

<b>Name:</b>		<b>Index Number:</b>		<b>Class:</b>	
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**DUNMAN HIGH SCHOOL**  
**Preliminary Examination**  
**Year 6**

MATHEMATICS (Higher 1)

8865/01

Paper 1

**September 2017**

**3 hours**

Additional Materials:      Answer Paper  
List of Formulae (MF26)

**READ THESE INSTRUCTIONS FIRST**

Write your Name, Index Number and Class on all the work you hand in.  
Write in dark blue or black pen on both sides of the paper.  
You may use a soft pencil for any diagrams or graphs.  
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

*For teachers' use:*

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Total
<b>Score</b>													
<b>Max Score</b>	<b>4</b>	<b>5</b>	<b>8</b>	<b>10</b>	<b>13</b>	<b>3</b>	<b>6</b>	<b>7</b>	<b>10</b>	<b>9</b>	<b>10</b>	<b>15</b>	<b>100</b>

**Section A : Pure Mathematics [40 marks]**

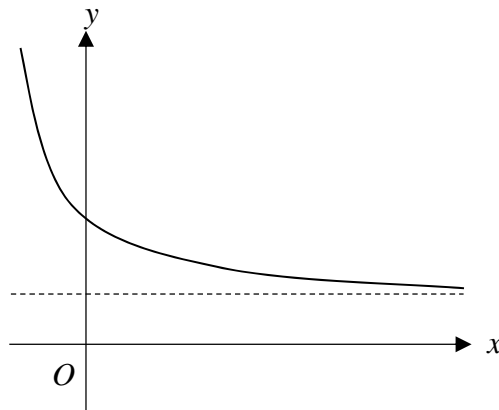
- 1** Abel, Ben and Caleb travelled from Raffles Place to Bishan using the same route. They hired private transport from different car companies, Car X, Y and Z, respectively. Ben and Caleb had cash vouchers which could be used to offset their final fares. Ben's cash voucher was twice as much in value as Caleb's.

The fare comprises 3 components: A fixed base fare, the distance travelled and the time taken for the journey. The rates of the different companies are shown in the following table.

	Car X	Car Y	Car Z
Base fare (\$)	3.20	3.00	3.00
Per kilometre (\$)	0.55	0.80	0.45
Per minute (\$)	0.29	0	0.20

Abel, Ben and Caleb paid \$15.60, \$6.60 and \$9.40 respectively. Find the distance travelled and the time taken from Raffles Place to Bishan. Assume that the traffic is the same for all 3 journeys. [4]

**2**



The diagram shows the curve  $C$  with equation  $y = \ln 2 + 2^{-x}$ .

- (i) State the exact equation of the asymptote and the exact coordinates of the point of intersection with the  $y$ -axis. [2]
- (ii) Find the  $x$ -coordinate of the point of intersection of  $C$  and the line  $y = 1$ , giving your answer correct to 4 decimal places. [1]
- (iii) Write down as an integral an expression for the area of the region bounded by  $C$ ,  $y = 1$  and the  $y$ -axis and evaluate this integral. [2]

- 3** (a) Differentiate  $\frac{\pi^2}{\sqrt{(3-\pi x)}}$ . [2]

- (b) Show that  $\frac{d}{dx}(e^{-x+2\ln x}) = xe^{-x}(2-x)$ . [3]

Hence find the exact solution of  $\int_1^e xe^{-x+a}(x-2) dx$  in terms of the constant  $a$ . [3]

- 4 [It is given that the volume of a cone is  $\frac{1}{3}\pi r^2 h$  where  $h$  is the vertical height and  $r$  the radius of the circular base of the cone.]

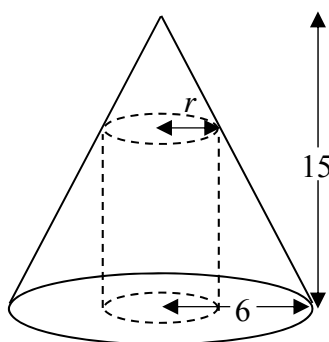
A hollow cone has a base radius 6 cm and height 15 cm. It is made of material with negligible thickness.

- (a) The cone is inverted. Initially, the cone is empty and water is poured into it at a rate of  $8 \text{ cm}^3 \text{ s}^{-1}$ . The depth of water in the cone is  $x$  cm at time  $t$  seconds.

(i) Show that the volume  $V \text{ cm}^3$  of the water in the cone is given by  $V = \frac{4}{75}\pi x^3$ . [2]

- (ii) Find the exact rate of increase of the depth of water at the instant when the depth is 5 cm. [3]

- (b)



The same cone is now placed in an upright position and a solid cylinder is to be inscribed in the cone (see diagram). Show that the total surface area  $A \text{ cm}^2$  of the cylinder of radius  $r$  cm is given by  $A = 30\pi r - 3\pi r^2$ , and hence find the exact value of the maximum  $A$ . [3]

- 5 At time 0000, a physicist observes the behaviour of 2 particles  $E_1$  and  $E_2$  for a period of 20 minutes.  $E_2$  is stationary for a few minutes before it starts moving. The speed  $v$ , in m/min after  $t$  min, of  $E_1$  and  $E_2$  satisfies the equation  $v = \frac{1}{3}t$  and  $v = \sqrt{(2t - 5)}$  respectively.

- (i) On the same diagram, sketch the speed-time graphs of  $E_1$  and  $E_2$  during the period of observation of 20 minutes, stating the exact coordinates of any points of intersection with the axes and points of intersection of the two graphs. [2]
- (ii) State the duration for which  $E_2$  moves faster than  $E_1$ . [1]
- (iii) The distance travelled is represented by the area under a speed-time graph. Determine which particle travels a longer distance for the period of observation of 20 minutes. [4]
- (iv) The derivative of speed with respect to time is known as acceleration. Without using a calculator, find the time at which  $E_1$  and  $E_2$  have the same acceleration. [3]

The speed of a third particle  $E_3$  satisfies the equation  $v = \sqrt{(a - (t - 5)^2)}$ , where  $a$  is a positive constant. Find the set of values of  $a$ , given that  $E_1$  and  $E_3$  will not travel at the same speed at any time  $t$ . Show your working clearly. [3]

**Section B : Statistics [60 marks]**

6 A sample of 5 people is chosen from a village of large population.

- (i) The number of people in the sample who are underweight is denoted by  $X$ . State, in context, the assumption required for  $X$  to be well modelled by a binomial distribution. [1]
- (ii) On average, the proportion of people in the village who are underweight is  $p$ . A total of 1000 samples of 5 people are chosen at random from the village and the results are shown in the table below.

$x$	0	1	2	3	4	5
Number of samples	93	252	349	220	75	11

Using the above results, find  $\bar{x}$ . Hence estimate the value of  $p$ . [2]

7 The Tan family has 2 children while the Wong family has 3 children. The children, together with both their parents, catch a movie at the cinema. At the ticket counter, they realise only 2 rows of consecutive empty seats are left, where Row L has 5 seats and Row M has 4 seats.

Find the number of different possible arrangements if

- (i) there are no restrictions, [1]
- (ii) the Tan siblings must not sit together. [3]

The 9 of them are randomly seated. Find the probability that Tan siblings sit between their parents given that the Wong family takes Row L. [2]

8 The insurance company Adiva classifies 10% of their car policy holders as ‘low risk’, 60% as ‘average risk’ and 30% as ‘high risk’. Its statistical database has shown that of those classified as ‘low risk’, ‘average risk’ and ‘high risk’, 1%, 15% and 25% are involved in at least one accident respectively.

Find the probability that a randomly chosen policy holder

- (i) is not involved in any accident if the policy holder is classified as “average risk”, [1]
- (ii) is not involved in any accident, [2]
- (iii) is classified as “low risk” if the policy holder is involved in at least one accident. [2]

Two policy holders are chosen at random.

- (iv) Find the probability that one is not involved in any accident while the other is involved in at least one accident. [2]

- 9 Students pursuing a particular university course are required to take  $m$  modules in each semester. At the end of each module, the students have to take an examination which comprises  $n$  questions. It may be assumed that for each examination, the number of questions answered correctly by a randomly chosen student follows a binomial distribution  $B(n, 0.6)$  with variance 24.

(i) Verify that  $n = 100$ . [1]

To pass a module, a student must answer at least 50 questions correctly in the examination.

(ii) Find the most probable number of questions that a randomly chosen student answers correctly in an examination. [2]

(iii) Show that the probability that a randomly chosen student passes a module is 0.983. [1]

(iv) Given that a randomly chosen student is at most 90.4% confident of passing all his modules in a semester, find the least value of  $m$ . [3]

Forty students in this course are randomly selected and their marks for a particular examination are recorded. Use a suitable approximation to find the probability that on average at most 58 questions are answered correctly. [3]

- 10 The students in a college are separated into two groups of comparable sizes, Group X and Group Y. The marks for their Mathematics examination are normally distributed with means and variances as shown in the following table.

	Mean	Variance
Group X	55	20
Group Y	34	25

- (i) Find, to 1 decimal place, the maximum passing mark if at least 60% of students from Group Y are to pass the examination. [2]
- (ii) Find the probability that the total marks of 4 students from Group Y is less than three times the mark of a student from Group X. State clearly the mean and variance of the distribution you use in your calculation. [3]
- (iii) The marks of 40 students, with 20 each randomly selected from Group X and Group Y, are used to compute a new mean score,  $\bar{M}$ . Given that  $P(-k < \bar{M} - 44.5 < k) = 0.9545$ , find the value of  $k$ . [3]

State a necessary assumption for your calculations to hold in parts (ii) and (iii). [1]

- 11** A retail manager of a large electrical appliances store wants to study the relationship between advertising expenditure,  $x$  hundred dollars, and the sales of their refrigerators,  $y$  thousand dollars, on a monthly basis. The data is shown in the following table.

$x$	5	8	12	16	18	20	23
$y$	12.5	12.9	14.6	15.8	17.0	19.3	20.8

- (i) Draw a scatter diagram to illustrate the above data and calculate the product moment correlation coefficient between  $x$  and  $y$ . [3]
- (ii) Find the equation of the regression line of  $y$  on  $x$ , in the form  $y = a + bx$ . Sketch this line on your scatter diagram. [2]
- (iii) Use a suitable regression line to estimate the advertising expenditure of a particular month if \$15000 was made from the sale of refrigerators. Comment on the reliability of this estimate. [3]
- (iv) Explain the meaning of  $b$  in the context of this question. [1]
- (v) There is an additional expenditure of \$200 for every month of advertising. Without any further calculations, state any change you would expect in the values of the constant  $b$  found in part (ii). [1]
- 12** (a) The centre thickness,  $X$  micrometres, of soft contact lenses from a certain company is a normally distributed random variable with mean  $\mu$ . The company claims that the centre thickness of their lenses is at most 30 micrometres. A random sample of 60 contact lenses is measured. The results are summarised as follows.

$$\sum(x - 30) = 24 \quad \sum(x - 30)^2 = 144$$

Test, at the  $2\frac{1}{2}\%$  significance level, whether the claim is justified. [6]

Explain, in the context of the question, the meaning of “at the  $2\frac{1}{2}\%$  significance level”. [1]

- (b) The company Snatch provides a ride-hailing service comprising taxis and private cars in Singapore. Snatch claims that the mean waiting time for a passenger from the booking time to the time of the vehicle’s arrival is 7 minutes. A random sample of 30 passengers’ waiting times is obtained and the standard deviation of the sample is 2 minutes. A hypothesis test conducted concludes that there is sufficient evidence at the 1% significance level to reject the claim.
- (i) State appropriate hypotheses and the distribution of the test statistic used. [3]
- (ii) Find the range of values of the sample mean waiting time,  $\bar{t}$ . [3]
- (iii) A hypothesis test is conducted at the 1% significance level whether the mean waiting time of passengers is more than 7 minutes. Using the existing sample, deduce the conclusion of this test if the sample mean waiting time is more than 7 minutes. [2]

Qn	Suggested Solution		
1	<p>Let <math>a</math> be the distance in km, <math>b</math> the time taken in minutes, <math>c</math> be the value of promo discount that Caleb had.</p> $\begin{cases} 3.2 + 0.55a + 0.29b = 15.6 \\ 3 + 0.8a = 6.6 + 2c \\ 3 + 0.45a + 0.2b = 9.4 + c \end{cases}$ $\begin{cases} 0.55a + 0.29b = 12.4 \\ 0.8a - 2c = 3.6 \\ 0.45a + 0.2b - c = 6.4 \end{cases}$ <p>Using GC,  <math>a = 12, b = 20, c = 3</math></p> <p>Hence the time taken was <u>20 minutes</u> and the distance travelled was <u>12 km</u>.</p>		
2i	<p>Equation of asymptote: <math>y = \ln 2</math>  Coordinates of point of intersection with y-axis: <math>(0, 1 + \ln 2)</math></p>		
ii	<p>Using GC,  x-coordinate = <math>1.70438 = 1.7044</math> (4 d.p.)</p>		
iii	<p>Area</p> $= \int_0^{1.70438} (\ln 2 + 2^{-x} - 1) dx$ $= 0.477006$ $= 0.477 \text{ unit}^2$		
3(a)	$\frac{d}{dx} \left( \frac{\pi^2}{\sqrt{3 - \pi x}} \right)$ $= \pi^2 \left( -\frac{1}{2} \right) (3 - \pi x)^{-\frac{3}{2}} (-\pi)$ $= \frac{\pi^3}{2} (3 - \pi x)^{-\frac{3}{2}}$ $= \frac{\pi^3}{2 \sqrt{(3 - \pi x)^3}}$		
(b)	<table style="width: 100%; border: none;"> <tr> <td style="width: 50%; vertical-align: top;"> <math display="block">\frac{d}{dx} (e^{-x+2 \ln x})</math> <math display="block">= (e^{-x+2 \ln x}) \left(-1 + \frac{2}{x}\right)</math> <math display="block">= (e^{-x})(e^{\ln x^2}) \left(-1 + \frac{2}{x}\right)</math> <math display="block">= x^2 e^{-x} \left(-1 + \frac{2}{x}\right)</math> <math display="block">= x e^{-x} (2 - x) \text{ (shown)}</math> </td> <td style="width: 50%; vertical-align: top;"> <p>Alternatively (use of product rule)</p> <math display="block">e^{-x+2 \ln x} = e^{-x} \cdot e^{2 \ln x} = e^{-x} \cdot e^{\ln x^2} = x</math> <math display="block">\therefore \frac{d}{dx} (e^{-x+2 \ln x})</math> <math display="block">= \frac{d}{dx} (x^2 e^{-x})</math> <math display="block">= e^{-x} (2x) + x^2 (-e^{-x})</math> <math display="block">= x e^{-x} (2 - x)</math> </td> </tr> </table>	$\frac{d}{dx} (e^{-x+2 \ln x})$ $= (e^{-x+2 \ln x}) \left(-1 + \frac{2}{x}\right)$ $= (e^{-x})(e^{\ln x^2}) \left(-1 + \frac{2}{x}\right)$ $= x^2 e^{-x} \left(-1 + \frac{2}{x}\right)$ $= x e^{-x} (2 - x) \text{ (shown)}$	<p>Alternatively (use of product rule)</p> $e^{-x+2 \ln x} = e^{-x} \cdot e^{2 \ln x} = e^{-x} \cdot e^{\ln x^2} = x$ $\therefore \frac{d}{dx} (e^{-x+2 \ln x})$ $= \frac{d}{dx} (x^2 e^{-x})$ $= e^{-x} (2x) + x^2 (-e^{-x})$ $= x e^{-x} (2 - x)$
$\frac{d}{dx} (e^{-x+2 \ln x})$ $= (e^{-x+2 \ln x}) \left(-1 + \frac{2}{x}\right)$ $= (e^{-x})(e^{\ln x^2}) \left(-1 + \frac{2}{x}\right)$ $= x^2 e^{-x} \left(-1 + \frac{2}{x}\right)$ $= x e^{-x} (2 - x) \text{ (shown)}$	<p>Alternatively (use of product rule)</p> $e^{-x+2 \ln x} = e^{-x} \cdot e^{2 \ln x} = e^{-x} \cdot e^{\ln x^2} = x$ $\therefore \frac{d}{dx} (e^{-x+2 \ln x})$ $= \frac{d}{dx} (x^2 e^{-x})$ $= e^{-x} (2x) + x^2 (-e^{-x})$ $= x e^{-x} (2 - x)$		

	$\int_1^e x e^{-x+a} (x-2) dx$ $= -e^a \int_1^e x e^{-x} (2-x) dx$ $= -e^a \left[ e^{-x+2\ln x} \right]_1^e$ $= -e^a \left[ e^{-e+2\ln e} - e^{-1+2\ln 1} \right]$ $= -e^a \left[ e^{-e+2} - e^{-1} \right]$ $= e^{a-1} - e^{a-e+2}$
<b>4(i)</b>	<p>Let <math>r</math> be the radius of water surface area</p> <p>Using similar triangles, <math>\frac{r}{6} = \frac{x}{15} \Rightarrow r = \frac{2}{5}x</math></p> <p>Volume of water, <math>V = \frac{1}{3} \pi \left( \frac{2x}{5} \right)^2 x</math></p> $= \frac{4}{75} \pi x^3 \text{ (shown)}$
<b>(ii)</b>	<p>Given <math>\frac{dV}{dt} = 8</math></p> <p>From part (i), <math>\frac{dV}{dx} = \frac{4}{75} (3) \pi x^2 = \frac{4\pi x^2}{25}</math></p> <p>Using Chain Rule,</p> $\frac{dx}{dt} = \frac{dx}{dV} \cdot \frac{dV}{dt} = \frac{25}{4\pi x^2} \times 8 = \frac{50}{\pi x^2}$ <p>When <math>x = 5</math>,</p> $\frac{dx}{dt} = \frac{50}{\pi(5)^2} = \frac{2}{\pi} \text{ cm/s}$ <p>The rate of increase of the depth of water is <math>\frac{2}{\pi}</math> cm/s when <math>x</math> is 5 cm.</p>
<b>b</b>	<p>Let the height of the cylinder be <math>h</math>.</p> <p>By similar triangles, <math>\frac{r}{6} = \frac{15-h}{15} \Rightarrow h = 15 - \frac{5}{2}r</math></p>



$$\begin{aligned}
 \text{Total surface area of the cylinder, } A &= 2\pi r^2 + 2\pi rh \\
 &= 2\pi r^2 + 2\pi r \left( 15 - \frac{5}{2}r \right) \\
 &= 30\pi r - 3\pi r^2 \text{ (shown)}
 \end{aligned}$$

$$\frac{dA}{dr} = 30\pi - 6\pi r$$

$$\frac{dA}{dr} = 0$$

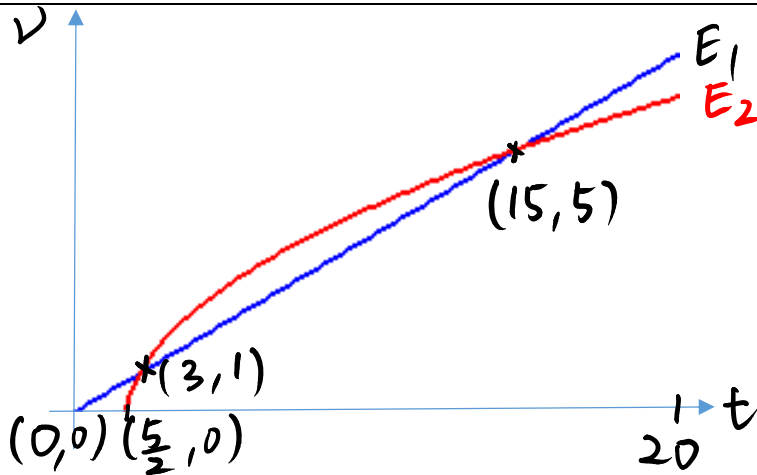
$$30\pi - 6\pi r = 0 \Rightarrow r = 5$$

$$\frac{d^2A}{dr^2} = -6\pi < 0$$

Total surface area is a maximum when  $r = 5$ .

$\therefore$  maximum value of the total surface area of the cylinder  
 $= 30\pi(5) - 3\pi(25) = 75\pi \text{ cm}^2$

5i



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NORMAL FLOAT AUTO REAL RADIAN MP
DISTANCE BETWEEN TICK MARKS ON AXIS
WINDOW
Xmin=0
Xmax=20
Xscl=1
Ymin=0
Ymax=7
Yscl=■
Xres=1
ΔX=.0757575757575757
TraceStep=.1515151515151515

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ii Points of intersection are at  $t = 3$  and  $t = 15$   
Hence duration =  $15 - 3 = \underline{12 \text{ minutes}}$

iii Let  $d_1$  and  $d_2$  be the distance travelled by  $E_1$  and  $E_2$  respectively.

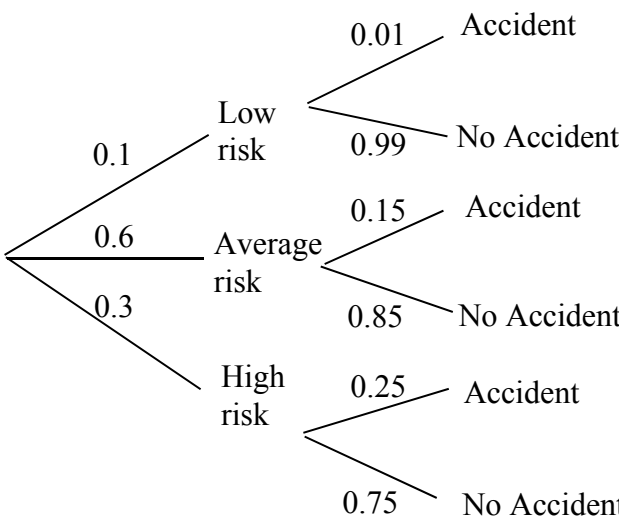
	$d_1 = \frac{1}{2}(20)\left(\frac{1}{3}(20)\right) = \frac{200}{3} \text{ m (or 66.7 m)}$ $d_2 = \int_{\frac{5}{2}}^{20} \sqrt{2t-5} dt$ $= \left[ \frac{\frac{2}{3}(2t-5)^{\frac{3}{2}}}{2} \right]_{\frac{5}{2}}^{20}$ $= \frac{1}{3} \sqrt{35^3} \text{ m (or 69.0 m)}$ <p>Since <math>d_2 &gt; d_1</math>, <math>E_2</math> travelled a longer distance.</p>
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<b>iv</b>	<p>Let <math>v_1</math> and <math>v_2</math> denote the speeds of <math>E_1</math> and <math>E_2</math>.</p> <p>To have the same acceleration,</p> $\frac{dv_1}{dt} = \frac{dv_2}{dt}$ $\frac{d}{dt}\left(\frac{1}{3}t\right) = \frac{d}{dt}\left(\sqrt{2t-5}\right)$ $\frac{1}{3} = \frac{1}{2}(2t-5)^{-\frac{1}{2}}(2)$ $\frac{1}{3} = \frac{1}{\sqrt{2t-5}}$ $2t-5 = 9$ $t = 7$ <p>Hence the time at which they have the same acceleration is <u>00 07</u></p>
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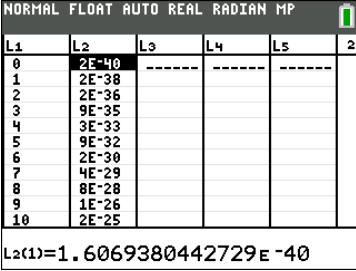
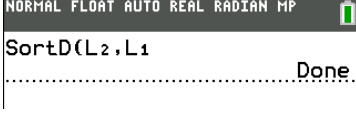
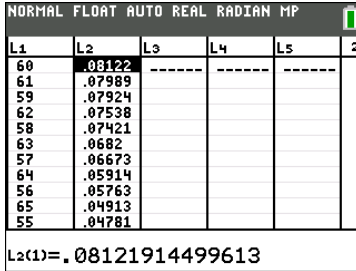
	<p>Suppose the speeds of both particles is the same,</p> <p>ie. <math>v_1 = v_3</math></p> $\frac{1}{3}t = \sqrt{a - (t-5)^2}$ $\frac{1}{9}t^2 = a - (t-5)^2$ $\frac{1}{9}t^2 = a - t^2 + 10t - 25$ $\frac{10}{9}t^2 - 10t + (25 - a) = 0$ <p>For the velocities to be always different,</p> $10^2 - 4\left(\frac{10}{9}\right)(25 - a) < 0$ $100 - \frac{1000}{9} + \frac{40}{9}a < 0$ $a < \frac{5}{2}$ <p>since <math>a</math> is positive,</p> <p>set of values of <math>a = \{a \in \mathbb{R}^+ : a &lt; \frac{5}{2}\}</math></p>
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<b>6i</b>	Weights of people in the village are independent of each other.
<b>(ii)</b>	$\bar{x} = 1.965$ (from GC) Since $n = 5$ , $np \approx 1.965 \Rightarrow p \approx 0.393$


<b>7(i)</b>	No. of ways = $9! = 362880$
<b>ii)</b>	No. of ways $= 9! - 7 \times 2 \times 7!$ $= 9! - 70560$ $= 292\,320$
	Probability $= \frac{2 \times 2}{4!}$ $= \frac{1}{6}$  Probability $= P(\text{Tan siblings sit between parents} \mid \text{Wong family takes Row L})$ $= \frac{P(\text{Tan siblings sit between parents and Wong family takes Row L})}{P(\text{Wong family takes Row L})}$ $= \frac{\frac{5 \times 2 \times 2}{9!}}{\frac{5!4!}{9!}}$ $= \frac{1}{6}$

<b>8(i)</b>	P(holder is not involved in any accident   the holder is classified as “average” risk) = 0.85
<b>(ii)</b>	 <p>Probability of a randomly chosen policy holder not involved in any car accident</p> $= (0.1)(0.99) + (0.6)(0.85) + (0.3)(0.75)$ $= 0.834$
<b>(iii)</b>	P(policy holder is “low risk”   has met at least one car accident)

	$= \frac{P(\text{holder is classified as "low" risk and met with at least 1 accident})}{P(\text{holder meets with at least 1 accident})}$ $= \frac{0.1(0.01)}{1 - 0.834}$ $= \frac{1}{166} = 0.00602 \text{ (3 s.f.)}$
(iv)	Probability $= 2(0.834)(1 - 0.834)$ $= 0.276888 \text{ (exact)}$

9	Let $X$ denote the no. of questions he can answer correctly out of $n$ .
(i)	$X \sim B(n, 0.6)$ if $n = 100$ , Variance $= npq = 100(0.6)(0.4) = 24$ (verified)
ii	$X \sim B(100, 0.6)$      <p> <math>L2(1) = 1.6069380442729E-40</math>   <math>P(X = 59) = 0.07924</math>  <math>P(X = 60) = 0.08122</math>  <math>P(X = 61) = 0.07989</math> </p> <p>The most probable number of questions answered correctly is <u>60</u>.</p>
(iii)	Required probability $= P(X \geq 50)$ $= 1 - P(X \leq 49)$ $= 0.98324$ (to 5sf) $= 0.983$ (to 3sf) (shown)
iv	Let $Y$ denote the no. of exams out of $m$ that he passed. $Y \sim B(m, 0.983)$

	$P(Y = m) \leq 0.904$ $\binom{m}{m} 0.983^m (1 - 0.983)^0 \leq 0.904$ $0.983^m \leq 0.904$ $m \lg 0.983 \leq \lg 0.904$ $m \geq 5.88621$ <p>least <math>m = 6</math></p>
	$X \sim B(100, 0.6)$ $E(X) = 60$ $\text{Var}(X) = 24$ By CLT, since $n = 40$ is large, $\bar{X} \sim N(60, \frac{24}{40})$ approximately $P(\bar{X} \leq 58) = 0.0049117 = 0.00491$ (3s.f.)

<b>10</b>	Let $Y$ be the score of Group Y students.
<b>(i)</b>	$P(Y \geq a) \geq 0.6$ $P(Y < a) < 0.6$ Thus $a < 32.733$ The maximum mark is 32.7
<b>(ii)</b>	$E(Y_1 + Y_2 + Y_3 + Y_4 - 3X) = 4E(Y) - 3E(X) = -29$ $\text{Var}(Y_1 + Y_2 + Y_3 + Y_4 - 3X) = 4\text{Var}(Y) + 9\text{Var}(X) = 280$ $\therefore Y_1 + Y_2 + Y_3 + Y_4 - 3X \sim N(-29, 280)$ $P(Y_1 + Y_2 + Y_3 + Y_4 < 3X) = P(Y_1 + Y_2 + Y_3 + Y_4 - 3X < 0) = 0.958$
<b>(iii)</b>	$\bar{M} = \frac{X_1 + \dots + X_{20} + Y_1 + \dots + Y_{20}}{40}$ $E(\bar{M}) = \frac{20E(X) + 20E(Y)}{40} = \frac{1}{2}(E(X) + E(Y)) = 44.5$ <p>Let <math>\sigma^2 = \text{Var}(\bar{M})</math></p> $= \frac{1}{1600}(20\text{Var}(X) + 20\text{Var}(Y))$ $= \frac{1}{80}(\text{Var}(X) + \text{Var}(Y)) = 0.5625$ <p><math>\bar{M} \sim N(44.5, 0.5625)</math></p> <p>Since <math>P(-k &lt; \bar{M} - 44.5 &lt; k) = 0.9545</math>  <math>\therefore 44.5 - k = 43.000</math>  <math>\Rightarrow k = 1.50</math> (3 s.f.)</p>  <p><b>Alternative</b></p>

	$\bar{M} \sim N(44.5, \sigma^2)$ <p>Since <math>P( \bar{M} - 44.5  &lt; 2\sigma) = 0.9545</math></p> <p><math>\therefore k = 2\sigma = 2\sqrt{0.5625} = 1.50</math> (3sf)</p>
	<i>Marks obtained by the students</i> are independent of one another.
11 i ii	<p><i>y/ thousands</i></p> <p><math>r = 0.97139 = 0.971</math> (3 s.f.)</p> <p>The equation of <math>y</math> on <math>x</math> :</p> <p><math>y = 9.3484 + 0.46531x</math></p> <p><math>y = 9.35 + 0.465x</math> (3 s.f.)</p>
iii	<p>Since <math>x</math> is the independent variable, <math>y</math> on <math>x</math> should be used for the estimation.</p> <p>For <math>y = 15</math>,</p> <p><math>x = 12.146 = 12</math></p> <p>The advertising expenditure is <u>\$12,000</u>.</p> <p>This estimate is reliable because :</p> <ul style="list-style-type: none"> <li>- <math>r</math> is close to 1 which indicates a strong positive linear correlation between <math>x</math> and <math>y</math>.</li> <li>- <math>y = 15</math> is within the given data range (interpolation), <math>12.5 &lt; y &lt; 20.8</math>.</li> </ul>
iv	$b$ is the gradient of the regression line which indicates that with every \$100 spent on advertising in a month, there is an increase of \$465 in the sale of refrigerators.
v	There would be no change to $b$ .

**12****a**

$$\bar{x} = \frac{\sum(x-30)}{60} + 30 = 30.4$$

$$s^2 = \frac{1}{59} \left[ \sum(x-30)^2 - \frac{(\sum(x-30))^2}{60} \right] = 2.2780 \text{ (5 s.f.)}$$

$$H_0 : \mu = 30$$

$$H_1 : \mu > 30$$

Conduct a 1-tail test at  $2\frac{1}{2}\%$  significance level.

Under  $H_0$ ,

$$\bar{X} \sim N\left(30, \frac{2.2780}{60}\right) \text{ approximately.}$$

Using a z-test,

$$\text{p-value} = P(\bar{X} > 30.4) = 0.020043 = 0.0200 \text{ (3 s.f.)}$$

Since p-value  $< 0.025$ , we reject  $H_0$  and conclude that there is sufficient evidence at  $2\frac{1}{2}\%$  significance level that the mean centre thickness of the soft contact lenses are more than 30  $\mu\text{m}$ . I.e. The claim is not justified.

It means that there is a probability of 0.025 of wrongly rejecting the claim that the mean centre thickness of the soft contact lenses is at most 30  $\mu\text{m}$ .

**b****(i)**

Let  $\mu$  be the mean of  $X$ .

$$H_0 : \mu = 7$$

$$H_1 : \mu \neq 7$$

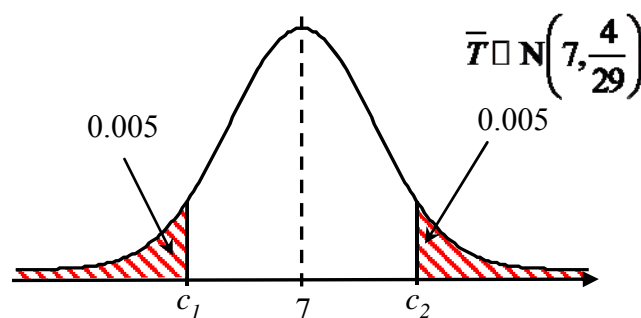
$$s^2 = \frac{30}{29} \text{ (sample variance)} = \frac{30}{29} (4) = \frac{120}{29}$$

Under  $H_0$ , since the sample size is large, the test statistic is

$$\bar{T} \square N\left(7, \frac{4}{29}\right) \text{ approximately by Central Limit Theorem.}$$

**(ii)**

Since the claim is rejected i.e. to reject  $H_0$  at 1% significance level.



From GC,  $c_1 = 6.04$  and  $c_2 = 7.96$ .

$$\bar{t} \leq 6.04 \text{ or } \bar{t} \geq 7.96 \text{ (3 s.f.)}$$

**(iii)** From the two tail test, we know that p-value (two tail)  $\leq 0.01$ .  
For a one-tail test,  
$$\text{p-value(one tail)} = \frac{\text{p-value (two tail)}}{2} \leq 0.005 < 0.01$$
, therefore we  
reject  $H_0$  and conclude that there is sufficient evidence at 1%  
significance level to say that mean waiting time is more than 7  
minutes.