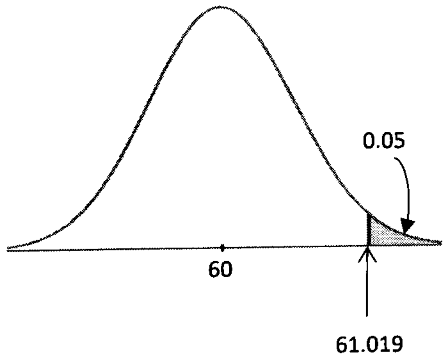
< 0.05

$\therefore$ Reject H_0.

Therefore, there is sufficient evidence at 5% level of significance to conclude that the mean vitamin C content in the bottles of the new juice line is greater than 60 mg.

Method 2: Critical Value method

Critical Region:



Reject H_0 if $\bar{y} \geq 61.019$

Since $\bar{y} = 62.167 > 61.019$,

Reject H_0.

Therefore, there is sufficient evidence at 5% level of significance to conclude that the mean vitamin C content in the bottles of the new juice line is greater than 60 mg.