VICTORIA JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION 2025

H2 MATHEMATICS

9758/01

PAPER 1

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer all questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

This document consists of **6** printed pages and **2** blank pages.

1 Using an algebraic method, solve the inequality $\dfrac{4x-3}{4x^2+3x-1} \le 1$. [3]

Hence, find the set of values of x that satisfy $\dfrac{4\ln x-3}{4(\ln x)^2+3\ln x-1} \le 1$. [2]

2 The function f is defined by

$$\mathrm{f}: x \mapsto \frac{x-1}{x}, \quad x \in \mathbb{R},\ x \neq 0,\ x \neq 1.$$

(a) Show that $\mathrm{f}^2(x) = \mathrm{f}^{-1}(x)$. [3]

(b) Find $\mathrm{f}^3(x)$ in simplified form. [1]

(c) Find $\mathrm{f}^{2030}(5)$. [2]

Functions g and h are defined by

$$\mathrm{g}: x \mapsto \frac{x-1}{x}, \quad x \in \mathbb{R},\ x \ge 1,$$

$$\mathrm{h}: x \mapsto -\sin ax, \quad x \in \mathbb{R},$$

where a is a positive constant.

(d) Find the value of a given that the range of hg is $(-1,0]$. [2]

3 Referred to the origin O, points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. Point C lies on OA, such that $OC:CA = 2:1$. Point D lies on OB, such that $OD:DB = \lambda:\mu$. It is given that the area of triangle ABD is half the area of triangle ABC.

(a) Show the area of triangle ABD is given by $\dfrac{\mu}{2(\lambda+\mu)}|\mathbf{a}\times\mathbf{b}|$. Hence find the ratio $\lambda:\mu$. [4]

(b) The point E has position vector $\dfrac{1}{4}\mathbf{a}+\dfrac{5}{8}\mathbf{b}$. Show that A, E and D are collinear. [3]

It is further given that the angle AOB is $\dfrac{\pi}{4}$ and O lies on the perpendicular bisector of the line segment AB.

(c) Find the length of projection of $\mathbf{a}$ on $\mathbf{b}$, giving your answer in terms of $|\mathbf{b}|$. Hence find the position vector of the point F, the foot of perpendicular from A to OB. [3]

4 **(a)** A sequence is such that $u_1 = p$, where p is a constant and $u_{n+1} = \dfrac{5u_n}{8u_n + 1}$, for $n..1$.

(i) Describe how the sequence behaves when $p = 1$. [2]

(ii) Find the value of p for which $u_6 = \dfrac{3125}{6253}$. [2]

(b) Another sequence v_1, v_2, v_3, … is such that for all $n..1$, $v_{n+2} - 2v_{n+1} + v_n = k$, where k is a constant.

Let $w_n = v_{n+1} - v_n$ for $n..1$. Explain why the sequence $\{w_n\}$ is an arithmetic progression. [2]

5 The function f is given by

$$\mathrm{f}(x) = \begin{cases} 2 + \sqrt{2^2 - (x-2)^2}, & 0 < x \le 4, \\ 2 - \sqrt{2^2 - (x-6)^2}, & 4 < x \le 8. \end{cases}$$

It is given that $\mathrm{f}(x) = \mathrm{f}(x+8)$ for all real values of x.

(a) On the diagram in the Printed Answer Book, sketch the graph of $y = \mathrm{f}(x)$ for $-6 \le x \le 7$, indicating clearly the coordinates of the end points and the points where the graph cuts the axes. [3]

(b) Without integrating, write down the exact area of the region bounded by $y = \mathrm{f}(x)$, the line $x = 4$, the x-axis and the y-axis. [1]

The curve C has equation $\dfrac{(y-3)^2}{a^2} - (x+2)^2 = 1$, where a is a positive real constant.

(c) State the equations of the asymptotes of C in terms of a. [1]

(d) Determine the range of values of a if there is at most one intersection between C and the graph of $y = \mathrm{f}(x)$. [2]

6 It is given that $\displaystyle\sum_{r=2}^{n} \frac{1}{r(r^2-1)} = \frac{1}{4} + \frac{1}{2(n+1)} - \frac{1}{2n}$.

(a) Show that $\displaystyle\sum_{r=2}^{n} \frac{1}{r(r^2-1)}$ is less than $\dfrac{1}{4}$. [2]

(b) Give a reason why the series $\displaystyle\sum_{r=2}^{\infty} \frac{1}{r(r^2-1)}$ converges, and write down its value. [2]

(c) Find the smallest value of n for which $\displaystyle\sum_{r=2}^{n} \frac{1}{r(r^2-1)}$ differs from $\displaystyle\sum_{r=2}^{\infty} \frac{1}{r(r^2-1)}$ by less than 0.0007. [2]

(d) Find $\displaystyle\sum_{r=m+1}^{N} \frac{1}{(r-m)(r-m+1)(r-m+2)}$, where m and N are integers with $N > m > 0$.
(There is no need to express your answer as a single algebraic fraction.) [2]

7 **Do not use a calculator in answering this question.**

(a) Find the complex number z which satisfies the equation $\frac{4|z|}{15-z^*}=5\mathrm{i}$. [3]

(b) The complex number w is such that $(w-\mathrm{i})^3=-\mathrm{i}$.

(i) Given that one possible value of w is $2\mathrm{i}$, find the two other possible values of w. Give your answers in cartesian form $a+b\mathrm{i}$. [4]

The points W_1, W_2 and W_3 on the Argand diagram represent the three roots of the equation $(w-\mathrm{i})^3=-\mathrm{i}$, and the point A represents the complex number $k\mathrm{i}$, where k is a positive real number.

(ii) Show that the points W_1, W_2 and W_3 lie on a circle with centre A for some value of k, stating the value of k. [2]

8 **(a)** A curve C with equation $y=\mathrm{f}(x)$ undergoes in succession, the following transformations.

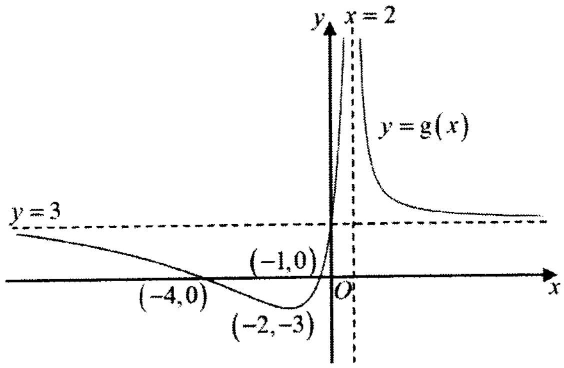
A: A reflection in the x-axis.

B: A stretch parallel to the x-axis with scale factor $\frac{1}{2}$, with the y-axis invariant.

The resulting curve has equation $y=ax^2+\frac{b}{x}$, where a and b are real constants.

Given that $\left(-1,\frac{1}{3}\right)$ is a turning point of $y=\frac{1}{\mathrm{f}(x)}$, find the values of a and b and state the equation of C. [5]

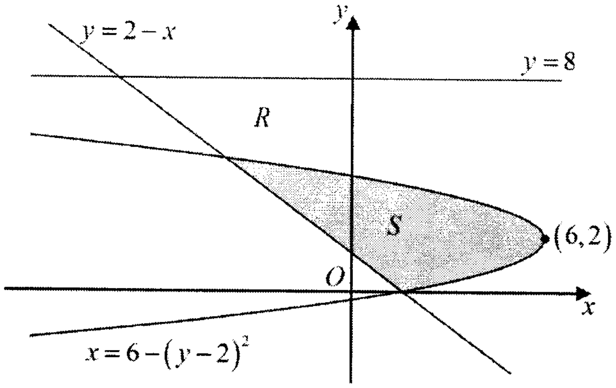
(b) The diagram below shows the curve of $y=\mathrm{g}(x)$. The curve has a minimum point at $(-2,-3)$ and crosses the x-axis at $(-1,0)$ and $(-4,0)$. The line $x=2$ is the vertical asymptote and the line $y=3$ is the horizontal asymptote.



(i) Sketch the graph of $y=\mathrm{g}'(x)$, labelling the coordinates of all relevant point(s) and state the equations of any asymptotes. [2]

(ii) Find the area of the region bounded by the graph of $y=\mathrm{g}'(x)$, the lines $x=-4$, $x=-2$ and the x-axis. [2]

9 In the diagram below, the region R is bounded by the curve C with equation $x=6-(y-2)^2$, the lines $y=8$, $y=2-x$ and the y-axis. The region S is bounded by C and the line $y=2-x$.



(a) Find the exact area of region R. [5]

(b) Find the volume of the solid of revolution formed when region S is rotated through 360° about the x-axis, leaving your answer to 2 decimal places. [3]

10 (a) (i) Express $1+4x$ in the form $A(2x-2)+B$, where A and B are constants to be determined. [1]

(ii) Hence, find $\int \frac{1+4x}{x^2-2x+5}\,\mathrm{d}x$. [4]

(b) Find the exact value of $\int_0^{\frac{\pi}{3}} x\sin 2x\,\mathrm{d}x$. [3]

11 It is given that $2\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{2}{x}-\ln x=y$.

(a) Use the substitution $y=\ln\left(\frac{u}{x}\right)$ to show that the differential equation can be reduced to $\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{f}(u)$, where the function $\mathrm{f}(u)$ is to be found. [3]

(b) Given that y has a minimum value at $x=3$, solve the differential equation $2\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{2}{x}-\ln x=y$, to find the particular solution for y in terms of x. [5]

(c) Sketch the graph of this particular solution. [2]

12 A chemical processing plant uses two types of automated dosing pumps, Pump A and Pump B, to regulate the flow of a catalyst into a reaction chamber. Each pump is removed from the plant and tested over a 10-hour trial period, and the volume of liquid dosed per hour is recorded.

The following data were collected.

- Pump A: The volume of liquid dosed was 4.5 litres in the first hour, and for each subsequent hour, the volume of liquid dosed decreased by a constant percentage of $r\%$.
- Pump B: The volume of liquid dosed was 4.7 litres in the first hour, and for each subsequent hour, the volume of liquid dosed decreased by 0.1 litres.

(a) Show that the total volume of liquid dosed by Pump A at the end of the trial period is

$$\frac{450}{r}\left[1-\left(1-\frac{r}{100}\right)^k\right] \text{ litres,}$$

where k is a constant to be determined. [3]

(b) It would be ideal if at the end of the trial period, the total volume of liquid dosed by Pump A is equal to that of Pump B. Find the value of r for this ideal situation. [2]

After the trial period, the two pumps were cleaned and reset, and were subsequently installed in the actual plant. Assume that the two pumps perform consistently as what was observed during the trial period.

(c) Pump A is installed for dosing in Reaction Line 1. To prevent underdosing, Pump A will cease to operate if the volume dosed in the next hour falls below 4.0 litres. With the assumption that $r = 1$, find the number of complete hours that Pump A should be in operation. [2]

Before operations begin in Reaction Line 2, Pump A breaks down.

(d) To minimise disruption, the company considers buying a new motor for Pump A from a third-party supplier. The supplier claims that with the new motor, Pump A can dose at least 360 litres of liquid in total if it ran continuously without stopping. Determine the range of values of r for which the supplier's claim is invalid. [2]

(e) The company eventually decides to use Pump B in Reaction Line 2. Using an algebraic method, determine if Pump B can dose at least 150 litres of liquid if left to run continuously without stopping. [3]

BLANK PAGE

BLANK PAGE

VICTORIA JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION 2025

H2 MATHEMATICS

9758/02

PAPER 2

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer all questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

This document consists of **8** printed pages.

Section A: Pure Mathematics [40 marks]

1 It is given that $f(x)=\ln(a+x)$, $x\in\mathbb{R}$, $x>-a$, where a is a constant.

(a) Using the standard series from the List of Formulae (MF27), find the series expansion for $f(x)$, up to and including the term in x^3. [2]

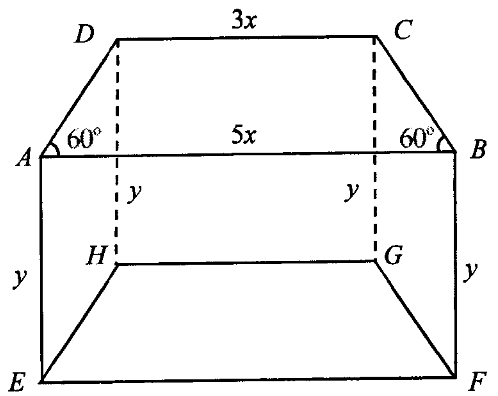
It is given that $a=1$.

(b) Hence, or otherwise, show that the series expansion of $\sin[f(x)]$, up to and including the term in x^3 is given by $x-\frac{1}{2}x^2+\frac{1}{6}x^3$. [2]

(c) Deduce the Maclaurin series for $\cos[f(x)]$ up to and including the term in x^2. [2]

(d) Find $\int_1^3\left(x-\frac{1}{2}x^2+\frac{1}{6}x^3\right)dx$. Without the use of a calculator or any further calculation, explain, whether this value is a good approximation to the value of $\int_1^3\sin[f(x)]dx$. [2]

2 The following diagram shows the dimensions of a trapezoidal prism with fixed volume $4k\sqrt{3}$ units^3, with variables x and y.



The top surface of the prism, *ABCD*, is an isosceles trapezoid with *AB* of length $5x$ units, *DC* of length $3x$ units, $AD=BC$ and $\angle ABC=\angle BAD=60°$. The rectangular sides *ABFE* and *BCGF* are perpendicular to both the top surface *ABCD* and the bottom surface *EFGH*, with $AE=BF=DH=CG=y$ units.

(a) Show that the total external surface area A of the trapezoidal prism is given by $A=8x^2\sqrt{3}+\frac{12k}{x}$. [4]

(b) Using differentiation, find the value of x in terms of k at which A is a minimum. [4]

(c) It is given instead that the volume of the prism is 1000 units^3 and its external surface area is 800 units^2. Find the two possible values of x. [2]

3 With reference to the point O as the origin and the x-y plane as a horizontal plane, the pyramid $OPQRV$ has a parallelogram base $OPQR$ and height OV. The position vectors of the points P and R are $-3\mathbf{i}+4\mathbf{j}+3\mathbf{k}$ and $5\mathbf{i}-2\mathbf{j}-\mathbf{k}$ respectively.

(a) Find the coordinates of the point S that lies on the line PR such that the distance from O to S is a minimum. [3]

(b) Find the cartesian equations of the planes such that the perpendicular distance from each plane to the base $OPQR$ is $2\sqrt{86}$ units. [3]

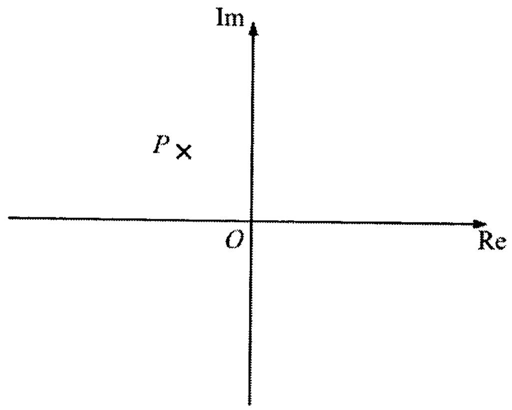
(c) Find the acute angle between OV and the vertical. [2]

(d) Given that QV is parallel to the vector $-5\mathbf{j}+8\mathbf{k}$, find the position vector of the point V. Hence find the exact volume of the pyramid $OPQRV$. [5]

[Volume of a pyramid $= \dfrac{1}{3} \times$ base area $\times$ height]

4 **Do not use a calculator in answering this question.**

The complex number z has modulus 1 and argument θ, where $\dfrac{\pi}{2} < \theta < \pi$, and the complex number w is given by $w = \mathrm{i}\sqrt{3}z$. The point P on the Argand diagram represents z.



(a) On the copy of the Argand diagram with origin O in the Printed Answer Booklet, plot the points Q and R to represent w and $z-w$ respectively. Show clearly the geometrical relationship between the points P, Q and R. [3]

(b) Find the area of the quadrilateral $ORPQ$. [1]

It is given that $\theta = \dfrac{3\pi}{4}$.

(c) Find z in the form $x + y\mathrm{i}$, where x and y are real numbers. [2]

(d) Show that $z - w = k\left[\left(\sqrt{3}-1\right)+\left(\sqrt{3}+1\right)\mathrm{i}\right]$, where k is a constant to be determined. [2]

(e) Hence show that $\tan\dfrac{5\pi}{12} = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$. [1]

Section B: Probability and Statistics [60 marks]

5 A code consists of 10 characters.

The first 5 characters of the code is formed using 5 letters chosen from the set {A, B, C, D, E, F, G, H} and the last 5 characters of the code is formed using 5 digits chosen from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 0\}$. The code allows for repetitions of letters and/or digits.

(a) Find the number of different codes that can be formed. [1]

(b) Find the probability that a code chosen at random

(i) contains the letter E exactly thrice and the number 5 exactly once, [2]

(ii) has E as the first character or 4 as its tenth character, but not both. [2]

6 Two events A and B are such that $\mathrm{P}(A) = p$ and $\mathrm{P}(B) = \frac{5}{4}p$. It is given that $\mathrm{P}(A \cup B) = \mathrm{P}(A \mid B) = 0.6$.

(a) Find the value of p. [3]

(b) Explain whether the events A' and B are independent. [1]

The events C and D are such that $\mathrm{P}(C) = 0.55$, $\mathrm{P}(D) = 0.6$ and $\mathrm{P}(A \cap C) = \mathrm{P}(A \cap D) = \mathrm{P}(C \cap D) = q$.

(c) Find the value of q that gives the minimum value of $\mathrm{P}(A \cup C \cup D)$ and state the minimum value of $\mathrm{P}(A \cup C \cup D)$. [2]

7 A shop sells apples in bags of 12. The average number of unripe apples in a bag is 2.16.

(a) State two assumptions needed for the number of unripe apples in a bag to be well modelled by a binomial distribution. [2]

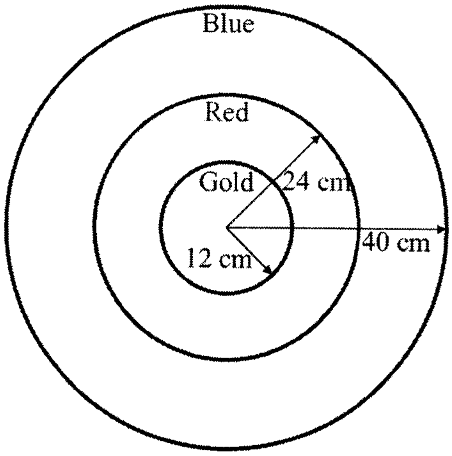
A bag of apples is sold at a reduced price if more than 3 apples are unripe.

(b) Find the probability that a randomly selected bag of apples is sold at a reduced price. [2]

(c) Twenty bags of apples are selected at random. Find the probability that fewer than 4 of these bags will be sold at a reduced price. [1]

(d) The shop also sells oranges in bags of n oranges. On average, the proportion of oranges that is unripe is 0.16. It is known that the modal number of unripe oranges in a bag is 2. Find the set of possible values of n. [2]

8 The diagram below shows a circular target board of radius 40 cm, divided by two concentric circles of radii 12 cm and 24 cm, into three regions, Gold, Red and Blue.



Arrows are shot at the target board and the scores obtained for hitting the Gold, Red and Blue regions are 10 points, 6 points and 3 points respectively. Arrows that land outside the target board score 0 points.

Alex shoots one arrow at the target board. The probability that his shot lands on the target board is p, where $p > 0.5$. If his shot lands on the target board, the arrow is equally likely to hit any position on the board. Let X represent the score obtained from a single shot.

You may assume that the arrow **does not land on the boundary** of any region.

(a) Show that the $\mathrm{P}(X = 6) = 0.27p$. [1]

(b) Given that $\mathrm{Var}(X) = 5.464476$, find the value of p. [4]

9 The random variable X has distribution $\mathrm{N}(\mu, \sigma^2)$.

(a) Given that $\mathrm{P}(X > k) = 0.45$, find the value of $\mathrm{P}(X > 2\mu - k)$. [2]

It is given that $\mu = 4$ and $\mathrm{P}(1 < X < 4) = 0.2475$.

(b) Find the value of σ. [2]

(c) Draw a sketch to show the distribution of X for x between -11 and 19. [2]

10 In this question you should state the parameters of any distributions you use.

An orchard sells kiwis of two varieties, Type A and Type B. The masses, in grams, of Type A and Type B kiwis follow the distributions $\mathrm{N}(110, 6^2)$ and $\mathrm{N}(85, 8^2)$ respectively. It is assumed that these two distributions are independent.

Kiwis are selected randomly and packed into bags. Type A kiwis are packed in bags of 30, while Type B kiwis are packed in bags of 60.

(a) In a randomly chosen bag of Type A kiwis, find the probability that the mean mass of kiwis is between 109 grams and 115 grams. [2]

The selling prices of Type A and Type B kiwis are \$25 per kilogram and \$15 per kilogram respectively.

(b) Find the probability that the average selling price of a kiwi from a bag of Type A kiwis is more than the average selling price of a kiwi from a bag of Type B kiwis by at most \$1.50. [3]

The orchard also produces genetically modified Type G kiwis, which are packed in bags of 50. The masses of Type G kiwis are identically distributed with a mean of 90 grams and a standard deviation of 5 grams.

Each Assortment Box contains one bag of Type A kiwis and one bag of Type G kiwis. A box is fit for sale if the total mass of the kiwis in the box is at least 7.85 kilograms.

(c) Ten Assortment Boxes are selected at random. Find the probability that none of the boxes are fit for sale. [3]

(d) State, in context, the approximation(s) and assumption(s) needed for the distribution(s) used in part **(c)**. [2]

11 An energy company is studying the relationship between the average daily temperature, x (in degrees Celsius), and the amount of electricity consumed, y (in megawatt-hours MWh). For a random sample of ten days, the records are shown in the table below.

Average daily temperature (x°C)	20	22	24	26	28	30	32	34	36	38
Electricity consumption (y MWh)	12.5	12.8	13.2	14.0	14.7	16.2	16.2	19.9	22.3	29.5

(a) Draw a scatter diagram for these values, labelling the axes clearly. [1]

It is thought that the electricity consumption and average daily temperature can be modelled by one of the formulae

$$y = a + bx^2, \qquad y = c + dx,$$

where a, b, c and d are constants.

(b) Find the value of the product moment correlation coefficient between

(i) x and y,

(ii) x^2 and y. [2]

(c) Using your answers in parts **(a)** and **(b)**, explain which of $y = a + bx^2$ and $y = c + dx$ is the better model and find the equation of a suitable regression line for this model. [3]

(d) Use the equation of your regression line to estimate the electricity consumption on a day with an average temperature of $37°\text{C}$. Comment on the reliability of your estimate. [2]

In some regions, temperature is measured in degrees Fahrenheit instead of degrees Celsius. A temperature of C degrees Celsius is equivalent to F degrees Fahrenheit, where $F = \frac{9}{5}C + 32$.

(e) Rewrite the equation of your regression line found in part **(c)** in terms of y and F, where F is the average daily temperature in degrees Fahrenheit. [1]

12 A hospital introduces a new medication regimen to help patients recover from a viral infection more quickly. Previously, the average recovery time was 7.2 days.

After adopting the new regimen, a random sample of 40 patients is observed, and their recovery times, x, in days, are recorded. The results are summarised below.

$$\sum(x-7.2)=-42 \quad \text{and} \quad \sum(x-7.2)^2=648$$

(a) Explain whether the hospital should carry out a one-tailed test or a two-tailed test. [1]

(b) Calculate unbiased estimates of the population mean and variance of the recovery times under the new regimen. [2]

(c) Carry out an appropriate test at the 5% significance level. You should state clearly the hypotheses for your test and define any parameters that you use. [4]

(d) Explain, in the context of the question, the meaning of 'at the 5% significance level'. [1]

(e) Explain whether it would have been sufficient for the hospital to take a random sample of 10 patients and record their recovery times under the new regimen in order to carry out the hypothesis test. [1]

It was discovered that an intern at the hospital had made an error in recording the recovery times. A random sample of another 30 patients was taken and their recovery times under the new regimen was taken and recorded by a senior researcher. The sample standard deviation was found to be 1.7 days and the sample mean time was denoted by k.

(f) A test at the 1% significance level concludes that the average recovery time did not improve from 7.2 days. Find the set of values of k. [3]