H

ST ANDREW'S JUNIOR COLLEGE

General Certificate of Education Advanced Level

Higher 2

MATHEMATICS 9758/01

Paper 1 **1 September 2025 (Monday)**

Preliminary Examination **3 hours**

Additional Materials: **Printed Answer Booklet**

List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions. Total marks : **100**

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.

You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **7** printed pages and **1** blank page.

[Turn Over

1 A function f is defined by $f(x) = ax^3 + bx^2 + cx + d$ where a, b, c and d are constants. The graph of $y = f(x)$ has a turning point at $x = -1$ and passes through the points $(2,10)$ and $(-2,-2)$. Given also that $\int_{-2}^{2} f(x)\,dx = 16$, find the values of a, b, c and d. [4]

2 A closed cylinder is designed to have a fixed external surface area of p cm^2 such that its volume is maximum. Find the radius of the cylinder in terms of p. [5]

3 The lines l_1 and l_2 have equations

$$l_1:\ \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix},$$

$$l_2:\ \mathbf{r} = \begin{pmatrix} 0 \\ 5 \\ -8 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix},$$

where λ and μ are parameters.

(i) Show that l_1 and l_2 are skew lines. [2]

The point P has coordinates $(1,5,0)$ and the plane π_1 has equation $y - z - 2 = 0$.

(ii) Find the coordinates of P', the reflection of P in π_1. [4]

(iii) Find the cartesian equation of the plane π_2 containing P and l_1. [3]

4 **(a)** The function f is defined by

$$f(x)=\begin{cases} a^2-x^2, & 0<x\le a, \\ a(x-a), & a<x\le 2a, \end{cases}$$

and $f(x) = f(x + 2a)$ for all real values of x, where a is a positive constant.

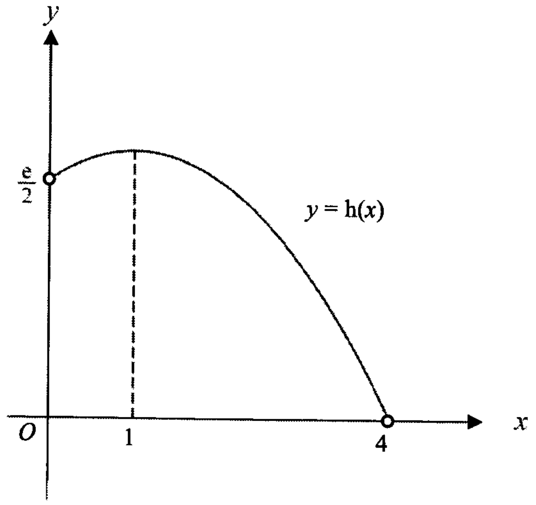
(i) Sketch the graph of $y = f(x)$ for $-2a \le x \le 3a$. [3]

(ii) Find $f\left(\dfrac{101a}{2}\right)$ in terms of a. [2]

(b) The function g is defined by

$$g: x \mapsto \ln x, \text{ where } x \in \mathbb{R},\ 0 < x \le e.$$

The diagram below shows the graph of the quadratic function h with domain defined where $0 < x < 4$. The graph has a maximum point at $x = 1$ and has end-points $\left(0, \dfrac{e}{2}\right)$ and (4,0).



(i) Determine whether hg exist. Justify your answer. [2]

(ii) State the other value of x for which $h(x) = \dfrac{e}{2}$. Given that h^{-1} exists, deduce the restricted domain of h such that domain of h^{-1} is $\left(0, \dfrac{e}{2}\right]$. [2]

(iii) Using the restricted domain of h found in **(ii)**, find the range of gh. [2]

5. An arithmetic progression has first term a and common difference d, where a and d are non-zero. The 3rd, 9th and 11th terms of the arithmetic progression are three distinct consecutive terms of a geometric progression.

(a) Find d in terms of a. [3]

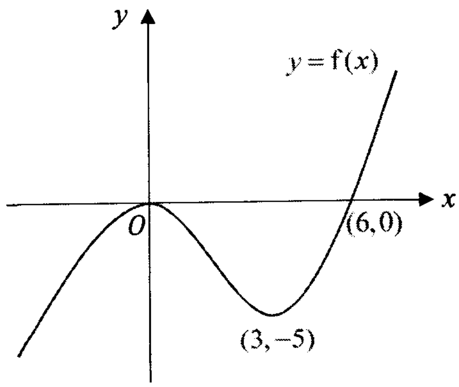
(b) Let the sum of the first n terms of the arithmetic progression be S_n where $S_1 = 143$. Find the largest possible value of S_n. [3]

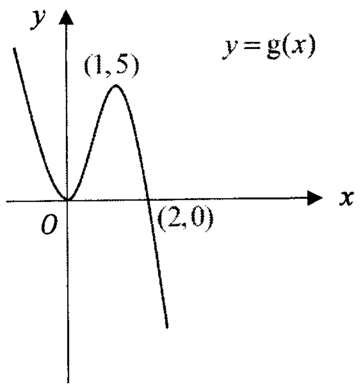
The first term of the geometric progression is b, where b is positive.

(c) Determine whether the geometric series is convergent. [2]

(d) Show that the sum of all terms after the n^{th} term of the geometric progression is at most $\frac{1}{2}b$, where n is a positive integer. [2]

6. The graph of $y = \mathrm{f}(x)$ cuts the x-axis at the point (6,0) and has a maximum point and a minimum point at (0,0) and (3,−5) respectively. The graph of $y = \mathrm{g}(x)$ cuts the x-axis at the point (2,0), and has a minimum point and a maximum point at (0,0) and (1,5) respectively. The diagrams below show the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$.





(i) Describe fully a sequence of two transformations which transforms the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{g}(x)$. [2]

It is given that $\mathrm{g}(x)$ can be expressed in the form $a\mathrm{f}(bx + c) + d$ where a, b, c and d are constants.

(ii) State the values of a, b, c and d. [2]

(iii) The derivative of the function $y = \mathrm{f}(x)$ with respect to x is denoted by $\mathrm{f}'(x)$. It is given that $\mathrm{f}'(6) = 4$. Find the exact value of $\mathrm{g}'(2)$. [3]

[Turn Over

7. **(a)** **(i)** On a single diagram, sketch the graphs of $y = \frac{3x}{x-1}$ and $y = 3x$, stating the equation(s) of any asymptote(s) and the coordinates of points of intersection. [4]

(ii) Hence find the area bounded by the curve $y = \frac{3x}{x-1}$ and the lines $y = 3x$ and $y = 8$. [2]

(b) A curve has the equation $y = \sqrt{x^2+1}$ where x is non-negative.

(i) With the aid of a sketch of the above curve, explain why the value of $S = \lim_{n\to\infty} \frac{1}{n}\left[1 + \sqrt{\frac{1}{n^2}+1} + \sqrt{\frac{4}{n^2}+1} + \ldots + \sqrt{\frac{(n-1)^2}{n^2}+1}\right]$ is $\int_0^1 \sqrt{x^2+1}\,\mathrm{d}x$. [4]

(ii) Hence find the value of S. [1]

8 A curve has equation $(x-y)^2 = 2\mathrm{e}^{xy}$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x - y - y\mathrm{e}^{xy}}{x - y + x\mathrm{e}^{xy}}$. [3]

(ii) Given that the curve cuts the positive y-axis at point A, find the equation of the normal to the curve at A. [2]

(iii) The normal to the curve at A meets the curve at another point B. Find the coordinates of B. [3]

9. **(a)** Use the substitution $x = 3\sec\theta$ to find $\int \frac{1}{x^2\sqrt{x^2-9}}\,\mathrm{d}x$. [5]

(b) The region R lies in the first quadrant and is bounded by the curve $y^2 = \frac{1}{x^2\sqrt{x^2-9}}$ and the lines $y = 0.1$ and $x = 4$. Find the exact volume of the solid generated when R is rotated through 2π radians about the x-axis. [4]

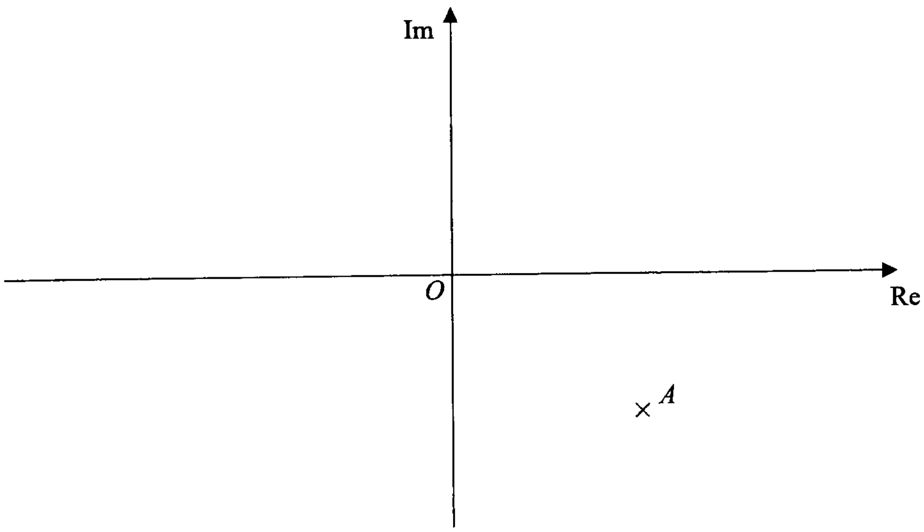
10. Do not use a calculator in answering this question.

(a) Find the complex numbers z and w which satisfy the following simultaneous equations.

$$|z|+5w=0$$
$$iz-4w=-4+7i$$

Give your answers in the form of $a+ib$, where a and b are real constants. [5]

(b)



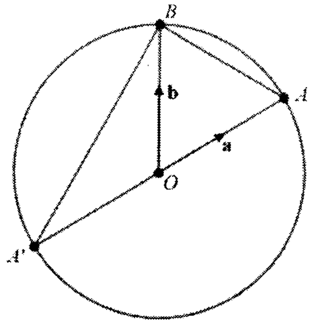
The point A on the Argand diagram represents the complex number u.

(i) On the copy of the Argand diagram in the Printed Answer booklet, plot the point B to represent the complex number $-u$. [1]

The points C and D represent the complex numbers $v-u$ and $(v-u)^*$ respectively, where v is an unknown complex number. It is also given that $\angle CDA = 90^\circ$.

(ii) By using the Argand diagram or otherwise, state the value of $\text{Im}(v)$ and justify your answer. [2]

11. The points A, B and A' lie on a circle with center O such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. It is also given that the line segment AA' is a straight line that passes through O (see diagram).



(i) Using a suitable scalar product, prove that $\measuredangle ABA' = 90^\circ$. [4]

(ii) Express the area of $\Delta ABA'$ in the form $k|\mathbf{a}\times\mathbf{b}|$, where k is a real constant. [2]

12. Welding fumes contain a dangerous amount of particulates. To protect workers in a workshop, an extraction device remove particulates in the air continuously. Let V mg represent the mass of particulates in the air in the workshop at time t min after the extraction device is activated.

Particulates are produced at a constant rate of 0.32 mg/min. The rate at which particulates are removed is proportional to the mass of particulates in the air.
When the mass of particulates in the air in the workshop is 18.75 mg, the mass of particulates in the air increases at a rate of 0.12 mg/min.

(i) Show that $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{4}{375}(30 - V)$. [2]

(ii) Solve the differential equation, given that the workshop is initially free of particulates. [5]

(iii) Sketch the graph of V against t. [2]

(iv) It is recommended that the mass of particulates in this workshop should be kept below 32 mg. Comment on whether the extraction device is effective enough to meet the recommendation. [1]

The extraction device becomes less efficient and now removes particulates at **half** its original rate.

(v) Write down a new differential equation to model this. [1]

(vi) The mass of particulates in the air in the workshop is said to reach a steady-state when it remains constant. Find the value of V at steady-state. [1]

End of Paper

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ST ANDREW'S JUNIOR COLLEGE

General Certificate of Education Advanced Level

Higher 2

MATHEMATICS 9758/02

Paper 2 **16 September 2025 (Tuesday)**

Preliminary Examination **3 hours**

Additional Materials: **Printed Answer Booklet**

List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions. Total marks: 100

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This document consists of **8** printed pages.

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Section A: Pure Mathematics [40 marks]

1 **(i)** Solve the inequality $3\left|x^2-4\right| \le \left|x+2\right|$. [4]

(ii) Hence, solve $\dfrac{3}{x^2}\left|1-4x^2\right| \le \left|\dfrac{1+2x}{x}\right|$. [2]

2 **(a)** It is given that $\mathrm{f}(x) = px^6 + qx^4 + r$, where p, q, and r are real constants.

(i) Show that if $x = \alpha$ is a root of $\mathrm{f}(x) = 0$, then $x = -\alpha$ is also a root. [1]

(ii) Given now that $x = 5$ and $x = \beta$ are roots of $\mathrm{f}(x) = 0$, where $\mathrm{Re}(\beta) \ne 0$ and $\mathrm{Im}(\beta) \ne 0$, write down all the remaining roots. [3]

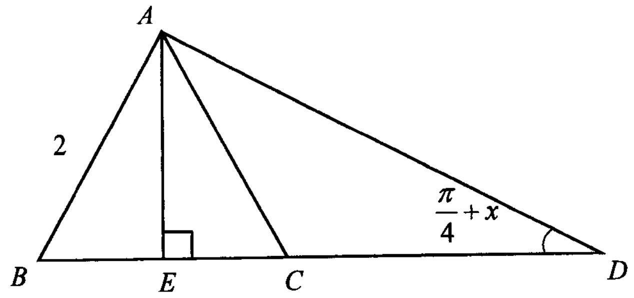
(b) The complex number z satisfies the equation

$$z^3 + (3-a)z^2 - (2+6\mathrm{i})z - 6 = 0,$$

where a is a complex number. It is given that one of the roots is $1 + \mathrm{i}$.

Find a and the other roots of the equation. [5]

3 The diagram below shows ΔABC and ΔAED. ΔABC is an equilateral triangle with each side measuring 2 units and ΔAED is a right-angled triangle with $\angle ADE = \dfrac{\pi}{4} + x$. Points C and E both lie on the line BD.



(a) Show that $BD = 1 + \dfrac{\sqrt{3}}{\tan\left(\dfrac{\pi}{4} + x\right)}$. [2]

(b) Given that x is sufficiently small for x^3 and higher powers of x to be neglected, show that $BD \approx a + bx + cx^2$ where a, b and c are exact constants to be determined. [4]

4 A sequence u_1, u_2, u_3, ... is such that $23u_{r+1} = 19u_r + 16$, for $r > 0$ and $u_1 = 2$.

(i) Write down the value of u_{10}, giving your answer correct to 4 decimal places. [1]

(ii) It is given that as $r \to \infty$, $u_r \to l$. Show that $l = 4$. [1]

(iii) Hence, find the smallest integer r for which u_r exceeds 99.9% of l. [3]

It is known that the kth term of this sequence is given by $u_k = 4 - 2\left(\frac{19}{23}\right)^{k-1}$.

(iv) Given that a new series S_n is defined by $S_n = \sum_{k=1}^{n}(u_k - c)$, where c is a real constant.

Express S_n in the form of $An + B\left[1 - \left(\frac{19}{23}\right)^n\right]$, where A is a constant in terms of c and B is a real number. [3]

(v) Determine the possible values of c for S_n to diverge as $n \to \infty$. [1]

5 A curve C has parametric equations

$$x = \tan\theta - \cot\theta, \quad y = \tan\theta + \cot\theta, \quad \text{for } 0 < \theta < \frac{\pi}{4}.$$

(i) Show that $\frac{dy}{dx} = -\cos 2\theta$. [3]

The line L is the tangent to C at point P with parameter p.

(ii) Show that the equation of L is $y + (\cos 2p)x = 2\sin 2p$. [3]

Let Q and R be the points where L cuts the x-axis and y-axis respectively.

(iii) Given that p increases at a constant rate of 0.2 radians per second, find the exact rate of change of the area of $\Delta\, OQR$ when $p = \frac{\pi}{6}$, where O is the origin. [4]

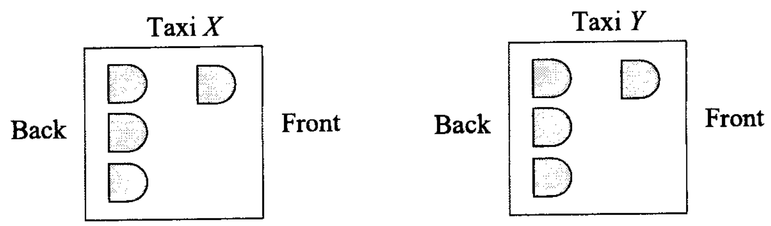
Section B: Statistics [60 marks]

6 A group of 8 tourists – 2 married couples and 4 singles – travels to the airport in two taxis, X and Y. Each taxi can take 4 passengers.

The 8 tourists divide themselves into two groups of 4, one group for taxi X and one group for taxi Y.

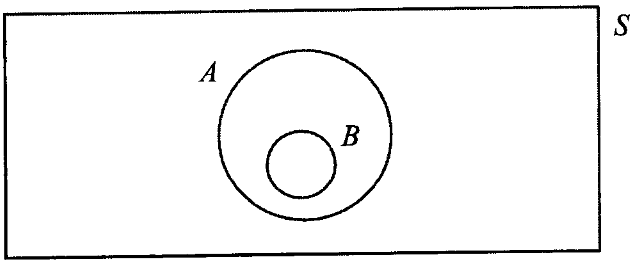
(i) Find the number of different ways in which this can be done if each married couple must travel together in the same taxi. [2]

Each taxi can take 1 passenger in the front and 3 passengers in the back (see diagram).



(ii) Find the number of seating arrangements so that each married couple sit next to each other at the back of the taxi. [3]

7 **(a)** With reference to the Venn diagram below, S is the universal set and A and B are non-empty proper subsets of S. Explain whether A and B are independent, and write down the value of $P(A \mid B)$.



[2]

(b) A fair die has three sides numbered 2, two sides numbered 1 and one side numbered 0. A game is played by throwing this fair die three times and the score, X, is the sum of the numbers obtained.

(i) Show that $P(X = 2) = \frac{7}{72}$. [2]

(ii) The table shows an incomplete probability distribution of X. Find the missing probabilities, showing your workings clearly. [3]

x	0	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{216}$	$\frac{1}{36}$	$\frac{7}{72}$		$\frac{7}{24}$		

The game is played 5 times.

(iii) Find the probability that exactly one of the games obtained a score of 0 and exactly 2 games obtained a score of more than 2. [3]

8 A company sells toys in blind boxes. Each blind box contains exactly one toy. On average, 2% of such blind boxes contain a faulty toy.

A collector buys 15 such blind boxes at random.

(i) State, in context, two assumptions needed for the number of faulty toys he gets to be well modelled by a binomial distribution. [2]

Assume now that the number of faulty toys the collector gets has a binomial distribution.

(ii) Find the probability that the collector gets no less than 3 faulty toys. [1]

(iii) Find the probability that the fourteenth box is the second box with a faulty toy. [2]

The company also sells keychains. The probability of a keychain being faulty is p. The number of faulty keychains follows a binomial distribution. Faults on keychains are independent of faults on toys in blind boxes.

(iv) Write down an expression, in terms of p, for the probability that in a random sample of 3 keychains, exactly one is faulty. [1]

A surprise pack contains 2 randomly chosen blind boxes and 3 randomly chosen keychains. Given that a randomly selected surprise pack contains at most 1 faulty item, the probability that one of the toys is the only faulty item in the pack is 0.035.

(v) Write down an equation satisfied by p. Hence, find the value of p. [4]

9 (i) Sketch a scatter diagram that might be expected when x and y are related approximately as given in each of the cases **(A)** and **(B)** below. In each case your diagram should include 6 points, approximately equally spaced with respect to x, and with all x- and y- values positive. The letters a, b, c and d represent constants.

(A) $y = a + bx^2$, where a is positive and b is negative,

(B) $y = c + \frac{d}{x}$, where c is positive and d is positive. [2]

Alvin is investigating the relationship between the amount of screen time per day and the sleep quality of youths. He recorded the sleep quality score, y, on a scale of 1 to 10 with 1 being the worst and 10 being the best, of particular youths with x hours of screen time per day.

x	1.6	1.7	1.9	2.0	2.5	2.8	3.9
y	9	8	7	6	4	3	1

(ii) Draw a scatter diagram for these values, labelling the axes. [1]

(iii) Explain which of the two cases in part **(i)** is the most appropriate for modelling these values and calculate the product moment correlation coefficient for this case. [2]

(iv) Alvin wants to estimate the sleep quality score when the screen time per day is 3 hours. Use the case that you have identified in part **(iii)** to find the equation of a suitable regression line and use your equation to find the required estimate. Explain whether you would expect this estimate to be reliable. [4]

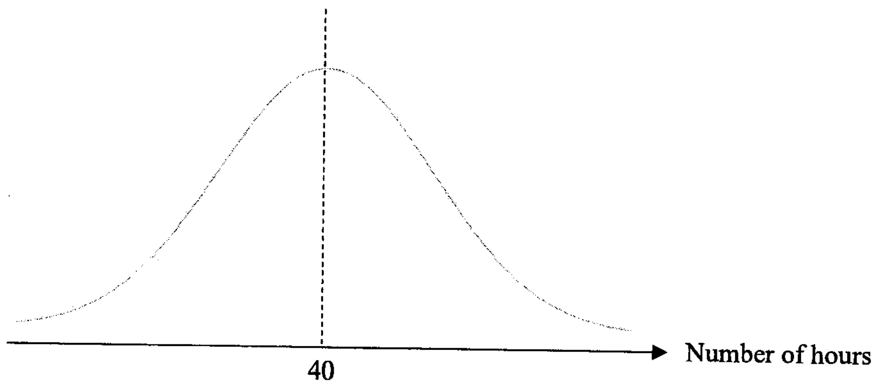
(v) Write the equation from part **(iv)** in terms of y and t, where t is the amount of screen time per day in minutes. Explain whether the product moment correlation coefficient for this case differs from the one found in part **(iii)**. [3]

(vi) Alvin concluded that this study shows that increased amount of screen time per day causes poorer sleep quality. Explain whether you agree with his conclusion. [1]

10 **In this question, you should state the parameters of any distributions you use.**

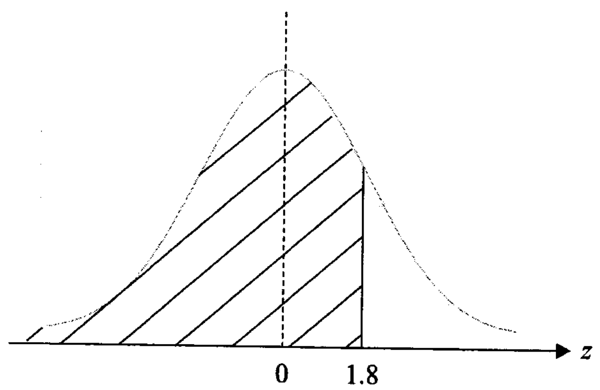
(a) The number of hours worked per week by employees of Company A follow a normal distribution with mean 40 hours and standard deviation α hours, while the number of hours worked per week by employees of Company B follow a normal distribution with mean 40 hours and standard deviation β hours, where $\alpha > \beta$.

The diagram shows the probability density function of X, where X is the random variable denoting the number of hours worked per week by a randomly chosen employee of Company A.



(i) On the copy of the diagram in the **Printed Answer Booklet**, sketch on the same diagram the probability density function of Y, where Y is the random variable denoting the number of hours worked per week by a randomly chosen employee of Company B. [1]

The probability that a randomly chosen employee of Company A works less than t hours per week is represented by the shaded area of the standard normal curve in the diagram below.



(ii) Write down an equation involving t and α. [1]

(iii) Two employees of company A are randomly chosen. Find the probability that both employees work more than t hours per week. [1]

(iv) Given that the probability that a randomly chosen employee of Company B works more than 42 hours per week is at least 0.25, find the range of values of β. [2]

(b) A shop sells small and large bags of rice. The masses of small bags of rice are normally distributed with mean 1 kg and standard deviation 0.16 kg. The masses of large bags of rice have mean 5 kg and standard deviation 0.5 kg. The masses of the small and large bags of rice are independent of one another.

(i) In a random sample of 300 small bags of rice, find the expected number of bags that have a mass greater than 1.25 kg. [2]

(ii) Find the probability that the total mass of 5 randomly chosen small bags of rice exceed the mean mass of 30 randomly chosen large bags of rice by more than 0.25 kg. [4]

11 A beverage company produces bottled orange juice, claiming that the mean vitamin C content in each 250ml bottle is 50 mg. It is known that the vitamin C content is normally distributed with a standard deviation of 2 mg.

Due to feedback from consumers, the quality control manager wants to test, at the 4% level of significance, whether the mean vitamin C content is, in fact, 50 mg. He randomly samples 12 bottles and measures the vitamin C content.

(i) State the hypotheses and find the critical region for this test. [3]

Recently, the company launched a new "Vitamin Boost" orange juice line, advertised to contain an average of 60 mg of vitamin C content per 250 ml bottle. After some complaints that the juice is too sour, the manager suspects that the vitamin C content might be higher than advertised. He decides to conduct a hypothesis test to investigate. This time, he randomly samples 30 bottles from the new juice line.

(ii) Explain why the manager takes a sample of 30 bottles for this test when he only took a sample of 12 bottles in his earlier test. [2]

The vitamin C content, y mg, in the sample of 30 bottles is summarized as follows:

$$\sum y = 1865, \sum y^2 = 116275$$

(iii) Calculate unbiased estimates of the population mean and variance for the vitamin C content in the bottles of the new juice line. [2]

(iv) At the 5% level of significance, test whether the mean vitamin C content in the bottles of the new juice line is greater than 60 mg. Clearly state your hypotheses and define any symbols you use. [4]