Name	( )	Class	

RIVER VALLEY HIGH SCHOOL
2017 Year 6 Preliminary Examination II
Higher 1

MATHEMATICS

8865/01

Paper 1

18 Sept 2017

3 hours

Additional Materials: Answer Paper
List of Formulae (MF26)
Cover Page

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number in the space at the top of this page.
Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, place the cover page on top of your answer paper and fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1 The curve C has equation $y = 4 + \mathrm{e}^{x^2}$. Without using a calculator, find the equation of the tangent to C at the point where $x = 1$, giving your answer in the form $y = mx + c$, where m and c are constants. [4]

Find also the area of the region bounded by the curve C, the tangent line at the point $x = 1$ and the y-axis. [2]

2 **(i)** Differentiate $\ln(x^2 + 1)$ with respect to x. [2]

(ii) Express $\dfrac{(x+1)^2}{x^2+1}$ in the form $a + \dfrac{bx}{x^2+1}$, where a, b are real constants. [3]

(iii) Hence find $\displaystyle\int \frac{(x+1)^2}{x^2+1}\,\mathrm{d}x$. [2]

3 The line $y = x$ and curve C with equation $y = \dfrac{1}{2}\left(\mathrm{e}^x - 1\right)$ intersect at the point $P(a, a)$, where $a > 0$.

(i) Show that $\mathrm{e}^a = 1 + 2a$ [2]

(ii) Given that the area of the region enclosed by the curve C, the x-axis and the line $x = k$ is $\dfrac{a}{2}$. Find the value of k in terms of a. [3]

(iii) Find the range of values of x such that $\mathrm{e}^x - 1 > 2x$. [2]

4 Elman invested \$2000, \$1500 and \$1000 into funds A, B and C respectively. The total yearly interest he earned from these three funds is \$309. Furthermore, the interest earned from fund A was half the sum of the interest earned from B and C.

At the start of the following year, Elman reviewed his portfolio and withdrew all his investment from the three funds. He then invested \$2500 and \$2000 in funds A and B respectively. He managed to earn a yearly interest of \$320. The interest rates from the three funds remained unchanged. Find which fund gave Elman the highest interest rate, stating its value. [6]

5 In order to improve his performance, Johnson who is a professional sprinter decides to monitor his running speed, S metres per second. His personal trainer believes that the relationship between S and the time, t seconds, can be modelled by the equation $S = t^3 - 10t^2 + 28t,$ for $0 \le t \le 5$.

(i) Use differentiation and this model to find the maximum value of S, justifying that this value is a maximum. [5]

(ii) Sketch the graph of S against t, giving the coordinates of any points of intersection with the axes and the stationary points. [2]

(iii) Find the area of the region bounded by the curve, the t-axis and the line $t = 5$. Give an interpretation of the area that you have found, in the context of the question. [2]

Johnson's trainer also records his oxygen intake, σ measured in cm^3 per second during the run. The relationship between σ and t is given by $\sigma = 5\ln(t+4) - 2,$ for $0 \le t \le 5$.

(iv) Find the exact value of t for which $\sigma = 23$. [2]

(v) Sketch the graph of σ against t, indicating the range of values of σ. [2]

(vi) Find the rate at which the oxygen intake is increasing when $t = 1$. [1]

Section B: Probability and Statistics [60 Marks]

6 The school ICT assistant needs to schedule a group of 4 Science, 3 Mathematics and 2 Humanities Department teachers for upgrade of their laptops. He decides to arrange the 9 teachers in random order. On a particular day, find the probability that

(i) Miss Tan from the Science Department is first and Mr Ng from the Humanities Department is the last teacher to have their laptops upgraded, [2]

(ii) teachers from the same department are randomly arranged to have their laptops upgraded before teachers from another department to have their laptops upgraded in random order, [2]

(iii) given that the 5 teachers have their laptops upgraded by noon time, there are exactly 2 from the Science Department and exactly 2 from the Mathematics Department. [2]

7 A game is played with a fair die and two bags of red and blue marbles, labelled A and B. Bag A contains 3 red marbles and 9 blue marbles while bag B contains 7 red marbles and 5 blue marbles. The die is thrown, and if it shows a number more than 4, a marble is drawn at random from bag A. Otherwise, a marble is drawn from bag B. Events X and Y are defined as follows:

X : A marble is drawn from bag A,

Y : A blue marble is drawn.

(i) Find $\mathrm{P}(X)$. [1]

(ii) Draw a tree diagram to illustrate the events X and Y. [2]

(iii) Given that a blue marble is drawn, find the probability that the marble is chosen from bag A. [2]

(iv) Hence or otherwise, determine if event X and Y are independent. [2]

8 Ah Hao food manufacturing company is famous for its high quality curry puff produced for distribution to market places and supermarkets. On average, in a batch of 100 curry puffs produced, 8 of them are identified as too salty by laboratory test. In a monthly routine check, Ministry of Health (MOH) officials randomly pick a sample of n curry puffs from the company for laboratory test.

(i) Denoting the number of curry puff that are too salty in the sample picked by MOH officials by X , state in context, two assumptions needed for X to be well modelled by a binomial distribution. Explain why one of these assumptions may not be true. [3]

(ii) Assuming that X follows a binomial distribution, find the least value of n if the probability that only one of the curry puff checked by the MOH officials is too salty is less than 0.1. [3]

(iii) Using the value of n determined in part **(ii)**, find the probability of having at least 3 but less than 10 curry puffs, being too salty in the random sample chosen by the MOH officials. [2]

9 **(a)** Eight pairs of values of variables x and y are measured. Draw a sketch of a possible scatter diagram of the data for each of the following cases:

(i) The product moment correlation coefficient is approximately zero. [1]

(ii) The product moment correlation coefficient is approximately -0.95. [1]

(b) Scientists are investigating the relationship between the amount of a chemical compound (KH_2) added in x mg and the production of a cough syrup in y ml in a newly invented medicine production process. The following table shows a series of research results collected.

x/mg	5	10	15	20	25	30	35	40	45	50	55	60
y/ml	1.2	1.5	1.7	2.0	2.4	2.7	3.0	3.4	4.2	4.5	4.9	5.4

(i) The equation of the estimated least squares regression line of y on x for a set of bivariate data is $y = a + bx$. Explain what do you understand by the least square regression line of y on x. Hence find the regression line based on the data in the above table. [3]

(ii) Interpret the values for a and b found part **(b)(i)** in the context of the question. [2]

(iii) It is required to estimate the amount of the chemical compound (KH_2) added when the amount of cough syrup produced is 4.0 ml. By using an appropriate regression line, find the amount of the chemical compound added. State the reason for the use of the regression line. [2]

(iv) Comment on the reliability of the estimate found in part **(b)(iii)**. [2]

10 A group of young physicists is conducting an experiment involving collisions between protons and anti-protons. The amount of energy, x MeV, released in each collision is recorded for 50 collisions. The results are summarized by

$$\sum(x-100) = 720 \quad \text{and} \quad \sum(x-100)^2 = 30500.$$

(i) Find the unbiased estimates for the population mean and variance of the amount of energy released in each collision. [3]

(ii) The group of young physicists claims that the mean amount of energy released in each collision is 108 MeV. Test, at the 5% level of significance, if their claim is correct. State with a reason, whether it is necessary to assume that the amount of energy released in each collision follows a normal distribution. [4]

[Question 10 continues on the next page]

(iii) A Nobel Prize winner physicist then predicts that the mean amount of energy released in each collision is μ_0 MeV. However, the group of young physicists proposed it should be higher. Find the range of value of μ_0 such that the proposal by the group of young physicists is rejected at the 3% level of significance based on the data in part **(i)**. [4]

11 Each month the amount of electricity, measured in kilowatt-hours (kWh), used by a household in Singapore has mean 500 and standard deviation 80. Assuming normal distribution, find the probability that

(i) in a randomly chosen month, less than 600 kWh is used for a household, [1]

(ii) out of 10 households, there are at most 8 which use less than 600 kWh given that there are at least 4 households use less than 600 kWh, [3]

(iii) there are least 5 months with more than 600 kWh usage per month in a year for a household, [2]

(iv) the amount of electricity used in two randomly chosen months for a household differ by less than 100 kWh. [3]

(v) State any assumption(s) made in the calculation for part **(iv)** and explain briefly why this assumption may not be valid in the real-world. [2]

The charges for electricity used is $0.22 per kWh.

(vi) Write down the distribution of the total charges of electricity used in any one month. Hence find the probability that, in a randomly chosen month, the total charge is more than $150. [2]

(vii) Each household receives a bill every three months. Find the least value of m such that the probability that a randomly chosen bill is less than $$m$ is at least 0.96. [3]

(viii) A campaign was launched to raise awareness of the importance of saving electricity. At the end of the campaign, it was found that the mean monthly electricity usage for a household dropped drastically to 185 kWh with standard deviation 80 kWh. Explain whether a normal distribution is appropriate here. [1]

End of Paper

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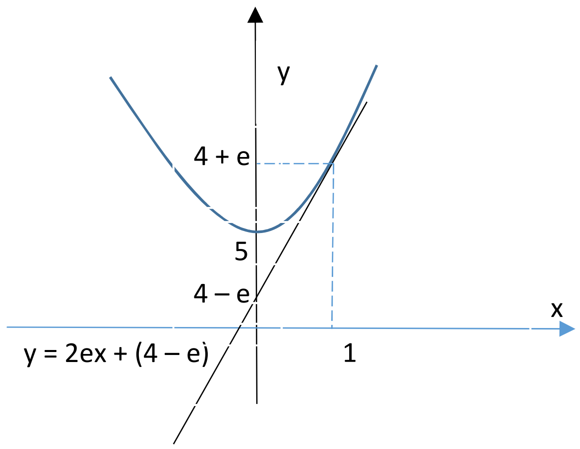
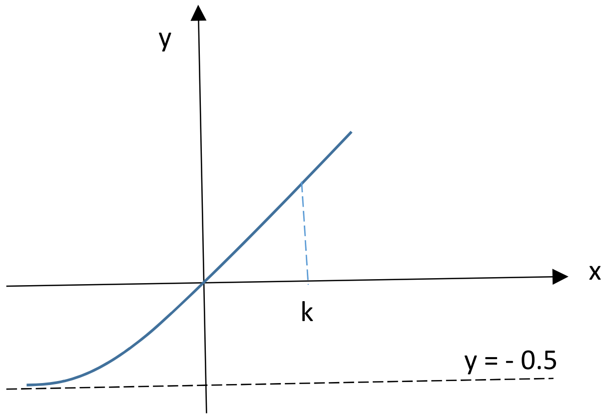
Solutions to 2017 Y6 H1 Maths Preliminary Exam II

1	Working	
	$y=4+e^{x^2} \Rightarrow \dfrac{dy}{dx}=2xe^{x^2}$ when $x=1, y=4+e, \dfrac{dy}{dx}=2e$ $y-(4+e)=2e(x-1)$ $y=2ex+(4-e)$  $\int_0^1 4+e^{x^2}-(2ex+4-e)\,dx=1.46$	

2	Working	
i)	$\dfrac{d\ln(x^2+1)}{dx}=\dfrac{1}{x^2+1}.2x=\dfrac{2x}{x^2+1}$	
ii)	$\dfrac{(x+1)^2}{x^2+1}=\dfrac{x^2+2x+1}{x^2+1}$ $=\dfrac{x^2+1}{x^2+1}+\dfrac{2x}{x^2+1}$ $=1+\dfrac{2x}{x^2+1}$	
iii)	$\int \dfrac{(x+1)^2}{x^2+1}\,\mathrm{d}x=\int 1+\dfrac{2x}{x^2+1}\,\mathrm{d}x=x+\ln(x^2+1)+c$	

3	**Working**	
i)	Let $x = a$ $\frac{1}{2}\left(e^{a}-1\right)=a$ $e^{a}-1=2a \Rightarrow e^{a}=1+2a$	
ii)	 $\int_{0}^{k}\frac{1}{2}\left(e^{x}-1\right)dx=\frac{1}{2}\left[e^{x}-x\right]_{0}^{k}$ $=\frac{1}{2}\left(e^{k}-k-1\right)$ $\frac{1}{2}\left(e^{k}-k-1\right)=\frac{a}{2}$ $e^{k}-k=1+a=e^{a}-a$ (from (i)) $\therefore k=a$	
iii)	Using GC, the points of intersection between y = x and the curve is $x=0$ and 1.256 The range of values of x: $x<0$ or $x>1.26$	

4 **Working**

Let a, b and c be the interest rates of each fund in per cent.

$$\frac{a}{100}(2000)+\frac{b}{100}(1500)+\frac{c}{100}(1000)=309$$

$$20a+15b+10c=309$$

$$2.\frac{a}{100}(2000)=\frac{b}{100}(1500)+\frac{c}{100}(1000)$$

$$40a-15b-10c=0$$

$$\frac{a}{100}(2500)+\frac{b}{100}(2000)=320$$

$$25a+20b=320$$

Solving the 3 equations, a = 5.15, b = 9.56 and c = 6.26

Fund B gave the highest rate = 9.56%

5 **Working**

i)

$$\frac{dS}{dt}=3t^2-20t+28=0$$

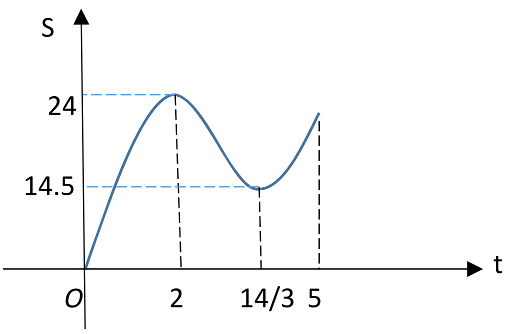
$$t=\frac{20\pm\sqrt{64}}{6}=2 \text{ or } \frac{14}{3}$$

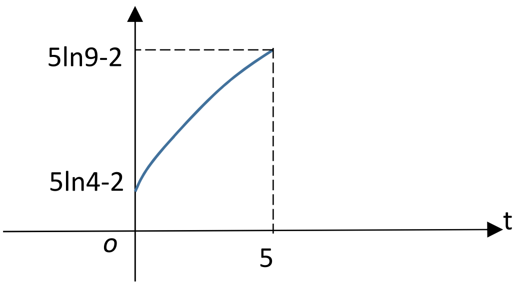
	1.995	**2**	**2.005**
gradient	+	**0**	-
shape			

	4.665	**14/3**	**4.675**
gradient	-	**0**	+
shape			

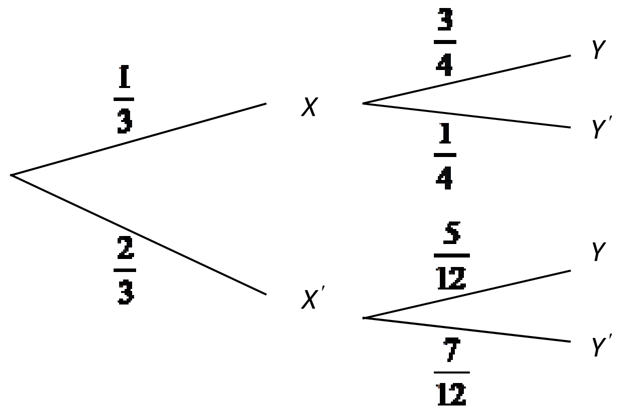
When $t=2, S=24$ which is maximum.

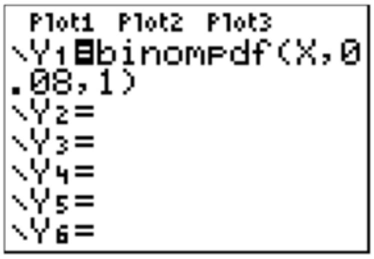
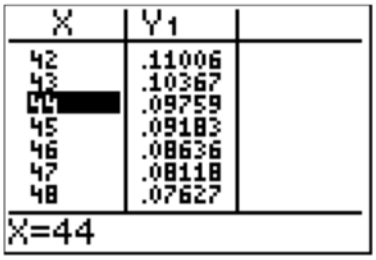
ii)

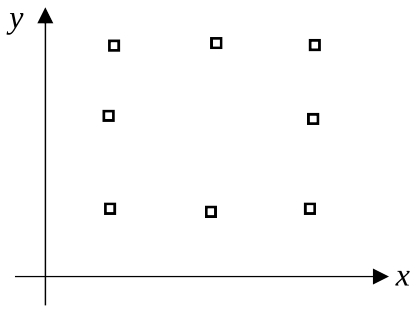
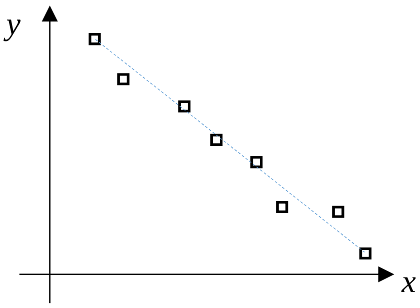
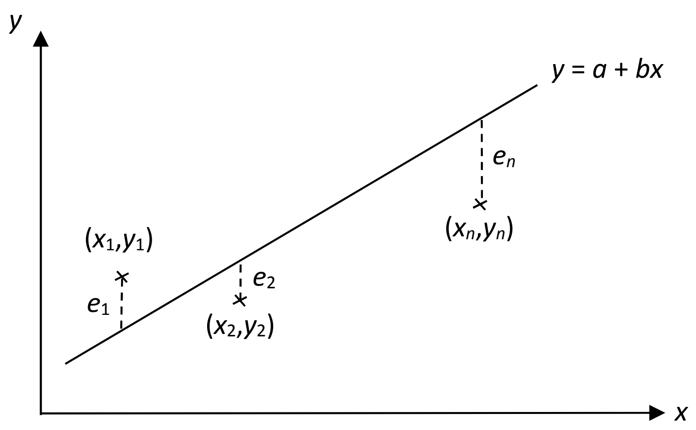


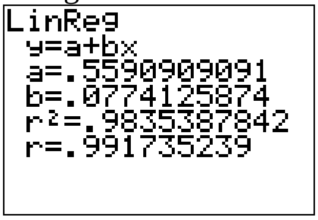
iii)	$\int_0^5 t^3 - 10t^2 + 28t\,dt \approx 89.6$ **89.6 metres** refers to the total distance covered by Johnson during 5 seconds.	
iv)	$5\ln(t+4) - 2 = 23$ $\ln(t+4) = 5$ $t + 4 = e^5$ $t = e^5 - 4$ seconds	
v)		
vi)	When t = 1, $\frac{d\sigma}{dt} = \frac{5}{1+4} = 1$ cm^3/s	

6	**Working**	
6(i)	The probability = $\frac{1!\times 7!\times 1!}{9!} = \frac{1}{8\times 9} = \frac{1}{72}$	
6(ii)	The probability = $\frac{(4!\times 3!\times 2!)\times 3!}{9!}$ $= \frac{1728}{362880} = \frac{1}{210}$	
6(iii)	The probability = $\frac{\binom{4}{2}\binom{3}{2}\binom{2}{1}\times 5!}{\binom{9}{5}\times 5!}$ $= \frac{36}{126} = \frac{2}{7}$	

	Working	Marks
7(i)	$\mathrm{P}(X)=\frac{2}{6}=\frac{1}{3}$	
7(ii)	**The tree diagram for the events *X* and *Y*:**  	
7(iii)	$\mathrm{P}(X\mid Y)=\frac{\mathrm{P}(X\cap Y)}{P(Y)}$ $=\frac{\frac{1}{3}\times\frac{3}{4}}{\frac{1}{3}\times\frac{3}{4}+\frac{2}{3}\times\frac{5}{12}}$ $=\frac{9}{19}$	
7(iv)	**Since** $\mathrm{P}(X\mid Y)=\frac{9}{19},\ \mathrm{P}(X)=\frac{1}{3}$ **and thus,** $\mathrm{P}(X\mid Y)\neq\mathrm{P}(X)$ **we can conclude that event *A* and *B* are not independent events.** **Alternative:** **P(*Y*) =** $\frac{1}{3}\times\frac{3}{4}+\frac{2}{3}\times\frac{5}{12}=\frac{19}{36}$ **Then P(*X*∩*Y*) =** $\frac{1}{4}\neq\mathrm{P}(X)\times\mathrm{P}(Y)$ **Hence, event *A* and *B* are not independent events.**	

8	Working	Marks
8(i)	The two assumptions are (i) The event of a particular chosen curry puff being too salty is independent of other curry puffs chosen by the MOH officials. (ii) The probability of choosing a curry puff that is too salty is constant for all curry puffs chosen by the MOH officials. (i) may not be true in general as curry puffs are manufactured in batches, thus if 1 curry puff is found to be too salty, curry puffs from the same batch will be salty too. (or any other similar reasoning)	
8(ii)	Assume $X \sim B(n, 0.08)$, Then we need to have $\mathrm{P}(X = 1) < 0.01$. Using binomial pdf in GC, we can set the necessary and obtain the following probability table for checking:  Hence, from the above table, we deduce that the least value of n should be 44.	
8(iii)	Let n = 44. Then $X \sim B(44, 0.08)$, Then we have $\mathrm{P}(3 \le X < 10)$ $= \mathrm{P}(X \le 9) - \mathrm{P}(X \le 2)$. $= 0.692$	

9	Working	Marks
9 (i)	The required scatter diagram are as follows: (i)  (ii) 	
9b(i)	Let the sample of bivariate data be (x_i, y_i) where $i = 1, 2, \ldots n$. Let $e_i = y_i - (a + bx_i)$ be the vertical deviation between the point (x_i, y_i) and the line $y = a + bx$.  The line $y = a + bx$ is the least square regression line for the sample of bivariate data if the sum of the squares of the vertical deviations, i.e. $\sum_{i=1}^{n} (e_i)^2$, is the minimum.	

	Using GC and the table of values,  the regression line of *y* on *x* is $y = 0.559 + 0.0774x$	
9b(ii)	*a* is the amount of cough syrup in ml produced with no amount of the chemical compound KH_2 added in the production process. *b* is the amount of cough syrup in ml produced with the addition of 1 mg of the chemical compound KH_2 added in the production processs.	
9b(iii)	As *x* is the independent variable and *y* is the dependent variable in the data set, we will need to use the regression line of *y* on *x* to estimate the value of *x* given *y* value. Thus, when *y* = 4.0, $4.0 = 0.55909 + 0.077413x$ $\Rightarrow x = 44.448736 \approx 44.4$ mg (3 sig fig)	
9b(iv)	The estimate for *x* value in part 9b(iii) is reliable for the following reasons: i. the correlation coefficient *r* = 0.992 is very close to 1, ii. the given *y* value of 4.0 is within the data range of the *y* value iii. the correct line *y* on *x* is used as *y* is the dependent variable.	

10	**Working**	**Marks**
10(i)	The unbiased estimate for population mean = $\bar{x}$ $= \frac{720}{50} + 100 = 114.4$ The unbiased estimate for population variance = s^2 $= \frac{1}{50-1}\left(\sum(x-100)^2 - \frac{\left(\sum(x-100)\right)^2}{50}\right)$ $= \frac{1}{49}\left(30500 - \frac{720^2}{50}\right) = \frac{2876}{7} = 410.8571429 \approx 411$ (3 sig. fig.)	
10(ii)	Let μ be the mean of the amount of energy released in each collision. For testing of the new claim by the young physicists, Let $H_0 : \mu = 108$ $H_1 : \mu \neq 108$ We then perform a one tail test at 5% level of significance i.e. $\alpha = 0.05$	

	Under H_0, $\overline{X} \sim N\left(108, \frac{2876/7}{50}\right)$ Then test statistics is $Z = \frac{\overline{X} - 108}{\sqrt{\frac{2876/7}{50}}} \sim N(0, 1)$ Using GC, the p-value = 0.0255723191. Since p-value < α, we reject H_0 and conclude that there is sufficient evidence at 5% level of significance that the population mean of the energy released in each collision is not 108 MeV. As the sample size n is large (50), it is not necessary to assume that X, the amount of energy released in each collision follows a normal distribution as by the Central Limit Theorem, $\overline{X}$ can be approximated by a normal distribution for the test to be valid.	
10(iii)	For testing of the new claim by the young physicists, Let $H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$ We then need to perform a one tailed test at 3% level of significance i.e. $\alpha = 0.03$ Under H_0, $\overline{X} \sim N\left(\mu_0, \frac{2876/7}{50}\right)$ Then test statistics is $Z = \frac{\overline{X} - \mu_0}{\sqrt{\frac{2876/7}{50}}} \sim N(0, 1)$. For H_0 not to be rejected, $z_{\text{calculated}}$ must not be in the critical region, hence, $\frac{114.4 - \mu_0}{\sqrt{\frac{2876/7}{50}}} \le 1.88079361$ $\Rightarrow 114.4 - \mu_0 \le 1.88079361 \times \sqrt{\frac{2876/7}{50}}$ $\Rightarrow \mu_0 \ge 114.4 - 1.88079361 \times \sqrt{\frac{2876/7}{50}}$ $\Rightarrow \mu_0 \ge 109.0085999$ $\Rightarrow \mu_0 \ge 109$ (3 sig fig)	

11	Working	Marks
i)	Let W be the r.v. of the amount of electricity used by a household in a month. $W \sim N(500, 80^2)$ $P(W < 600) = 0.89435 \approx 0.894$	
ii)	Let H be the number of households out of 10 which use less than 960 kWh, H ~B(10, 0.89435) $P(H \le 8 \mid H \ge 4) = \frac{P(4 \le H \le 8)}{P(H \ge 4)} = \frac{P(H \le 8) - P(H \le 3)}{1 - P(H \le 3)}$ $= \frac{0.28583}{0.9999868} = 0.28583 \approx 0.286$	
iii)	Let X be the r.v. of number of months that use more than 600 kWh, $X \sim B(12, 0.10565)$. $P(X \ge 5) = 1 - P(X \le 4) \approx 0.00550$	
iv)	$W_1 - W_2 \sim N(0, 12800)$ $P(\lvert W_1 - W_2 \rvert < 100) = P(-100 < W_1 - W_2 < 100) = 0.62324 \approx 0.623$	
v)	Assume that the amount of electricity used by a household of the two months is independent. This assumption may not be true if the family knows that they have over-used energy in the first month, they may use less in the later months. (or any other reasonable explanations)	
vi)	Let C = 0.22W be the r.v. of the cost of electricity for a household in each month. $C \sim N(110, 309.76)$ $P(C > 150) = 0.011521 \approx 0.0115$	
vii)	$T = C_1 + C_2 + C_3 \sim N(330, 929.28)$ $P(T < m) \ge 0.96$ $m \ge 383.368 \approx 383$ (3 sig. fig)	
viii)	Let V be the r.v of the amount of electricity used by a household per month at the end of the campaign. Supposed $V \sim N(185, 80^2)$ Consider: a) $P(V < 0) = 0.0104$ Since the probability that a household uses negative amount of electricity is 0.0104 (1.04%) which is quite significant, Normal distribution is deemed as unsuitable. OR b) $P(185 - 3(80) < W < 185 + 3(80)) = 0.997$ Since probability of 0.997 of amount of electricity used is within 3 standard deviation and there is quite significant probability that the amount of electricity used is negative, Normal distribution is deemed unsuitable.	