YISHUN INNOVA JUNIOR COLLEGE
JC 2 PRELIMINARY EXAMINATION
Higher 2

CANDIDATE NAME

CG

MATHEMATICS
Paper 1

9758/01

29 August 2025

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Write your CG, index number and name on the work you hand in.

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

1 Show that the differential equation $y\left(\frac{dy}{dx}+2y\right)=\frac{x}{e^{4x+x^2}}$ can be reduced by the substitution $z=y^2e^{4x}$ to $\frac{dz}{dx}=\frac{2x}{e^{x^2}}$. Hence, find the general solution in the form $y^2=f(x)$. [4]

2 Ronald is writing a novel. He began his new novel on 1 March 2025, writing 23 pages on the first day. On each subsequent day, he writes 90% of the number of pages he has written the previous day.

(a) Find the total number of pages Ronald will have written by 31 March 2025. [1]

Sam is also writing a novel. He began writing his novel on 8 March 2025, writing 2 pages on the first day. On each subsequent day, he writes 1 more page than the day before.

(b) Find the first date on which Sam writes more pages in a day than Ronald. [2]

(c) Find the first date on which Sam's total number of pages written exceeds Ronald's. [2]

3 **(a)** Find the series expansion of $\frac{1}{\sqrt{1+5x}}$, up to and including the term in x^2. State the range of values of x for which the expansion is valid. [3]

(b) Hence, use the substitution $x=\frac{1}{20}$ to obtain an approximation of $\sqrt{5}$, expressing your answer as a fraction. [2]

(c) Without any further calculation, explain whether using the substitution $x=-\frac{4}{25}$ gives a better approximation of $\sqrt{5}$ than the substitution used in part **(b)**. [1]

4 A curve has parametric equations $x=\frac{4t^3}{3}-t$, $y=t-3t^2$.

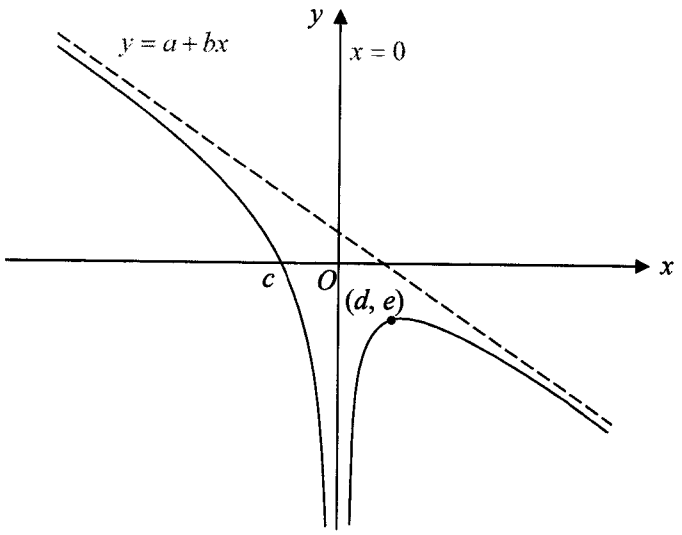
(a) For the part of the curve where $x>0$, find the equation of the tangent to the curve which is parallel to the y-axis. [3]

(b) Find the coordinates of the point where the tangent meets the curve again. [2]

5 **(a)** Without using a calculator, solve the inequality $\frac{9x-8}{x^2+2x-3}\geq 2$. [4]

(b) Hence solve the inequality $\frac{9\ln x-8}{(\ln x)^2+2\ln x-3}\geq 2$. [2]

6 The diagram below shows the graph of $y = \mathrm{f}(x)$. The graph has asymptotes $y = a + bx$ and $x = 0$, intersects the x-axis at the point $(c, 0)$ and has a turning point (d, e).



Sketch the following graphs on separate diagrams, labelling the equations of any asymptotes and the coordinates of any points where the graphs cross the axes and of any turning points.

(a) $y = 3\mathrm{f}(x + c)$, [2]

(b) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

(c) $y = \mathrm{f}'(x)$. [2]

7 **(a)** On the same diagram, sketch the graphs $y = |2x - 1|$ and $y = x^2 - 4x - 2$. [2]

(b) Hence find the set of values of x for which $|2x - 1| < x^2 - 4x - 2$. [2]

(c) Without using a calculator, find the range of values of k for which $|2x - 1| = x^2 - 4x - 2 + k$ has only positive real roots. [4]

8 **(a)** Find the exact value of $\int_0^{\frac{1}{4}\pi} \mathrm{e}^x \cos 2x \, \mathrm{d}x$. [4]

(b) **(i)** Show that $\dfrac{\mathrm{d}}{\mathrm{d}x} \ln(\tan x) = \dfrac{k}{\sin 2x}$, where k is a constant to be determined. [2]

(ii) Hence find $\int \mathrm{cosec}\, 2x \left[\ln(\tan x) + \cos 2x\right] \mathrm{d}x$. [2]

9 **Do not use a calculator in answering this question.**

(a) The complex numbers z and w satisfy the following equations.

$$iw+z=3i$$

$$2w^*-3z=12-i$$

Find z and w, giving your answers in the form $x+iy$ where x and y are real numbers. [4]

(b) It is given that $f(x)=ax^4+x^3+bx^2-9x+35$, where a and b are real numbers, and $f(x)=0$ has no repeated roots. The graph of $y=f(x)$ intersects the x-axis exactly once.

(i) Explain why $a=0$. [2]

One of the roots of $f(x)=0$ is $1+2i$.

(ii) Find the other roots and the value of b. [3]

10 Planes p and q are perpendicular. Plane p has equation $\mathbf{r}\cdot(\mathbf{i}+2\mathbf{j}-\mathbf{k})=6$. Plane q contains the line l with equation $\mathbf{r}=\mathbf{j}-12\mathbf{k}+\lambda(4\mathbf{i}+\mathbf{j}+10\mathbf{k})$ where λ is a parameter. The point A has coordinates $(0,1,-12)$.

(a) Find a cartesian equation of q. [2]

(b) Find a vector equation of the line m, where p and q meet. [2]

l passes through p at point B. Point C is the foot of the perpendicular of A to p.

(c) Find the position vector of B. [2]

(d) Find the position vector of C. [2]

(e) Find the exact area of triangle ABC. [2]

11 The function f is defined by

$$f: x \mapsto \frac{1}{x^2+2x}, \quad x\in\mathbb{R}, \ x<-1, x\neq -2.$$

(a) Show that f has an inverse. [1]

(b) Define f^{-1} in similar form. [4]

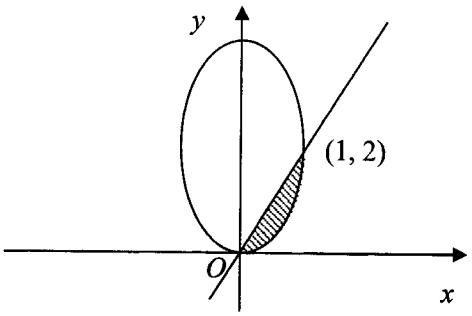
(c) The function g is such that $fg(x)=x^2+1$. Find $g(x)$. [2]

The function h is defined by

$$h: x \mapsto \frac{1}{x^2}, \quad x\in\mathbb{R}, \ x\neq 0.$$

(d) Only one of the composite functions fh and hf exists. Give a definition (including the domain) of the composite that exists, and explain why the other composite does not exist. [3]

12 **(a)** Use the substitution $x = \sin\theta$ to show that $\int_0^1 \sqrt{1-x^2}\,\mathrm{d}x = \frac{\pi}{4}$. [4]



The diagram shows the region R bounded by the ellipse with equation $4x^2 + (y-2)^2 = 4$ and the line $y = 2x$. The ellipse and the line intersect at the origin and the point (1, 2).

(b) Find the area of R, giving your answer in terms of π. [4]

(c) Find the volume of solid generated when R is rotated 2π radians about the x-axis. [2]

13 Around November 2019, there was an outbreak of COVID-19 in Wuhan, China, caused by the coronavirus SARS-CoV-2 virus, which subsequently spread to the rest of the world. On 23 January 2020, there was 1 confirmed case in Singapore. 70 days later, there were 1 049 confirmed cases.

A student is interested to study the spread of the virus in Singapore and uses the model $\frac{\mathrm{d}N}{\mathrm{d}t} = rN$, where r is a positive constant, N denotes the total number of confirmed cases, and t is the number of days after 23 January 2020.

(a) Solve the differential equation to express N in terms of t. [4]

(b) Use the model to estimate the number of days it will take to reach 1 000 000 cases. [1]

To contain the spread of the virus, Singapore implemented strict circuit breaker lockdown measures starting on 7 April 2020, when the number of confirmed cases stood at 1,481. By the time the measures were lifted 55 days later, the number of confirmed cases had surged to 35 292.

To study the spread of the virus during this period, the student uses a second model $\frac{\mathrm{d}N}{\mathrm{d}u} = sN$, where s is a positive constant, and u is the number of days after 7 April 2020.

(c) Find the value of s. [2]

(d) By comparing the values of r and s, comment on whether the lockdown measures were effective in containing the spread of the virus. [1]

(e) Give a reason why neither model can estimate the actual number of confirmed cases accurately. [1]

On 30 December 2020, Singapore became the first country in Asia to start its COVID-19 vaccination campaign. The student now uses a third model $\frac{\mathrm{d}N}{\mathrm{d}v} = aN\left(1 - \frac{N}{b}\right)$, where a and b are positive constants, and v is the number of days after 30 December 2020.

(f) Find the maximum rate of change of N in terms of a and b. [3]

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JC 2 PRELIMINARY EXAMINATION
Higher 2

CANDIDATE NAME

CG

MATHEMATICS
Paper 2

9758/02

17 September 2025

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3 hours

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Section A: Pure Mathematics [40 marks]

1 The acute angle between two non-zero vectors **a** and **b** is α radians, and the vectors **b** and $2\mathbf{a}-\mathbf{b}$ are perpendicular. Find the value of α if $|\mathbf{b}|=\sqrt{2}|\mathbf{a}|$. [3]

2 **Do not use a calculator in answering this question.**

It is given that $z=\sqrt{3}-\mathrm{i}$ and $w=2\mathrm{i}z$.

(a) Find the modulus and argument of z. [2]

(b) Deduce the modulus and argument of w. [2]

O, A, B and C represent the complex numbers 0, z, $z+w$ and w respectively.

(c) Illustrate these four points on a single diagram. [2]

(d) Identify the shape of the quadrilateral $OABC$ and state its area. [2]

3 A closed cylindrical tank with radius r m and height h m is to be constructed to hold 1000 m^3 of water. The material for the base and lid costs \$50 per m^2, while the material for the curved surface costs \$30 per m^2.

(a) Show that the total cost C (in dollars) can be expressed as $C=100\pi r^2+\dfrac{60000}{r}$. [2]

(b) Use differentiation to find the exact radius of the cylinder for which C is minimum. State the minimum cost. [4]

(c) Sketch the graph showing the total cost as the radius of the tank varies. [2]

A change to one of the materials used results in the cylinder's volume doubling to 2000 m^3, while maintaining the same minimum cost and radius found in part **(b)**.

(d) State which material's cost has changed and its value. [1]

4 **(a)** A sequence is such that $u_1=a$ and $u_{n+1}=bu_n$, for $n\geq 1$. Both a and b are constants.

(i) Explain why the sequence is a geometric progression. [1]

(ii) Given that $u_2=6$, $u_1+u_2+u_3=26$, and the sum to infinity exists, find the values of a and b. [4]

(b) It is given that $\sum_{r=1}^{n} r^2=\frac{1}{6}n(n+1)(2n+1)$.

(i) Find $\sum_{r=1}^{n} r(r+2)$ in terms of n. Leave your answer in factorised form. [2]

(ii) Hence find $\sum_{r=2}^{n}(r+1)(r+3)$ in terms of n. You do not need to leave your answer in factorised form. [3]

5 It is given that $e^y = 1 + e^x$.

(a) Show that $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{dy}{dx}$. [2]

(b) Hence find the first four non-zero terms of the Maclaurin series for y. [5]

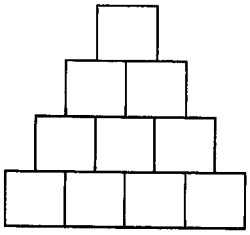
(c) Hence find the Maclaurin series for $\frac{e^{2x}}{1+e^{2x}}$, up to and including the term in x^3. [3]

Section B: Probability and Statistics [60 marks]

6 A children's game set consists of 1 red brick, 2 white bricks, 3 orange bricks and 4 blue bricks. All bricks are identical except for their colour.

(a) Find the number of ways to arrange the 10 bricks in a row if all the orange bricks are separated. [2]

A girl plans to arrange the 10 bricks in 4 rows as shown below.



First row

Second row

Third row

Fourth row

Find the number of arrangements if there are

(b) no restrictions, [1]

(c) exactly two orange bricks and only one white brick in the same row. [3]

All the 10 bricks are now numbered from 1 to 10.

(d) In how many ways can the bricks be arranged in a circle so that the ones with the same colour are next to each other? [2]

7 Customers at a shopping mall participate in a game involving 2 boxes of tokens, labelled A and B. Box A contains n black tokens and 4 gold tokens, while Box B contains 4 black tokens and 4 silver tokens. The tokens are identical in all aspects except for their colour.

In a game, a customer first tosses a fair coin. If a head is obtained, two tokens are drawn at random from Box B. Otherwise, one token is drawn from each box. A black token scores 1 point, a silver token scores 2 points, and a gold token scores 3 points. The total number of points is the customer's score X.

(a) Show that $P(X=2) = \frac{3}{28} + \frac{n}{4(n+4)}$. [1]

(b) Given that $P(X=2) = \frac{3}{14}$, determine the probability distribution of X. [4]

(c) Find $P(X_1 = 4 \mid X_1 + X_2 \geq 9)$, where X_1 and X_2 are independent observations of X. [3]

8 Andy is learning how to solve a Rubik's cube. The table shows his personal record time, y seconds, for solving a Rubik's cube, x days after he started.

x	5	10	15	20	25	30	35	40
y	220	150	100	70	48	35	25	18

(a) Draw a scatter diagram for these values, labelling the axes. [1]

(b) Use your diagram and the context of the question to explain whether the relationship between y and x should be modelled by $y = a + bx$ or by $y = a + \frac{b}{x}$, where a and b are constants. [2]

(c) Find the product moment correlation coefficient and the equation for the chosen model in part **(b)**. [2]

(d) Use your equation to estimate his personal record time, 28 days after he started. Explain whether your estimate is reliable. [2]

(e) Without further calculations, explain whether the product moment correlation coefficient calculated in part **(c)** would be different if y was recorded in minutes instead. Re-write the equation for the chosen model in this case. You do not need to simplify your answer. [2]

9 **In this question, you should state the parameters of any normal distributions you use.**

A confectionary bakes egg tarts for sale. The masses of these egg tarts have a normal distribution and 26% of them have a mass greater than 65 grams. An egg tart is equally likely to have a mass less than 50 grams as it is to have a mass greater than 70 grams.

(a) Find the mean and variance of this distribution. [3]

The confectionary also bakes donuts and buns. The masses in grams of the donuts have the distribution $\mathrm{N}(80, 10^2)$ and the masses in grams of the buns have the distribution $\mathrm{N}(70, 6^2)$.

(b) Find the probability that the mass of a randomly chosen donut is within 5 grams from its mean. [1]

(c) Find the probability that the mass of a randomly chosen bun is less than the mass of a randomly chosen donut. [2]

The confectionery also tops its donuts and buns with chocolate. A chocolate topping increases the mass of a donut by 10% and the mass of a bun by 20%.

(d) Find the probability that the total mass, with chocolate topping, of 3 randomly chosen donuts and 5 randomly chosen buns exceeds 700 grams. [4]

10 A company advertises that its rechargeable batteries can run non-stop for an average of 505 minutes after a full charge. 100 batteries are randomly selected, and their running time after a full charge, x minutes, are measured. The data obtained is summarised by

$$\sum(x-500)=320, \quad \sum(x-500)^2=8416.$$

(a) Suggest a reason why, in this context, the given data is summarised in terms of $(x-500)$ rather than x. [1]

(b) Calculate unbiased estimates of the population mean and variance of the running time of the batteries. [2]

(c) A hypothesis test is carried out at the 5% level of significance. Determine whether the company is overestimating the average running time of the batteries. [4]

(d) State, giving a reason, whether it is necessary to assume a normal distribution for the test to be valid. [1]

(e) After a change is made in the manufacturing process, another test at the 5% significance level is carried out using a new random sample of size 50. The sample standard deviation is found to be 8.5 minutes. Find the range of values of the sample mean in order to conclude that the average running time of the batteries after a full charge is more than 505 minutes. [4]

11 In a large shipment of second-hand T-shirts, 4% of the T-shirts are torn. The T-shirts are sold in boxes of 25 pieces each. Let X denote the number of torn T-shirts in a box.

(a) State, in the context of the question, two assumptions needed to model X by a binomial distribution. [2]

You are now given that X can be modelled by a binomial distribution.

(b) A box is randomly chosen. Find the probability that a box contains at least 2 torn T-shirts. [2]

(c) A box is deemed to be of inferior quality if it contains at least 2 torn T-shirts. Find the probability that, in a random sample of 10 boxes of T-shirts, more than 3 boxes are of inferior quality. [2]

A distributor purchases a batch of 60 boxes of T-shirts.

(d) Find the probability that the average number of torn T-shirts per box is more than 1.1. [3]

(e) Find the most probable number of boxes of inferior quality in the batch. [2]

In a large shipment of second-hand blouses, a proportion p of the blouses is torn. The blouses are also sold in boxes of 25 pieces each. The number of torn blouses in a box follows a binomial distribution. It is known that the probability of a box containing exactly 2 torn blouses is 0.206.

(f) Write down an equation satisfied by p. Hence, find the value of p, given that $p > 0.1$. [2]

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