

BEATTY SECONDARY SCHOOL **END-OF-YEAR EXAMINATION 2018**

SUBJECT: Mathematics

LEVEL

: Secondary 3 Express

PAPER

: 4048 / 01

DURATION: 1 hour 30 minutes

SETTER: Ms Joanna Chong

DATE

: 5 October 2018

CLASS: NAME: **REG NO:**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.

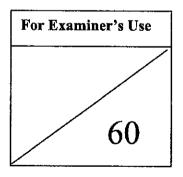
If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

You are expected to use a scientific calculator to evaluate explicit numerical expressions. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of

The number of marks is given in brackets [] at the end of each question or part question. The total number of marks for this paper is 60.



This paper consists of 13 printed pages (including this cover page)

[Turn over

Mathematical formulae

Compound Interest

Total amount =
$$P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle ABC = $\frac{1}{2}ab\sin C$

Arc length = $r \theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$Mean = \frac{\sum fx}{\sum f}$$

Standard deviation =
$$\sqrt{\frac{\Sigma f x^2}{\Sigma f} - \left(\frac{\Sigma f x}{\Sigma f}\right)^2}$$

3 Answer ALL questions.

Evaluate $\frac{\sqrt[3]{3.16 \times 10^{-6}}}{1.8(2.5 \times 10^{8})}$ and give your answer in standard form.

Answer[1]

 $2 \qquad \text{Simplify} \qquad \frac{3x}{x^2 - 4} - \frac{2}{2 - x}.$

Answer[3]

Given that the three points (3, 4), (2, k) and (7, 5) are collinear. Find the value of k.

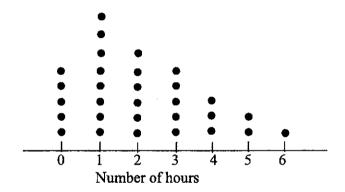
Answer $k = \dots [2]$

4	Andrew invested some money in a savings account for 5 years.
	The rate of compound interest was fixed at 0.3% per annum.
	At the end of 5 years, there was \$ 875 000 in his account.

How much interest did Andrew earn at the end of 5 years?

Answer	\$	[3]
--------	----	-----

5 The dot diagram shows the number of hours a group teenagers spend on the handphone in a day.

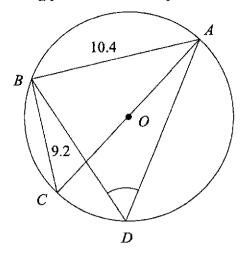


- (a) For the above distribution, find
 - (i) the mode,
 - (ii) the mean,
 - (iii) the median.

(b) If a teenager is picked at random from the group, what is the probability that this teenager spends at least 4 hours on the handphone in a day?

Answer (b)[1]

In the diagram, O is the centre of the circle ABCD. AB = 10.4 cm and BC = 9.2 cm. Find angle ADB, stating your reasons clearly.



Answer	angle <i>ADB</i> =	[3]

- 7 Factorise completely
 - (a) $75d^2-3$,
 - **(b)** $4p^2-4pr+r^2-2pq+rq$.

[Turn over

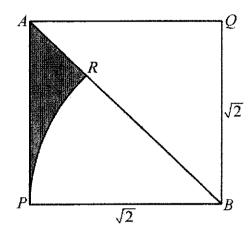
- 8 (a) Simplify $\left(\frac{a^3}{3b}\right)^{-2} \div a^2b^5 \times \left(\frac{125a^4b^2}{5}\right)^0$, leaving your answer in positive indices.
 - **(b)** Given that $\sqrt{\frac{k^2}{4}} = 8^{-\frac{1}{3}}$, find the value of k.

(b)
$$k = \dots [2]$$

- 9 (a) Solve the inequality $\frac{2}{3}x-1 < x \le 2(8-x)$.
 - (b) Write down largest rational number which satisfies $\frac{2}{3}x-1 < x \le 2(8-x)$.

In the diagram, AQBP is a square of side $\sqrt{2}$ cm. PR is an arc of a circle with centre at B and radius $\sqrt{2}$ cm.

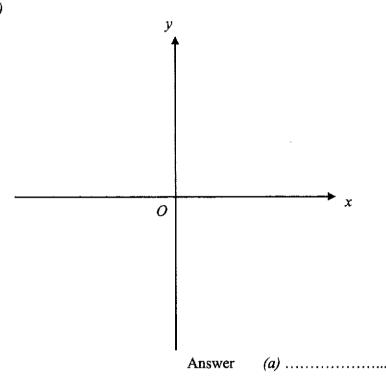
Find the area of the shaded region APR, leaving your answer in terms of π .



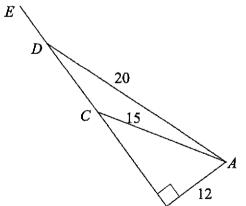
Answer	cm ²	Γ3 1
AIISIVEI		1-7

- Express $x^2 + 4x + 7$ in the form $(x+h)^2 + k$, where h and k are constants. 11 (a)
 - Hence, sketch the graph of $y = x^2 + 4x + 7$ on the axes below, indicating its turning point (b) and intercept(s) clearly.

Answer (b)



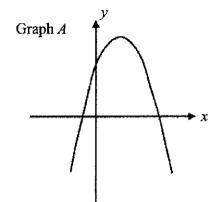
- 12 In the diagram, $\angle ABC = 90^{\circ}$, AB = 12 cm, AC = 15 cm, AD = 20 cm and BCDE is a straight line. Calculate, giving each answer as a fraction,
 - (a) $\sin \angle ADB$,
 - **(b)** $\cos \angle ACD$.

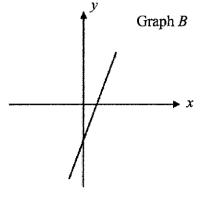


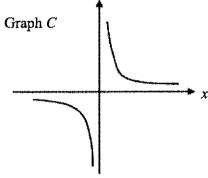
(a)[1] Answer

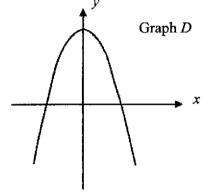
(b).....[2]

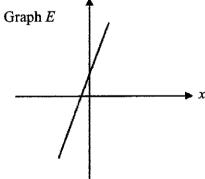
- 13 State the graph that corresponds to each of following equation.
 - y=3x-7,(a)
 - **(b)**
 - $y=2-x^2.$ (c)

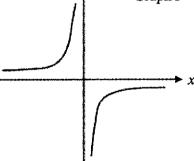










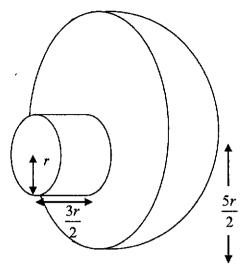


14 The solid shown is made from a cylinder and a hemisphere.

The cylinder has a radius of r and height $\frac{3r}{2}$.

The hemisphere has a radius of $\frac{5r}{2}$.

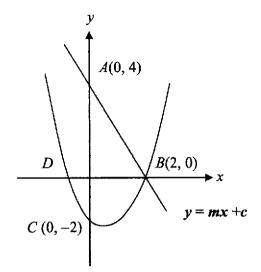
Find an expression, in terms of π and r, for the total surface area of the solid.



Answercm² [3]

The diagram below shows a straight line y = mx + c intersecting the y-axis at A(0, 4) and x-axis at B(2,0).

The curve y = (x-2)(x+k) cuts the x-axis at B and D and the y-axis at C(0, -2).



- (a) Find
 - (i) the value of m and of c.
 - (ii) the length of AB.
 - (iii) the value of k.

(b) A second curve with the equation $y = ax^2 + bx + d$ cuts the x-axis at points B and D. Determine whether the value of $b^2 - 4ad$ is positive, negative or zero. Explain your answer.

Answer	b^2-4ad	is	 	 	 	
			 	 	 	[1]

16 Diagram I shows an open metal pail which is in the shape of a frustum. The diameters of the two circular ends are 45 cm and 15 cm. The height of the pail is 40 cm.

The curved surface of the frustum can be found using the model in Diagram II.

- (a) By using appropriate triangles in Diagram II, find the value of h.
- (b) Hence, find the total area of metallic sheet used to make the pail.

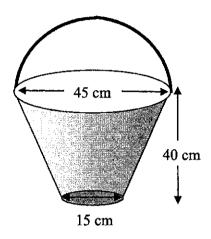


Diagram I

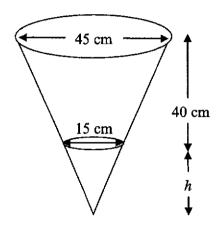


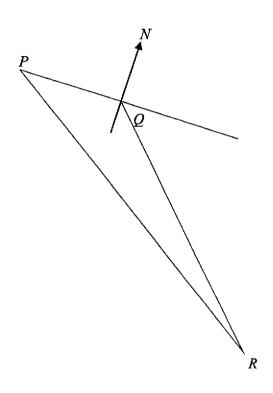
Diagram II

(a)
$$h =$$
cm [2]

In the diagram, P, Q and R are three ports. P is 900 m due west of Q. At 07 40, a ship, Pioneer, left P and travelled to R on a bearing of 130°. At the same time, another ship, Queen, left Q and travelled at an average speed of 15 km/h to R. It arrived at 08 10 on the same day.

Calculate

- (a) the distance QR, in metres,
- (b) the bearing of Q from R.



Answer	$(a)QR = \dots m$	[1]
	(b)	.[3

BEATTY SECONDARY SCHOOL END OF YEAR EXAMINATION 2018



SUBJECT: Mathematics

LEVEL

: Sec 3 Express

PAPER : 4048/2

DURATION: 2 hours

SETTER: Mrs Samsol

DATE

: 10 October 2018

CLASS: REG NO:

READ THESE INSTRUCTIONS FIRST

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Write in dark blue or black pen.

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Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.

The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any question, it must be shown in the space below that question.

Omission of essential working will result in loss of marks.

The total of the marks for this paper is 80.

You are expected to use a scientific calculator to evaluate explicit numerical expressions. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place. For π , use either your calculator value or 3.142.

2 Mathematical Formulae

Compound Interest

Total amount =
$$P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = $\pi r l$

Surface area of a sphere = $4\pi r^2$

Volume of a cone =
$$\frac{1}{3} \pi r^2 h$$

Volume of a sphere =
$$\frac{4}{3}\pi r^3$$

Area of triangle
$$ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

Sector area =
$$\frac{1}{2}r^2\theta$$
, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

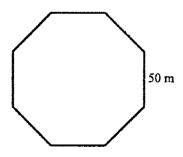
Statistics

$$Mean = \frac{\sum fx}{\sum f}$$

Standard Deviation =
$$\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 (a) Simplify
$$\frac{4xy-8x^2}{y^2-4x^2}$$
. [2]

- It is given that $a = \frac{3b+c}{b-5c}$. (b)
 - (i) Find a when b=3 and c=-1. [1]
 - (ii) Express b in terms of a and c. [2]
- Solve the equation $\frac{6}{2x+5} = x-3$. (c) [3]
- 2 (a) The force, F newtons, between two magnets is inversely proportional to the square of the distance, d centimetres, between them. When the force is 25 newtons, the distance is 3 centimetres.
 - Find the equation connecting F and d. (i) [2]
 - The force between the magnets is 10 newtons. (ii) Find the distance between the magnets. [2]
 - For two pairs of identical magnets, the ratio of the distances between them is 2:3. Find the ratio of the forces between the two pairs of magnets. [2]
 - The floor of a recreation hall is in the shape of a regular octagon with sides (b) of length 50 m.



Find the floor area of this hall. [5]

[Turn Over

3 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation

$$y = \frac{x}{2}(2+3x-x^2)$$
.

Some corresponding values of x and y are given in the table below.

x	-2	-1	-0.4	0	1	2	2.3	3	4
у	8	1	p	0	2	4	4.2	3	- 4

(a) Find the exact value of p.

[1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-2 \le x \le 4$.

Using a scale of 1 cm to represent 1 unit, draw a vertical y-axis of $-6 \le y \le 10$.

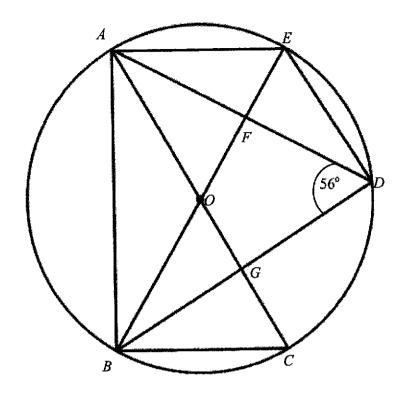
On your axes, plot the points given in the table and join them with a smooth curve.

[3]

- (c) Use your graph to find the solutions of the equation $\frac{x}{2}(2+3x-x^2)=4$. [2]
- (d) By drawing a tangent, find the gradient of the curve at the point (-1, 1). [2]
- (e) (i) On the same axes, draw the line with gradient $\frac{1}{3}$ that passes through the point with coordinates (3, 3).
 - (ii) Write down the x-coordinates of the points where this line intersects the curve. [2]

5

4



The diagram shows a circle ABCDE, centre O. AD and BE intersect at the point F while AC and BD intersect at the point G. AC and BE pass through O and BE bisects angle ABD. Angle $ADB = 56^{\circ}$

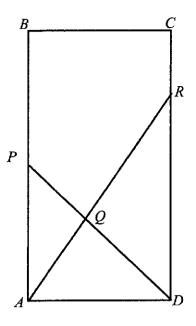
Find, giving reasons for each answer,

(i)	angle ACB,	[1]
(ii)	angle AOB,	[1]
(iii)	angle ADE,	[1]
(iv)	angle AED,	[2]
(v)	angle BGC.	[1]

[Turn Over

5 ABCD is a rectangle. P is the mid-point of AB and R is a point on CD.

AR and DP intersect at the point Q.



(a) Show that triangles AQP and RQD are similar.

[2]

(b) Given that DR = 3RC, find

(i)
$$\frac{AP}{DR}$$
,

[1]

(ii)
$$\frac{\text{Area of } \Delta AQP}{\text{Area of } \Delta RQD} ,$$

[1]

(iii)
$$\frac{\text{Area of triangle } ARD}{\text{Area of rectangle } ABCD}$$
.

[1]

(c) Given that the area of triangle $AQP = 25.5 \text{ cm}^2$, find the total area of triangles AQP, AQD and DQR.

[2]

6

(e)

second.

A water storage tank has a capacity of 8000 litres. (a) A large pump can empty water from the tank at a rate of x litres per minute. Write down an expression, in terms of x, for the number of minutes the pump would take to empty a full tank. [1] A small pump can empty water from the tank at a rate which is 50 (b) litres per minute less than the large pump. Write down an expression, in terms of x, for the number of minutes the small pump would take to empty a full tank. [1] It takes 35 minutes longer to empty the tank using the small pump (c) than it does to empty it using the large pump. Write down an equation in x to represent this information, and show [3] that it reduces to $7x^2 - 350x - 80\ 000 = 0$. (d) Solve the equation $7x^2 - 350x - 80\ 000 = 0$, giving your solutions correct to 2 decimal places. [3]

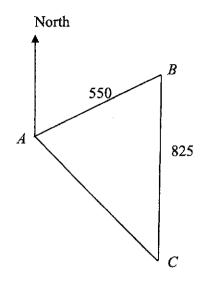
Find the time taken to empty a full tank of water using the small

Give your answer in minutes and seconds, correct to the nearest

[Turn Over

[2]

In the diagram, A is a point at sea level at the foot of a vertical cliff. Two buoys, B and C, are on the surface of the sea and C is due south of B. AB = 550 m and BC = 825 m. The bearing of B from A is 065° .



(a) Calculate

(i) AC,

[2]

(ii) the bearing of A from C,

[3]

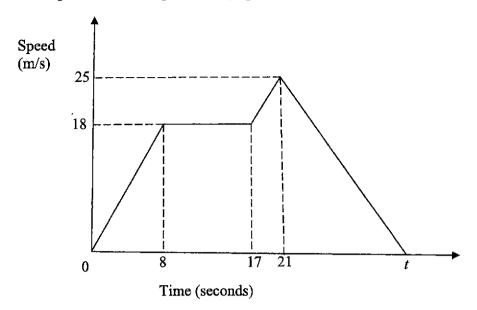
(iii) the shortest distance from B to AC.

[2]

(b) P is a point at the top of the cliff vertically above A.
 The angle of depression of B from P is 9°.
 Calculate the angle of elevation of P from C.

[3]

The diagram shows the speed-time graph for a car journey. 8

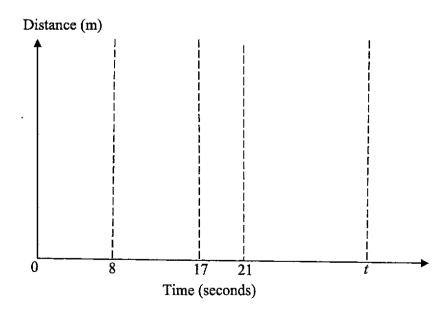


The maximum speed of the car was 25 m/s. (a) Change 25 m/s into km/h. [1]

Calculate the acceleration of the car after 6 seconds. **(b)** [1]

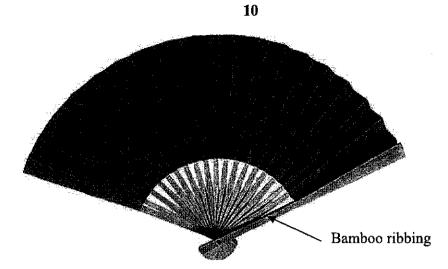
The total distance travelled for the whole journey was 420 m. (c) [2] Find the value of t.

Copy and complete the distance-time graph for the whole journey. (d)



[Turn Over

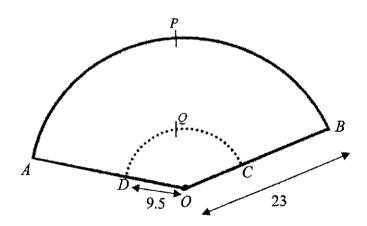
[4]



http://www.fantastica-uk.co.uk

The handheld fan above is made of paper with bamboo ribbing.

In this question, the fan is modelled as sectors of circles. APB and DQC are arcs of circles centre O with radii 23 cm and 9.5 cm respectively. The perimeter of APBCQD is 64 cm.



- (a) Calculate angle DOC in radians. [2]
- (b) Calculate the area of paper required to cover the region APBCQD. [3]

[4]

Amanda intends to make 100 identical paper fans to sell at a charity event. The material needed to make a paper fan are

- 4 sheets of A4 coloured papers,
- 14 bamboo sticks,
- 1 metal craft ring,
- a bottle of craft glue (for making 12 fans).

She finds the following information on the internet.

•	Material	Unit cost (subject to 7% GST)
1	A pack of 50 sheets of A4 coloured papers	\$7.10
2	A pack of 100 ice-cream sticks	\$5.35
3	A pack of 10 metal craft rings	\$2.40
4	A pack of 48 bamboo sticks	\$11.40
5	A bottle of craft glue	\$5.00
* G	ST stands for Goods and Service	e Tax

(c) Suggest a reasonable selling price for each fan in order for Amanda to make more than 150% profit of the cost of the material.

Justify your decision and show your workings clearly.

End of Paper

PartnerInLearning

BP~275

Answer key

1(a)
$$\frac{4x}{y+2x}$$
 1(b)(i) 1 1(b)(ii) $b = \frac{c+5ac}{a-3}$ 1(c) $x = 3\frac{1}{2}$ or $x = -3$

2(a)(i)
$$F = \frac{225}{d^2}$$
 2(a)(ii) $d = 4.74$ cm 2(a)(iii) 9:4 2(b) 12 100 m²

$$3(a) p = -0.128$$

3(c)
$$x = -1.55$$
 (± 0.05), $x = 2$ or $x = 2.55$ (± 0.05)

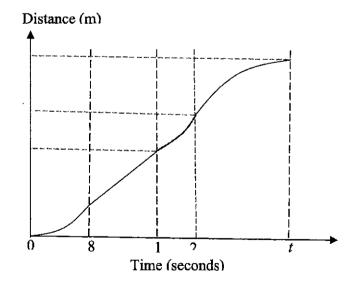
$$3(d)$$
 Gradient = $-3.5 (\pm 0.8)$

$$3(e)(ii)$$
 $x = -1.2$, $x = 1.2$ or $x = 3$ (±0.1)

5(a) Use of AA property 5(b)(i)
$$\frac{2}{3}$$
 5(b)(ii) $\frac{4}{9}$ 5(b)(iii) $\frac{3}{8}$ 5(c) 121.125 cm²

6(a)
$$\frac{8000}{x}$$
 6(b) $\frac{8000}{x-50}$ 6(d) $x=134.79$ or $x=-84.79$ 6(e) 94 min 21 seconds

8(d)



9(a)
$$\frac{74}{65}$$
 radians [accept 1.14 rad]

9(b)
$$249\frac{3}{4}$$
 cm² [accept 250 cm²]

9(c) Accept \$12.52 or more

PartnerInLearning



BEATTY SECONDARY SCHOOL END-OF-YEAR EXAMINATION 2018

MARK SCHEME

SUBJECT: Mathematics

LEVEL

: Secondary 3 Express

PAPER : 4048 / 01

DURATION: 1 hour 30 minutes

SETTER: Ms Joanna Chong

DATE

: 5 October 2018

CLASS: NAME: REG NO:
CLACC. NAME. DEC.NO.

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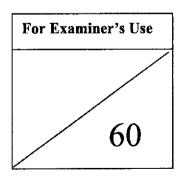
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Standard deviation =
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3 Answer ALL questions.

Evaluate $\frac{\sqrt[3]{3.16 \times 10^{-6}}}{1.8(2.5 \times 10^{8})}$ and give your answer in standard form.

$$3.26 \times 10^{-11} B1$$

Answer[1]

2 Simplify
$$\frac{3x}{x^2 - 4} - \frac{2}{2 - x}$$

$$\frac{3x}{(x - 2)(x + 2)} - \frac{2}{2 - x} M1(factorisation)$$

$$= \frac{3x}{(x - 2)(x + 2)} + \frac{2}{(x - 2)} M1(sign)$$

$$= \frac{3x + 2(x + 2)}{(x - 2)(x + 2)}$$

$$= \frac{5x + 4}{(x - 2)(x + 2)} A1$$

Answer[3]

Given that the three points (3, 4), (2, k) and (7, 5) are collinear. Find the value of k.

$$\frac{5-k}{7-2} = \frac{5-4}{7-3} \quad M1$$

$$\frac{5-k}{5} = \frac{1}{4}$$

$$k = \frac{15}{4} = 3\frac{3}{4} \quad A1$$

Answer[2]

Andrew invested some money in a savings account for 5 years. 4 The rate of compound interest was fixed at 0.3% per annum. At the end of 5 years, there was \$875 000 in his account.

How much interest did Andrew earned at the end of 5 years?

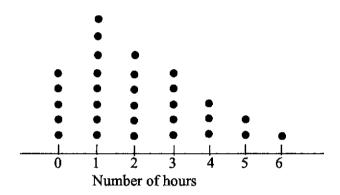
$$875000 = P\left(1 + \frac{0.3}{100}\right)^{5} \qquad M1$$

$$I = 875000 - their \frac{875000}{\left(1 + \frac{0.3}{100}\right)^{5}} \text{ or } 861992.30 \quad M1$$

$$= \$13007.70 \quad A1$$

Answer

5 The dot diagram shows the number of hours a group teenagers spend on the handphone in a day.



For the above distribution, find (a)

- (i) the mode,
- (ii) the mean,
- (iii) the median.

B1(i) 1

(ii) mean
$$=\frac{63}{20}$$

= 2.1 B1

(iii) 2 B1

> (a)(i)hour(s) [1] Answer

> > (ii)hour(s) [1]

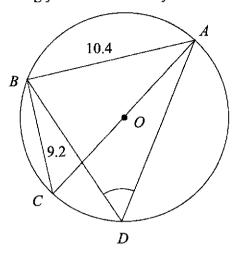
(iii) hour(s) [1]

(b) If a teenager is picked from the group, what is the probability that this teenager spends at least 4 hours on the handphone in a day?

$$\boxed{\frac{6}{30} = \frac{1}{5} \quad B1}$$

Answer (b)[1

In the diagram, O is the centre of the circle ABCD. AB = 10.4 cm and BC = 9.2 cm. Find angle ADB, stating your reasons clearly.



Since AC passes through O, $\angle ABC = 90^{\circ}$ (Angle in a semi-circle) B1

$$\tan \angle ACB = \frac{10.4}{9.2}$$

$$\angle ACB = 48.503^{\circ}$$
 M1

Hence, $\angle ADB = 48.5^{\circ}$ (Angles in the same segment)

Answer angle $ADB = \dots$ [3]

B1

7 Factorise completely

(a)
$$75d^2 - 3$$

(b)
$$4p^2-4pr+r^2-2pq+rq$$

(a)
$$\begin{vmatrix} 3(25d^2 - 1) & M1 \\ = 3(5d + 1)(5d - 1) & A1 \end{vmatrix}$$

(b)
$$\begin{cases} 4p^2 - 4pr + r^2 - 2pq + rq \\ = (2p - r)^2 - q(2p - r) & M1 \\ = (2p - r)(2p - r - q) & A1 \\ \text{Or } (r - 2p)(r + q - 2p) \end{cases}$$

Answer (a)[2]

(b)[2]

(a) Simplify $\left(\frac{a^3}{3b}\right)^{-2} \div a^2b^5 \times \left(\frac{125a^4b^2}{5}\right)^0$, leaving your answer in positive indices. 8

(b) Given that
$$\sqrt{\frac{k^2}{4}} = 8^{-\frac{1}{3}}$$
, find the positive value of k.

(a)
$$\left(\frac{a^3}{3b}\right)^{-2} \div a^2b^5 \times \left(\frac{125a^4b^2}{5}\right)^0$$

$$= \left(\frac{3b}{a^3}\right)^2 \times \frac{1}{a^2b^5} \quad M1(\text{for dealing with power zero})$$

$$= \frac{9b^2}{a^6} \times \frac{1}{a^2b^5} \quad M1(\text{for } \frac{9b^2}{a^6})$$

$$= \frac{9b^{-3}}{a^8}$$

$$= \frac{9}{a^8b^3} \quad A1$$

$$\sqrt{\frac{k^2}{4}} = 8^{-\frac{1}{3}}$$

$$\frac{k}{2} = 8^{-\frac{1}{3}} \quad M1$$

$$k = 2 \times 2^{-1}$$

$$k = 2^0$$

$$k = 1 \quad A1$$
Or $\frac{k^2}{4} = 8^{-\frac{2}{3}} \quad M1$

$$\frac{k^2}{4} = \frac{1}{4}$$

$$k^2 = 1$$

$$k = 1 \quad \text{or } k = -1 \text{(rejected)}$$

(a)[3] Answer

(b)[2]

- Solve the inequality $\frac{2}{3}x-1 < x \le 2(8-x)$.
 - Write down the largest rational number which satisfies $\frac{2}{3}x-1 < x \le 2(8-x)$. (b)

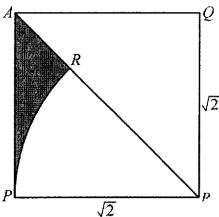
(b)
$$5\frac{1}{3} \quad B1$$

(a)[3] Answer

(b)[1]

In the diagram, AQBP is a square of side $\sqrt{2}$ cm. PR is an arc of a circle with centre at B and radius $\sqrt{2}$ cm.

Find the area of the shaded region APR, leaving your answer in terms of π .



Area of square
$$= (\sqrt{2})^2 = 2 \qquad M1$$
Area of quadrant
$$= \frac{1}{4}(\pi)(\sqrt{2})^2$$

$$= \frac{\pi}{2} \qquad M1$$
Area of shaded region
$$= \frac{1}{2} \times \left(2 - \frac{\pi}{2}\right)$$

$$= \left(1 - \frac{\pi}{4}\right) cm^2 \qquad B1$$

Area of
$$\triangle ABP = \frac{1}{2}(\sqrt{2})^2$$

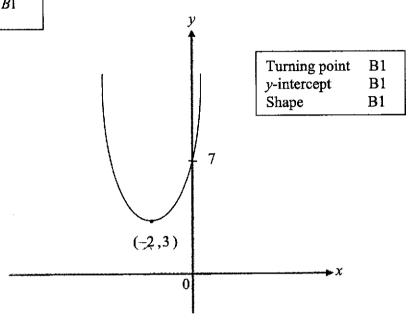
 $= 1 \text{ cm}^2$ M1
 $\angle ABP = 45 = \frac{\pi}{4}$
Area of sector $= \frac{1}{2}(\sqrt{2})^2 \left(\frac{\pi}{4}\right)$
 $= \frac{\pi}{4}$
Shaded area $= \left(1 - \frac{\pi}{4}\right) \text{ cm}^2$

Answercm² [3]

[3]

- 11 (a) Express $x^2 + 4x + 7$ in the form $(x+h)^2 + k$, where h and k are constants.
 - (b) Hence, sketch the graph of $y = x^2 + 4x + 7$ on the axes below, indicating its turning point and intercept(s) clearly.

(a) $x^2 + 4x + 7$ $= (x+2)^2 - 2^2 + 7$ $= (x+2)^2 + 3$ B1



Answer

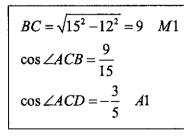
In the diagram, $\angle ABC = 90^{\circ}$, AB = 12 cm, AC = 15 cm, AD = 20 cm and BCDE is a straight line. Calculate, giving each answer as a fraction,

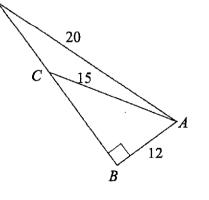
- (a) $\sin \angle ADE$,
- **(b)** $\cos \angle ACD$.

(a)

$$\sin \angle ADB = \frac{12}{20}$$
$$= \frac{3}{5} B1$$

(b)



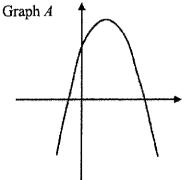


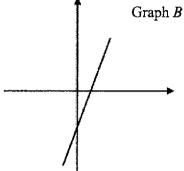
(a) [1]

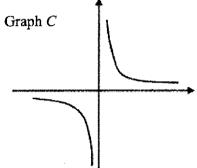
Answer (a)	****************	[1]
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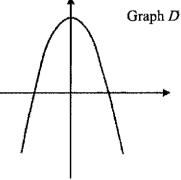
13 State the graph that correspond to each of following equation.

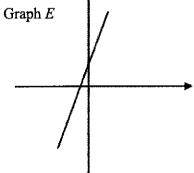
- **(i)** y = 3x - 7
- (ii)
- (iii)



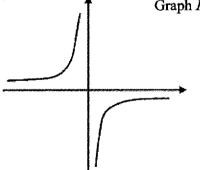








Graph F



Answer

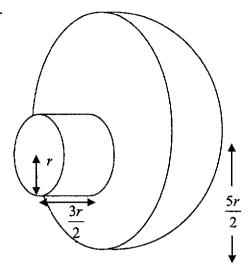
<i>(i)</i>			D	!								ı	Г1	
(i)	 	 	 D	١.,	 	 	 	٠.	٠		٠	٠	IJ	L,

14 The solid shown is made from a cylinder and a hemisphere.

The cylinder has a radius of r and height $\frac{3r}{2}$.

The hemisphere has a radius of $\frac{5r}{2}$.

Find an expression, in terms of π and r, For the total surface area of the solid.

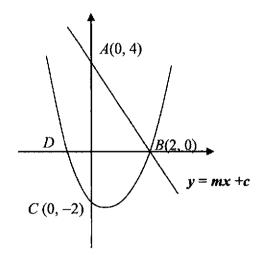


curved SA of hemisphere =
$$2\pi \left(\frac{5r}{2}\right)^2 = \frac{25}{2}\pi r^2$$
 M1
curved SA of cylinder = $2\pi r \left(\frac{3r}{2}\right) = 3\pi r^2$ M1
Total SA = $\frac{25}{2}\pi r^2 + 3\pi r^2 + \pi \left(\frac{5r}{2}\right)^2$
= $\frac{87}{4}\pi r^2 = 21\frac{3}{4}\pi r^2$ cm² A1

Answercm² [3]

15 The diagram below shows a straight line y = mx + c intersecting the y-axis at A(0, 4) and x-axis at B(2,0) and D.

The curve y = (x-2)(x+k) cuts the x-axis at B and the y-axis at C(0, -2).



- Find (a)
 - (i) the value of m and of c.
 - (ii) the length of AB.
 - (iii) the value of k.

(a)(i)
$$m = -2 B1$$

 $c = 4 B1$

$$\sqrt{2^2 + 4^2} = \sqrt{20}$$

(ii) $AB = = 4.4721...$
 $= 4.47 B1$

Sub (
$$(0,-2)$$

(iii) $\begin{array}{c} -2k = -2 \\ k = 1 & B1 \end{array}$

Answer	(a)(i) $m = \dots c = \dots [2]$
	(a)(ii)[1]
	(a)(iii)[1]

A second curve with the equation $y = ax^2 + bx + d$ cuts the x-axis at points (b)

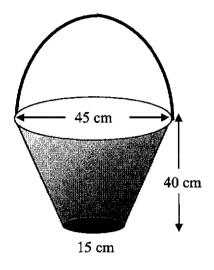
Determine whether the value of $b^2 - 4ad$ is positive, negative or zero. Explain your answer.

 b^2-4ad is positive because the curve cuts the x-axis at two intersection points. B1

Diagram I shows an open metal pail which is the shape of a frustum. The diameter of the two circular ends are 45 cm and 15 cm. The height of the pail is 40 cm.

The curved surface of the frustum can be found using the model in Diagram II.

- (a) By using appropriate similar triangles in Diagram II, find the value of h.
- (b) Hence, find the total area of metallic sheet used to make the pail.



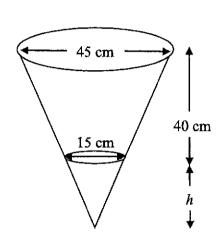


Diagram 1

Diagram II

$$\frac{h}{h+40} = \frac{15}{45} \quad M1$$

$$3h = h+40$$

$$2h = 40$$

$$h = 20 \quad A1$$

Slant height of cone =
$$\sqrt{60^2 + 22.5^2} = \sqrt{4106.25}$$
 M1
slant height of smaller cone = $\sqrt{20^2 + 7.5^2} = \sqrt{456.25}$
Curved SA of entire cone = $\pi(22.5)\sqrt{4106.25}$ M1
Curved SA of smallercone = $\pi(7.5)\sqrt{456.25}$
area of metallic cone = $\pi(22.5)\sqrt{4106.25}691 - \pi(7.5)\sqrt{456.25} + \pi(7.5)^2$ M1
= 4202.981497
= 4200 cm² (3 sf) A1

Slant height of cone = $\sqrt{60^2 + 22.5^2} = \sqrt{4106.25}$ M1 Curved SA of entire cone = $\pi(22.5)\sqrt{4106.25}$ M1

Curved SA of pail = $\frac{8}{9} \times \pi (22.5) \sqrt{4106.25} = 4026.26691$

Total area of metal sheet = $4026.26691... + \pi (7.5)^2$ M1 = 4202.981497= $4200 \text{ cm}^2 (3\text{sf}) \text{ A}1$

In the diagram, P, Q and R are three ports. P is 900 m due west of Q. At 07 40, a ship, Pioneer, 17 left P and travelled to R on a bearing of 130°. At the same time, another ship, Queen, left Q and travelled at an average speed of 15 km/h to R. It arrived at 08 10 on the same day.

Calculate

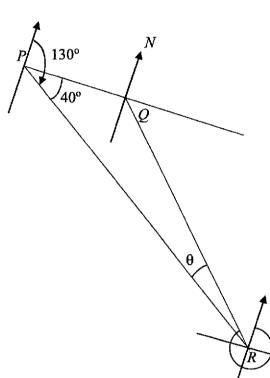
the distance QR, in metres, (a)

[1] the bearing of Q from R. [3] **(b)**

(a)
$$QR = \frac{1}{2} \times 15 \times 1000$$

= 7500 m B1

(b)



$$\frac{\sin \theta}{900} = \frac{\sin 40^{\circ}}{7500} \qquad M1$$

$$\sin \theta = \frac{900 \sin 40^{\circ}}{7500}$$

$$\theta = 4.42387...^{\circ}$$

Bearing of Q from $R = 270^{\circ} + 40^{\circ} + 4.42387...^{\circ}$ M1= 314.42387...° = 314.4° A1

Beatty Secondary School

End-of-Year Examination 2018

Secondary 3 Express

Mathematics Paper 2 (4048/02)

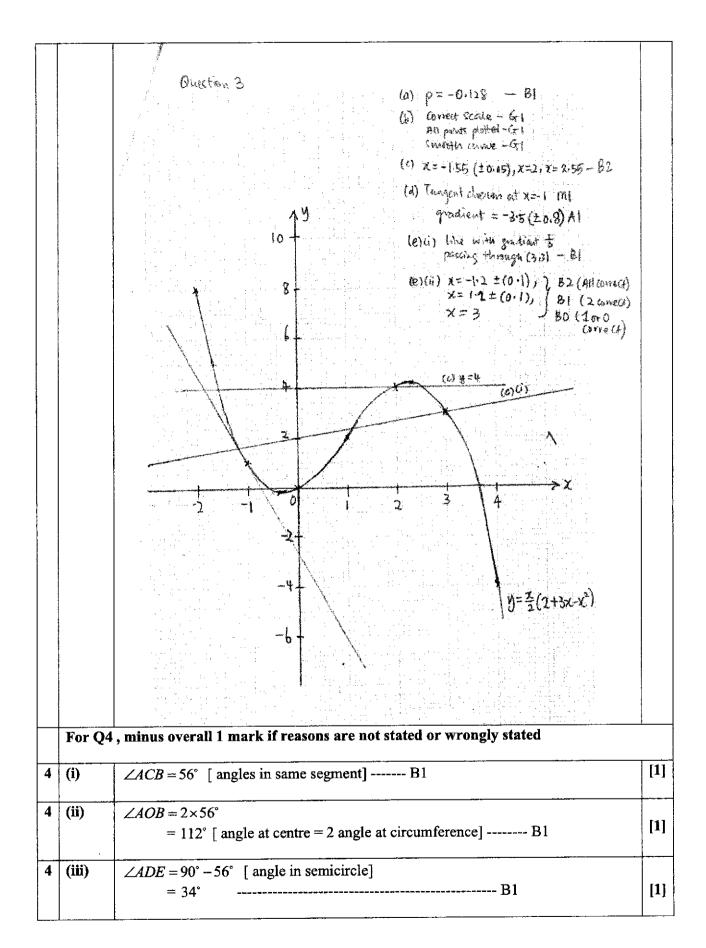
Setter: Mrs Samsol

Mark Scheme

1	(a)	$\frac{4xy - 8x^2}{y^2 - 4x^2}$ $= \frac{4x(y - 2x)}{(y + 2x)(y - 2x)}$	[2]
1	(b)(i)	$a = \frac{3b+c}{b-5c}$ $a = \frac{3(3)+(-1)}{3-5(-1)}$ $a = 1$ B1	[1]
1	(b)(ii)	$a = \frac{3b+c}{b-5c}$ $ab-5ac = 3b+c$ $ab-3b=c+5ac$ $b(a-3)=c+5ac$ $b = \frac{c+5ac}{a-3}$ A1	[2]
1	(c)	$\frac{6}{2x+5} = x-3$ $(2x+5)(x-3) = 6$ $2x^2 - x - 15 = 6$ $(2x-7)(x+3) = 0$ $x = 3\frac{1}{2}$ or $x = -3$ M1 [expansion] $x = 3\frac{1}{2}$ or $x = -3$ M1 [factorization]	[3]

2	(a)(i)	$F = \frac{k}{d^2}$, where k is a constant	
		$25 = \frac{k}{3^2}$ M1 [calculation of constant]	
		3^{2} $k = 25 \times 9$	
		$k = 25 \times 9$ $k = 225$	
		Equation is $F = \frac{225}{d^2}$ A1	[2]
2	(a)(ii)	$10 = \frac{225}{d^2}$	
		$\frac{10-\overline{d^2}}{d^2}$	
		$d^{2} = 22.5 - M1\sqrt{d}$ $d = \sqrt{22.5} [Accept \ d = \pm \sqrt{22.5} \]$ $d = 4.74 \text{ cm } (3 \text{ sf}) - A1$	
		$d = \sqrt{22.5}$ [Accept $d = \pm \sqrt{22.5}$]	
		[A0 if negative value is not rejected]	[2]
2	(a)(iii)	Given that $\frac{d_1}{d_2} = \frac{2}{3}$	
		$\frac{F_1}{F_2} = \left(\frac{k}{d_1^2}\right) \div \left(\frac{k}{d_2^2}\right)$	
		$\frac{F_1}{F_2} = \left(\frac{d_2}{d_1}\right)^2$ M1[calculation of ratio between force &	
		Distance]	
		$\frac{F_1}{F_2} = \left(\frac{3}{2}\right)^2$	
		$\frac{F_1}{F_2} = \frac{9}{4}$ or 9:4 A1	
		F_2 4	[2]
2	(b)	Divide octagon into 8 identical isosceles triangles, ABC.	
117.477		C = 50 m $A = B$	
		angle $ABC = \frac{360^{\circ}}{8}$ M1 $= 45^{\circ}$	

:		$\sin\left(\frac{45^{\circ}}{2}\right) = \frac{25}{AB} \qquad \qquad M1$	
	And the state of t	$AB = \frac{25}{\sin\left(\frac{45^{\circ}}{2}\right)}$ $AB = 65.328 \qquad A1$	
		area of $\triangle ABC = \frac{1}{2}(65.328)^2 \sin 45^\circ$ ————————————————————————————————————	
		Floor area of hall = 8×1508.87 =12070.96 = 12 100 m ² (3 sf) A1 Accept other alternatives	[5]
3	(a)	p = -0.128 B1	[1]
3	(b)	Correct scale	[3]
3	(c)	$x = -1.55 \ (\pm 0.05)$, $x = 2$ or $x = 2.55 \ (\pm 0.05)$	[2]
3	(d)	Tangent drawn at $(-1,1)$ M1 Gradient = -3.5 (± 0.8) G1	
3	(e)(i)	Straight line drawn B1	[2] [1]
3	(e)(ii)	x = -1.2, $x = 1.2$ or $x = 3$ (±0.1)	[2]



.4	Ges	(/DC //DE 0455 1 '	-
4	(iv) $\angle ABO = \angle ADE = 34^{\circ}$ [angle in same segment]		***************************************
		$\angle ABD = 2 \times 34^{\circ}$ [BE bisects angle ABD] M1	
:		= 68°	
		$\angle AED = 180^{\circ} - 68^{\circ}$ [angles in opposite segment]	
		= 112° A1	
			[2]
4	(22)	(DCC 2 248 F1 4) 1 2 / 1 3	
4	(v)	$\angle BGC = 3 \times 34^{\circ}$ [1 exterior angle = 2 interior opposite angles]	[1]
		$\angle BGC = 102$ B1	[*]
5	(a)	In triangles AQP and RQD,	
	(,	$\angle PQA = \angle DQR$ (vertically opposite angles)	
		$\angle APQ = \angle RDQ$ (PA parallel to RD, alternate angles) - any 2	
		$\angle PAQ = \angle DRQ$ (PA parallel to RD, alternate angles) B1	
		Therefore triangles AQP and RQD are similar. (AA property) B1	[2]
		Therefore triangles not una not are similar. (1211 property)	[2]
5	(b)(i)	AP 2	
		$\frac{AP}{DR} = \frac{2}{3} \qquad B1$	[1]
5	(b)(ii)	Area of $\triangle AOP = (2)^2$	
		$\frac{\text{Area of } \Delta AQP}{\text{Area of } \Delta RQD} = \left(\frac{2}{3}\right)^2$	
		$=\frac{4}{0}$	[11]
	(b)(iii)	1	[1]_
5	(-)()	$\frac{\text{Area of triangle } ARD}{\text{Area of rectangle } ABCD} = \frac{\frac{1}{2}(DR)(AD)}{(DC)(AD)}$	
		Area of rectangle $ABCD = \frac{2}{(DC)(AD)}$	
		$=\frac{1}{2}\left(\frac{DR}{DC}\right)$	
		1(2)	
		$=\frac{1}{2}\left(\frac{3}{4}\right)$	
		2(4)	
		$=\frac{3}{8}$ B1	[1]
		8	L-1
ı			1

5	(c)	Given triangle $AQP = 25.5 \text{ cm}^2$.		
		Area of triangle $AQD = \frac{3}{2}(25.5)$		
		$= 38.25 \text{ cm}^2$ M1 for knowing ratios of		
		triangles		
		Area of triangle $DQR = \frac{9}{4}(25.5)$		
		$= 57.375 \text{ cm}^2$		
		total area of triangles AQP , AQD and DQR = $25.5 + 38.25 + 57.375$		
		$= 121.125 \text{ cm}^2 - \text{A1}$	[2]	
		OR A ARCHA A COLLA DOR		
		$= \Delta APQ : \Delta AQD : \Delta DQR$ $= 4 : 6 : 9$ M1		
		Total area = $\frac{25.5}{4} \times 19$ [accept all other alternatives]		
		· .		
		$= 121\frac{1}{8} \text{ or } 121.125 \text{ cm}^2 A1$		
6	(a)	Time taken = $\frac{8000}{2}$ minutes B1	[1]	
		*		
6	(b)	Time taken for small pump = $\frac{8000}{x-50}$ minutes B1		
6	(c)	$\frac{8000}{r-50} - \frac{8000}{r} = 35 - \text{M1}$		
		x-50 x		
		8000x - 8000(x - 50)		
		$\frac{8000x - 8000(x - 50)}{x(x - 50)} = 35$		
		$400000 = 35x(x-50) M1\sqrt{35x^2 - 1750x - 400000} = 0$		
		Divide by 5		
		$7x^2 - 350x - 80000 = 0 \qquad A1$	[3]	
6	(d)	$-(-350) + \sqrt{(-350)^2 - 4(7)(-80000)}$		
	` ′	$x = \frac{-(-350) \pm \sqrt{(-350)^2 - 4(7)(-80000)}}{2(7)} $ M1		
		x = 134.7887 or $x = -84.78875$	[2]	
		x = 134.79 or $x = -84.79$ (correct to 2 d p) A2	[3]	
6	(e)	Time taken by small pump = $\frac{8000}{124.788 - 50}$		
		134.786-30		
	West of the second	= 94.35297 minutes B1 = 94 minutes 21 seconds B1 [2]		

	-			
7	(a)(i)	$AC = \sqrt{550^2 + 825^2 - 2(550)(825)\cos 65^\circ}$ M1		
		AC = 774.33		
		AC = 774 m (3 sf) A1		
_	() (4)			
7	(a)(ii)	$\frac{\sin \angle ACB}{\cos ACB} = \frac{\sin 65}{\cos ACB}$		
		550 774.33		
		$\angle ACB = \sin^{-1}\left(\frac{550\sin 65}{774.33}\right)$ M1		
		$\angle ACB = 40.071$ Bearing of A from $C = 360 - 40.071$ M1		
		= 319.929		
		= 319.9 (to 1 dp) A1	[2]	
7	(a)(iii)		[3]	
		$\sin 40.071 = \frac{d}{825}$		
		$d = 825 \sin 40.071$ M1 $$		
		d = 531.08		
		shortest distance = 531 m (to 3 sf) A1		
		Accept other $\frac{1}{2} \times 774.33 \times d = \frac{1}{2} (550)(825) \sin 65 \text{alternatives}$		
		$d = \frac{205618.5792 \times 2}{774.337}$		
		d = 531.08	[2]	
		d = 531 mA1		
7	(b)	$\tan 9 = \frac{h}{550}$		
		$\tan 9 = \frac{1}{550}$		
		$h = 550 \tan 9$ M1		
		550 tan 9		
		$\tan \theta = \frac{550 \tan 9}{774.33} M1$		
		$3 + 3 \left(550 \tan 9^{\circ} \right)$		
		$\theta = \tan^{-1}\left(\frac{550\tan 9^{\circ}}{774.33}\right)$		
		$\theta = 6.418^{\circ}$	[3]	
		Angle of elevation of P from $C = 6.4^{\circ}$ (to 1 dp) A1		
		- ·		

8	(a)	25 m/s = 25 x 3.6			
		= 90 km/h B1	[1]		
8	(b)	Acceleration = $\frac{18}{8}$			
		$= 2\frac{1}{4} \text{ m/s}^2 - B1$			
		4			
8	(c)	Distance travelled for 21 seconds:			
		$\frac{1}{2}(8)(18) = 72$			
		9×18=162 M1 [area under curve]			
		$\frac{1}{2}(18+25)(4) = 86$			
		2 2 -			
		Total distance = 72 +162 +86			
		= 320 m			
		Distance for 21 seconds			
		$= \frac{1}{2}(17+9)(18) + \frac{1}{2}(18+25)(4)$			
		$\frac{1}{2}(t-21)(25) = 420 - 320$	[2]		
		$\begin{bmatrix} \frac{1}{2}(t-21)(23)-420-320\\ t-21=8 \end{bmatrix}$			
		t-21=8 $t=29$			
8	(d)	Distance (m)			
		<u></u>			
		420			
		320			
		234			
		72 0 8 17 21	[4]		
		Time (seconds)			
B4 - I mark for each part of the graph					

		Γ				
		Giva D2	if distances are not indi	antad on vertical axis		
9	(a)	Give B3 if distances are not indicated on vertical axis. $23\theta + 9.5\theta + 2(23 - 9.5) = 64 M1 [\text{ sum of 2 arc} = 37]$			•	
	(4)					
		$32.5\theta = 37$				
		$\theta = \frac{74}{65}$				
		"	<i>.</i> 74			
		Angle DOC	$r = \frac{7}{65}$ radians [acce	pt 1.14 rad]	A 1	[2]
			03			
9	(b)	1	.(74) 1 .(74)	1	
		Area = $\frac{1}{2}$ (2	$\left \frac{3}{65} \right = \frac{1}{2} (9.5)^2 \left \frac{7}{65} \right $)	M2 √	
				•	1	
		= 249	$\frac{3}{4}$ cm ²		A1	
		Or	7			
			2/4 1004 1/0 5/2/4	1004)	1	
		Area = $-(2)$	$(1.1384) - \frac{1}{2}(9.5)^{3}(1$.1384) M2	. Y	
		= 250	cm ² (to 3 sf)	A1		[3]
9	(c)					[4]
		Material	Amount required	cost		
		A4 paper	$\frac{100\times4}{50} = 8 \text{ packs}$	$8 \times 7.10 = 56.80$		
		Marcal		$10 \times 2.40 = 24.00$		
		Metal ring	$\frac{100}{10} = 10 \text{ packs}$	$10 \times 2.40 = 24.00$		
		Bamboo		30 × 11,40=342.00		
		sticks	$\frac{100\times14}{48}$ = 29.16	30 × 11,40-342.00		
		Glue		$9 \times 5 = 45.00$		
			$\frac{100}{12} = 8.33$	7 / 3 = 45.00		
			P1 – calculation of	Q1 – calculation of		
			amount of materials	cost		
		Average o	a ost for 1 fan = $\frac{56.80 + 1}{100}$	24.00+342.00+45.00		
ŀ		/ I voiago c	OUT THE	100		
			$=\frac{467.80}{}$			
			100			
			= \$4.678	670 107 P4 F 6		
] :		price of 1 fa	$\sin \text{ with GST} = 2.5 \times 4$ = \$ 12.51365	.678 × 1.07 R1 [fo	r using 2.5]	
			- φ 12.313UJ			
		Accept sug	gested selling price of S	\$ 12.52 or more	S1	
			_			
			4.4		Terr Address	
	-	Accept alternative workings			į	
						į

Material	Amount required	Cost (including 7% GST)
A4 paper	$\frac{100\times4}{50} = 8 \text{ packs}$	$8 \times 7.10 \times 1.07 = 60.78$
Metal ring	$\frac{100}{10} = 10 \text{ packs}$	$10 \times 2.40 \times 1.07 = 25.68$
Bamboo sticks	$\frac{100 \times 14}{48} = 29.16$	30 × 11.40 × 1.07 = 365.94
Glue	$\frac{100}{12} = 8.33$	$9 \times 5 \times 1.07 = 48.15$
	P1 – calculation of amount of materials	Q1 – calculation of cost