H

TEMASEK JUNIOR COLLEGE

2025 PRELIMINARY EXAMINATION

Higher 2

TEMASEK JUNIOR COLLEGE

MATHEMATICS

9758/01

1 Sep 2025

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Answer all questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages and **2** blank pages.

1 The sequence $\{u_n\}$ is a geometric sequence with first term a and common ratio $\frac{3}{2}$, where $a > 0$.

Another sequence $\{v_n\}$ is defined by $v_n = \frac{9}{u_n}$ for all positive integers n.

(a) Show that $\{v_n\}$ is also a geometric sequence. [2]

(b) Given that $v_2 = u_2$, find the value of a. [2]

2 The curve C has equation

$$xy = 1 + (x-y)^3.$$

Find the gradient of the tangent to C at the point A where $y = 0$. [4]

3 A curve C has equation $y = \frac{a}{2bx - x^2}$, where $a, b > 0$.

(a) Describe the transformation that maps the graph of C onto the graph of $y = \frac{a}{b^2 - x^2}$. [2]

(b) Given that $a = 1$ and $b = 2$, sketch C stating the equations of any asymptotes and coordinates of any stationary points and of the points where the curve crosses the axes. [2]

4 In the triangle ABC, $AC = 1$, angle $BAC = \frac{\pi}{3}$ radians and angle $ABC = \left(\frac{\pi}{6} + \theta\right)$ radians.

(a) Show that $BC = \frac{\sqrt{3}}{\cos\theta + \sqrt{3}\sin\theta}$. [2]

(b) Given that θ is sufficiently small such that θ^3 and higher powers of θ may be neglected, show that

$$BC \approx \sqrt{3}\left(1 + a\theta + b\theta^2\right)$$

where a and b are constants to be determined. [3]

5 **(a)** Without using a calculator, solve the inequality $\frac{2x-25}{x^2-2x-3} \le 2$. [4]

(b) Hence solve the inequality $\frac{2|x+1|-25}{(x+1)^2 - 2|x+1| - 3} \le 2$. [3]

6 The points A and B lie on a circle with center O and radius λ unit. With reference to the origin O, points A and B have position vectors **a** and **b** respectively. The point X on the line segment AB is such that $AX : XB = 1 : 3$ and the point Y is the foot of perpendicular of X on OB.

(a) Find the position vector of X. [1]

It is given that the acute angle AOB is $\frac{\pi}{6}$.

(b) Find $\mathbf{a}\cdot\mathbf{b}$ in terms of λ. [1]

(c) Show that the position vector of Y is $\frac{2+3\sqrt{3}}{8}\mathbf{b}$. [2]

(d) Hence find the exact area of ΔOXY in term of λ. [4]

7 Mabel and Janice decided to start a 5-year savings plan beginning in January 2026.

Mabel saves using a piggy bank. At the start of January 2026, she deposits \$101. Each subsequent month, she increases her deposit by \$1—so she deposits \$101 in January, \$102 in February, \$103 in March, and so on, until \$112 in December. At the start of each new year, she resets her monthly deposit to \$101 in January and repeats the same pattern through December. She continues this routine from 2026 to 2030, inclusive.

Janice, on the other hand, deposits \$100 at the start of every month into a bank account that earns 0.3% interest per month, with interest calculated and added into the account at the end of each month.

(a) Show that Janice will have more money in her savings account than Mabel has in her piggy bank at the end of December 2030. [5]

(b) Find the month and year when Janice's savings first exceed Mabel's savings. [4]

8 (a) The complex numbers z and w satisfy the following equations.

$$2z+|w|=2-4\text{i}$$

$$\text{i}z-w=2\text{i}$$

Find z and w, giving your answers in the form $a+\text{i}b$, where a and b are real numbers. [5]

(b) It is given that $z_1=2-\text{i}$ and $z_3=-3+2\text{i}$. On an Argand diagram, mark the points A and C representing z_1 and z_3 respectively. [1]

The points B and D on the Argand diagram represent complex numbers z_2 and z_4 respectively. Given that $ABCD$ is a square, labelled in an anti-clockwise direction, find z_2 and z_4. [4]

9 A curve C is defined parametrically by the equations

$$x = t + \frac{1}{t}, \quad y = t - \frac{1}{t}, \quad t \neq 0.$$

(a) Sketch the curve C, showing clearly the coordinates of the axial intercepts. [2]

(b) Use differentiation to find the values of t for which the tangents to the curve are parallel to the y-axis. [3]

(c) Show that the equation of normal at the point where $t = 2$ is given by $y = -\frac{3}{5}x + 3$. [3]

(d) The normal at the point where $t = 2$ cuts the curve C again at the point Q. Determine the exact coordinates of Q. [3]

10 **(a)** Differentiate $\sqrt{1-x^2}$ with respect to x.

Hence evaluate $\int_{-1}^{1} \cos^{-1} x \, dx$, giving your answer in exact form. [5]

(b) The finite region R is bounded by the curve $y = \cos^{-1}(x-3)$, where $2 \leq x \leq 4$, the line $y = \pi$ and the axes.

(i) Describe a geometrical transformation which will map the graph of $y = \cos^{-1}(x-3)$ onto the graph of $y = \cos^{-1} x$. [1]

(ii) Hence or otherwise, find the area of R in exact form. [2]

(c) Find the exact value of the volume generated when R is rotated completely about the y-axis. [4]

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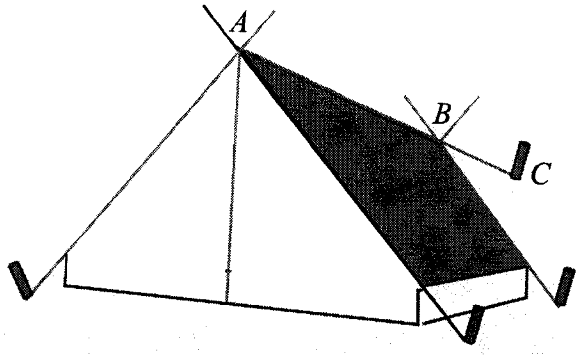

At a campsite, an A-frame tent is pitched on level ground. Its roof is formed by two slanted polyester sheets lying on two intersecting planes,

$$\pi_1: \ x + y + \alpha z = 6$$

$$\pi_2: \ 2x - y + z = 4$$

where α is a positive real constant. The intersection of π_1 and π_2 forms a central ridge in the form of a line AB. This ridge is extended with a rope, which is secured to the ground at the point C for stability. The ground is assumed to be the horizontal xy-plane.

(a) Find a vector parallel to the central ridge, giving your answer in terms of α. [2]

(b) If C lies on AB produced, show that the coordinates of C is $\left(\frac{10}{3}, \frac{8}{3}, 0\right)$. [3]

Hence, or otherwise, write down a vector equation of line AB in term of α. [1]

(c) If the angle between the two slanted polyester sheets is 60°, find the value of α. [3]

It is now given that $\alpha = 1$.

(d) A string is tied between two hooks inside the tent to hang decorations. One hook is fixed at the point D (2, 3, 1) which lies on π_1. The other hook is located at the point E which lies on π_2 such that the string DE is perpendicular to π_2. Find the coordinates of E. [3]

12 **(a)** Show, by means of the substitution $w = x^2 y$, that the differential equation

$$2y + x\frac{dy}{dx} = \frac{x^4 y^2}{\sqrt{x^2+1}}$$

can be reduced to the form $\frac{dw}{dx} = \frac{w^2 x}{\sqrt{x^2+1}}$. [2]

Hence find the general solution of the differential equation $2y + x\frac{dy}{dx} = \frac{x^4 y^2}{\sqrt{x^2+1}}$, leaving your answer in the form $y = f(x)$. [3]

(b) At a durian plantation, mature durians are susceptible to pests' infection. The spread of pests at the plantation resulted in durians being classified into two categories, either they are bad durians that have been infected by the pests or good durians that have not been infected. It is given that x denotes the number of bad durians, in thousands, in a fixed population size P, in thousands, where $P > 4$. The rate of spread of infection, $\frac{dx}{dt}$, where t represents the time taken in days, can be modelled as being proportional to the product of the number of bad durians and good durians that have not been infected. It is given that $x = 4$ and $\frac{dx}{dt} = P - 4$ when $t = 0$.

(i) Write down the differential equation involving $\frac{dx}{dt}$, x and P [2]

(ii) Solve the differential equation in part **(i)** and find x in terms of P and t. [5]

(iii) By using an appropriate graph, explain in context, the long-term implications if no additional measures are carried out to control the spread of the pests' infection. [2]

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Higher 2

MATHEMATICS

9758/02

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[Turn over

Section A: Pure Mathematics [40 marks]

1 It is given that $y = \ln\sqrt{1+e^{3x}}$.

(a) Show that $2e^{2y}\frac{dy}{dx} = 3e^{3x}$. [2]

(b) By further differentiation of the result in part (a), find the Maclaurin series for y, up to and including the term in x^2, giving the coefficients in exact form. [4]

(c) Deduce the Maclaurin series for $\ln\left(\frac{\sqrt{1+e^{-3x}}}{2}\right)$, up to and including the term in x^2. [2]

2 (a) Using the result $\sum_{r=2}^{n} \frac{2}{(2r+1)(2r+3)} = \frac{1}{5} - \frac{1}{2n+3}$, find

$\sum_{r=2}^{2n} \frac{2}{(2r-1)(2r+1)}$ in terms of n. [4]

(b) A sequence of positive numbers $x_1, x_2, x_3, ...$ satisfies the recurrence relation

$$x_{n+1} = \sqrt{4+5x_n} \quad \text{for } n = 1, 2, 3, \ldots.$$

(i) Given that the sequence converges to L, find the exact value of L. [2]

(ii) Prove that ${x_{n+1}}^2 - L^2 = 5(x_n - L)$. [1]

(iii) Use the result in part (ii) to show that if $x_n > L$, then $x_{n+1} > L$. [2]

3 The function f is defined by $f : x \mapsto \frac{1}{2} + e^{-x}$, $x \in \mathbb{R}$, $x \geq 0$.

(a) Sketch the graph of $y = f(x)$, $y = f^{-1}(x)$ and $y = f^{-1}f(x)$ on the same diagram, showing clearly their relationship. [3]

The function g is defined by $g : x \mapsto 1 + 4x - x^2$, $x \leq 2$.

(b) Explain why g^{-1} exists. [1]

(c) Find $g^{-1}(x)$ and state its domain. [3]

(d) Explain why the composite function gf exists. [2]

(e) Find the exact range of gf. [2]

4 [The arc length L of a sector of radius a and angle θ is given by $L = a\theta$;
The volume V of a cone of base radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$]

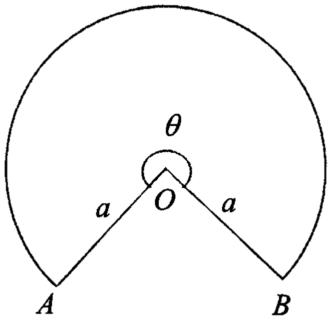

Figure 1

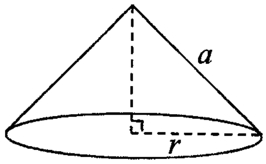

Figure 2

A metal sheet is shaped as a circular sector with angle θ and fixed radius a, as shown in Figure 1. This sector is then formed into a right circular cone with slant height a by joining the two radii OA and OB together as shown in Figure 2.

(a) If θ is measured in radians, state the base radius of the cone, r, in term of a and θ. [1]

(b) Show that the volume of the cone, V, is given by

$$V^2 = \frac{a^6}{576\pi^4}\theta^4\left(4\pi^2 - \theta^2\right).$$ [3]

(c) Using differentiation, find the exact value of θ that will maximise V. [4]

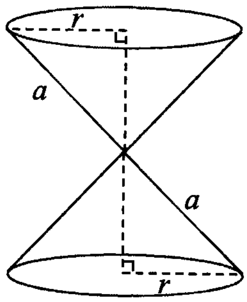

Figure 3

Two identical cones, as described in part **(c)**, are joined together tip-to-tip to form a new object, as shown in Figure 3. The upper cone is initially filled with water, which leaks out into the lower cone at a rate of $\frac{\pi}{15}$ cm^3 s^{-1}. At time t seconds, the height of the water in the upper cone is y.

(d) Show that the volume of water in the cone, W cm^3, is given by $W = \frac{2\pi y^3}{3}$. [2]

(e) Calculate the rate of change of y when $y = 2$. [2]

[Turn Over

Section B: Probability and Statistics [60 marks]

5 In a simple archery game, a player shoots an arrow at a target. The probability of hitting the target depends on the player's skill and the wind conditions for the day. The wind can either be good or bad, with the probability of a good wind day being 0.7.

According to past statistics:

- Dave has a 70% chance of hitting the target on a good wind day and a 60% chance on a bad wind day.
- Rafael has an 80% chance of hitting the target on a good wind day and a 50% chance on a bad wind day.

Dave and Rafael decide to compete in a game where each player shoots 2 arrows at the target. The player who hits the target more times wins the game.

(a) Find the probability that Dave wins the game on a good wind day. [3]

It is now known that the probability Dave wins the game on a bad wind day is 0.39.

(b) Given that Dave wins the game, find the probability that the day was a bad wind day. [3]

6 A treasure hunt game is to be played in a 3-storey building. There are 12 specific locations where 5 treasures are to be hidden. Among the 5 treasures, 4 of them are identical. The locations are distributed across the storeys as follows:

- 3 locations on the 1st storey,
- 3 locations on the 2nd storey, and
- 6 locations on the 3rd storey.

Each location can hold at most one treasure.

(a) Find the number of ways to distribute the 5 treasures among the 12 locations for each of the following separate cases:

(i) The 5 treasures can be hidden in any of the 12 locations. [1]

(ii) Exactly 2 treasures must be hidden on the 3rd storey. [2]

(iii) There must be at least one treasure on each of the 1st and 2nd storeys, and at least 2 treasures on the 3rd storey. [3]

Five teams of 2 players each take part in the game. The 10 players and 2 game masters are to be seated around a round table for a briefing.

(b) Find the number of different seating arrangements such that players from the same team must be seated next to each other but the two game masters must not be seated next to each other. [3]

7 A factory produces porcelain bowls. It is known that, on average, 8% of the bowls are faulty. The bowls are packed in boxes of 20. A box is considered imperfect if it contains more than 2 faulty bowls. Assume that the number of faulty bowls in a box follows a binomial distribution.

(a) Find the probability that a randomly chosen box is imperfect. [2]

These boxes are packed into cartons of 12 boxes each.

(b) Find the probability that, out of 3 randomly chosen cartons, there are 2 cartons that contains fewer than 15 faulty bowls each. [3]

Let Y be the number of imperfect boxes in a randomly chosen carton.

(c) State the values of $\mathrm{E}(Y)$ and $\mathrm{Var}(Y)$. [2]

(d) Hence using a suitable approximation, find the probability that the total number of imperfect boxes in a random sample of 35 cartons is at most 85. [2]

8 An examination consists of two parts: a written paper and a lab-based practical. Marks obtained by a randomly chosen candidate follow normal distribution with means and standard deviations as shown in the following table.

	Mean	Standard deviation
Written paper	62	σ
Lab-based practical	56	12

It is given that the 95th percentile score for the written paper is 85 marks.

(a) Show that the value of σ is 13.983, correct to 5 significant figures. [1]

(b) Find the probability that the difference between the marks obtained in the written paper and in the lab-based practical of a randomly chosen candidate is at least 9. [3]

The overall mark obtained for the examination is the total of 60% of the mark obtained from written paper and 40% of the mark obtained from lab-based practical.

(c) Find the probability that the overall mark of a randomly chosen candidate is less than 60. [3]

(d) Find the smallest value of n such that the probability that the mean overall mark of n randomly chosen candidates being at least 58 is at least 0.95. [3]

(e) State an assumption needed for the calculations in **part (b) to (d)** to be valid and explain why the assumption may not be valid in practice. [2]

[Turn Over

9 At a school canteen, the average queuing time (in minutes) for a student to be served at a stall is known to be 4.5 minutes. The school operation manager claims that with a recent change in students' timetable, there is an improvement in the average queuing time.

A random sample of 10 students' queuing time is observed after the change in students' timetable and the data is shown below.

4.2, 5.2, 3.1, 3.5, 4.6, 4.8, 3.8, 3.7, 4.5, 5.0

Given that the population variance of queuing time is known to be 0.48 minutes^2, a one-tailed test is carried out to determine whether the manager's claim is justified.

(a) State an assumption needed to carry out the hypothesis test. [1]

(b) Test whether the manager's claim is justified at 5% level of significance. [5]

A student leader Jason suspected that with the change in students' timetable, the average queuing time for a student to be served at a stall is in fact more than λ minutes and also that the population variance of queuing time is no longer 0.48 minutes^2. To investigate this, Jason recorded the queuing time of the first 50 students who visited the stall from 12 pm to 1 pm.

(c) Give a reason why the sample collected is not a random sample. [1]

Jason decided to collect another set of data to do his investigation. He recorded the queuing time, x, in minutes, of a random sample of 50 students. The summarized data is as shown below.

$$\bar{x} = 4.6 \quad \text{and} \quad \sum x^2 = 1068$$

(d) Find an unbiased estimate of the population variance. [1]

Jason carried out a hypothesis test at 2% level of significance with an alternative hypothesis of $\mu > \lambda$ where μ is the population mean queuing time.

(e) Explain why the assumption in **(a)** is no longer necessary. [1]

(f) By finding the critical region in terms of z-values, find the range of values of λ such that the Jason's suspicion is justified. [3]

10 Anand signed up for a privilege card with Temasek Airlines where cardholders earn more reward points as they fly further with the airline. From the airlines' website, Anand identified 8 different destinations and recorded the flight distance, x miles, of each destination from Singapore. He also recorded the corresponding reward points, y, credited into his card after each flight. The data are shown below.

Destination	A	B	C	D	E	F	G	H
Flight distance, x	200	400	600	800	1000	1200	1400	1600
Reward points, y	110	160	190	200	210	215	220	225

(a) Sketch a scatter diagram of the data. [2]

(b) Calculate the value of the product moment correlation coefficient, and explain why this value does not necessarily mean that a linear model is the best for the relationship between x and y. [2]

(c) Without calculating the product moment correlation coefficient, explain which of the following equations, where a and b are constants, and $b > 0$, can be used to model the relationship between x and y.

(I) $y = a + bx^2$

(II) $y = a + b\ln x$

(III) $y = a + be^{-x}$ [3]

(d) Using the model identified in **(c)**, find the equation of the corresponding regression line. [2]

(e) Hence estimate the reward points that can be earned for a destination 700 miles away from Singapore. Comment on the reliability of your estimation. [2]

(f) It is given that 1 mile is approximately 1.609 kilometres. State what will happen to the product moment correlation coefficient found in **(b)** if the flight distance is measured in kilometres instead of miles. [1]

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