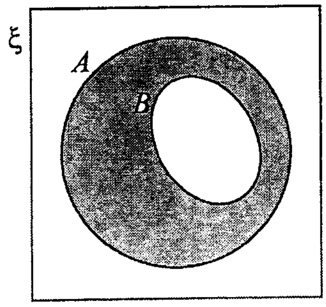
Answer key St. Gabriel

1a	$\dfrac{11x-6}{(x-2)^2}$
1b	$x=2,\ y=-4$
1c	$\dfrac{-1}{x-3}$ or $\dfrac{1}{3-x}$
1d	$x=-12$
1e	$x \le -\dfrac{1}{7}$
1f	$x=-\dfrac{1}{7}$
2ai	540 m
2aii	381 m
2b	236.6°
3a	$\begin{pmatrix} 6 \\ 8 \\ 10 \end{pmatrix}$
3bi	$\begin{pmatrix} 64 \\ 190 \end{pmatrix}$
3bii	The elements represent the total amount collected from the sales of men's and womens' t-shirts respectively.
3ci	$\begin{pmatrix} 115 & 202.5 & 51.5 \end{pmatrix}$
3cii	$\begin{pmatrix} 115 & 202.5 & 51.5 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 369 \end{pmatrix}$
4a	$b=4.75$
4d	$-2.4 \le x_1 \le -2.2,\ x_2=0,\ 2.2 \le x_3 \le 2.4$
4e	$A=3,\ B=16$
5ai	$603\ \text{cm}^3$
5aii	$17.0\ \text{cm}^2$
5bi	$7.33\ \text{cm}^3$
5bii	82
6ai	$\angle ADE = 90° + \angle ADG$ $\angle CDG = 90° + \angle ADG$ $\therefore \angle ADE = \angle CDG$
6aii	$AD = CD$ (sides of square $ABCD$) $\angle ADE = \angle CDG$ [from 6(a)(i)] $DE = DG$ (sides of square $DEFG$) $\therefore \Delta ADE \equiv \Delta CDG$ (SAS)
6bi	$\angle ADE = 26°$ ($\angle$s in same segment)
6bii	51°
6biii	103°

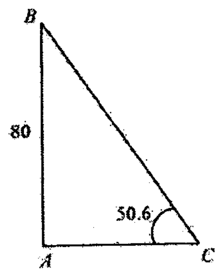
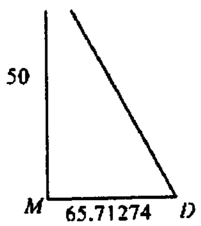
7a	48.1°
7b	14.7 cm
8a	$a=3$
8bi	0.949 kg
8bii	The **mid-values** of the masses are used instead as the **exact** masses were not known.
8c	The masses of durians from shop *B* are **heavier (or have higher mass)** than shop *A* because the mean mass of durians from shop *B* (4.4kg) is **greater** than shop *A* (3.5kg). The masses of durians from shop *B* have a **greater** spread than shop *A* because the standard deviation of the masses of durians from shop *B* (1.2kg) is **greater** than shop *A* (0.949kg).
9ai	$\begin{pmatrix} 3 \\ 10 \end{pmatrix}$
9aii	10.4 units
9bi	$-2\mathbf{a}$
9bii	$\mathbf{a}-\mathbf{b}$
9d	$\overrightarrow{DF}=\frac{1}{5}(5\mathbf{b}+\mathbf{a})$ $\overrightarrow{DE}=\frac{1}{3}(5\mathbf{b}+\mathbf{a})$ Since $\overrightarrow{DF}=k\overrightarrow{DE}$, $\overrightarrow{DF}$ is parallel to $\overrightarrow{DE}$ and ***D* is a common point**, hence *D*, *E*, and *F* lie on the same straight line.
9e	$\frac{4}{9}$
10ai	$\frac{1}{2}$
10aii	$\frac{\pi}{25}$ or 0.04π
10bi	$\frac{1}{4}$
10bii	$\frac{1}{4}$
10biii	$\frac{7}{16}$
11a	363 miles
11b	6 hrs 10 mins
11c	Recommend Bus Advantage: Lower cost by bus ($71.52) than train ($136) Disadvantage: Longer travelling time by bus (7 hours 40 min) than train (6 hours 10 min) OR Recommend Train Advantage: Shorter travelling time by train (6 hours 10 min) than bus (7 hours 40 min) Disadvantage: Higher cost by train ($136) than bus ($71.52)

2022 Sec 4E5N/4NA(OOS)
EM PRELIM P1
Marking Scheme with Marker's Report

Solutions:			
1	$\sqrt{-\frac{35}{27}-\left(\frac{-11^2}{81}\right)}=\frac{4}{9}$		
2	Different scales used for the vertical axes.		
3a	Diagram 2		
3b	Height of cylindrical part of the container / Depth of water when the cylindrical part of the container is fully filled up		
4a	$4+7x-x^2=-(x^2-7x-4)$ $=-\left[x^2-7x+\left(\frac{-7}{2}\right)^2-\left(\frac{-7}{2}\right)^2-4\right]$ $=-\left[\left(x-\frac{7}{2}\right)^2-\frac{65}{4}\right]$ $=-\left(x-\frac{7}{2}\right)^2+\frac{65}{4}$		
4b	Max value $=\frac{65}{4}$		
5a	$\frac{10+n}{16+n}$		
5b	$\frac{10+n}{16+2n}=\frac{9}{16}$ $160+16n=144+18n$ $2n=16$ $n=8$ Total number of cubes = 32		
6	$\angle AOC=360°-216°$ ($\angle$ at a point) $=144°$ $\angle BOC=\angle OBA$ $=90°$ (tan ⊥ rad, alt. $\angle$s, $OC // AB$) $\angle AOB=144°-90$ $=54°$		
7	$\frac{3-2x}{4}=6-\frac{x+5}{7}$ $\frac{3-2x}{4}=\frac{42-x-5}{7}$ $21-14x=148-4x$ $10x=-127$ $x=-12.7$		

Solutions:			
8a			
8b	1, 9, 25, 49, 81		
8c	0		
9a	$x=15, y=19, z=23$		
9b	$4n+7$ or $11+4(n-1)$		
10a	$2x^{-3n}=2\left(x^{n}\right)^{-3}$ $=\frac{2}{1000}$ $=\frac{1}{500}$ or 0.02 o.e		
10b	$\left(\frac{8m^3}{n^{-6}}\right)^{\frac{1}{3}} \div \frac{m^{-4}}{n^3}=\frac{2m}{n^{-2}}\times\frac{n^3}{m^{-4}}$ $=2m^5n^5$		
11a	7		
11b	$y=\sqrt{9-4x}$ $y^2=9-4x$ $4x=9-y^2$ $x=\frac{9-y^2}{4}$ or $x=-\frac{y^2-9}{4}$ or $x=\frac{(3+y)(3-y)}{4}$		
12	$p=\frac{360}{10}=36$ $q=\frac{36}{2}=18$ $r=36+18=54$		
13a	smallest $x=27$ corresponding $y=15$		
13b	$y=50-(26+13)$ $=11$		
14a	Total in US\$ $=5000\left(1+\frac{2.5}{100}\right)^2$ $=5253.125$		

Solutions:			
	$= US\$5253.13$		
14b	In Jan 2020, US\$5000 = 1.3453 × 5000 = S\$6726.50 In Jan 2022, US\$5253.125 = 5253.125 ÷ 0.7415 = S\$7084.46 Mr Lim made a gain		
15a	$\sqrt{36}$ cm : $\sqrt{9}$ km 6cm : 3km 6cm: 300000cm 1:50000 $n = 50000$		
15b	$1.64 \text{ km} = 164000 \text{ cm}$ Length of road $= \dfrac{164000}{50000}$ $= 3.28$ cm		
16a	$(2a-3)^2 - 4a(a-4)$ $= 4a^2 - 12a + 9 - 4a^2 + 16a$ $= 4a + 9$		
16b	$14x^2 - 7xy + 3ay - 6ax$ $= 7x(2x-y) + 3a(y-2x)$ $= 7x(2x-y) - 3a(2x-y)$ $= (7x-3a)(2x-y)$ or any equivalent form		
17a	$1188 = 2^2 \times 3^3 \times 11$		
17b	$m = 2 \times 11^2$ $= 242$		
17c	$360 = 2^3 \times 3^2 \times 5$ HCF of 360 and 1188 = $2^2 \times 3^2$ $= 36$		
18ai	$\angle ECD = 76° - 34°$ $= 42°$ $\angle EBA = \angle DCA$ (Corresponding Angles, $BE//CD$) Or $\angle BED = \angle ECD$, Alternate Angles, $BE//CD$		
18aii	$\angle EDC = 180° - 34° - 76°$		

Solutions:			
	$= 70°$ **Angle sum of triangle**		
18b	EC is the longest side because it is <u>opposite the largest interior angle.</u> $\angle EDC = 70°$ $\angle CED = 68°$ $\angle ECD = 42°$ Or $\frac{EC}{\sin \angle EDC} = \frac{DC}{\sin \angle DEC} = \frac{ED}{\sin \angle ECD}$ $\frac{EC}{\sin 70^o} = \frac{DC}{\sin 68^o} = \frac{ED}{\sin 42^o}$ Since $\sin 70^o > \sin 68^o > \sin 42^o$ $EC > DC > ED$		
19	 $\frac{AB}{AC} = \tan 50.6°$ $AC = \frac{80}{\tan 50.6°}$ $=65.71274$  $\frac{50}{65.71274} = \tan x^o$ $x^o = \tan^{-1} \frac{50}{65.71274}$ $=37.267^0$ $=37.3^0$ (1d.p)		

Solutions:			
20a	$VX^2 = OX^2 + VO^2$ (Pythagoras' Theorem) $VX = \sqrt{3.5^2 + 8^2}$ $VX = \sqrt{76.25}$ or 8.7321 (5 s.f.) Total surface area of the pyramid = $4\left(\frac{1}{2} \times 7 \times \sqrt{76.25}\right) + 7^2$ $= 171.2497444$ $= 171 \text{ cm}^2$ (correct to 3 s.f.)		
20b	Vol. of the Pyramid $= \frac{1}{3} \times 7^2 \times 8$ $= 130\frac{2}{3} \text{ cm}^3$ Vol. of 1 sphere $= \frac{4}{3} \times \pi \times (0.2)^3$ $= \frac{4}{375}\pi \text{ cm}^3$ or $\frac{32}{3}\pi \text{ mm}^3$ Maximum number of spheres $= 130\frac{2}{3} \div \frac{4}{375}\pi$ = 3899.29 (5 s.f.) = 3899 (round down)		
21i	\$2.70		
21ii	IQR = \$3.70-\$2.00 \$1.70		
21iii	**Acceptable Answers** School B, since the median of donations of School B (\$3.00) is higher the median of donations of School A (\$2.70). School B has a lower interquartile range of (\$1.30) compared to School A (\$1.70), donations are more consistent/less widespread with the greater donations than School A. School B has a higher upper quartile (\$4) than School A (\$3.60), so 25% of students in School B donated \$4 of more compared to less than 25% of students in School A.		

Solutions:			
22ai	$\overrightarrow{DC}=3\mathbf{a}$		
22aii	$\overrightarrow{DA}=\overrightarrow{DC}+\overrightarrow{CA}$ $=3\mathbf{a}+2\overrightarrow{CM}$ $=3\mathbf{a}+4\mathbf{b}$		
22aii	$\overrightarrow{DX}=\frac{1}{5}\overrightarrow{DA}$ $=\frac{1}{5}(3\mathbf{a}+4\mathbf{b})$		
22b	$\overrightarrow{BX}=\overrightarrow{BD}+\overrightarrow{DX}$ $=\mathbf{a}+\frac{1}{5}(3\mathbf{a}+4\mathbf{b})$ $=\mathbf{a}+\frac{3}{5}\mathbf{a}+\frac{4}{5}\mathbf{b}$ $=\frac{8}{5}\mathbf{a}+\frac{4}{5}\mathbf{b}$ $=\frac{4}{5}(2\mathbf{a}+\mathbf{b})$ $\overrightarrow{BX}=\frac{4}{5}(2\mathbf{a}+\mathbf{b})$ (Shown)		
22c	$\overrightarrow{BM}=\overrightarrow{BC}+\overrightarrow{CM}$ $=4\mathbf{a}+2\mathbf{b}$ $=2(2\mathbf{a}+\mathbf{b})$		
22di	$\frac{BX}{BM}=\frac{\lvert\frac{4}{5}(2\mathbf{a}+\mathbf{b})\rvert}{\lvert 2(2\mathbf{a}+\mathbf{b})\rvert}=\frac{2}{5}$		
22dii	$\frac{\text{area of }\Delta ABX}{\text{area of }\Delta AMX}=\frac{\frac{1}{2}\times BX\times\perp h}{\frac{1}{2}\times MX\times\perp h}=\frac{BX}{MX}$ $=\frac{2}{3}$		
22iii	$\frac{\text{area of }\Delta ABX}{\text{area of }\Delta ABC}=\frac{\text{area of }\Delta ABX}{\text{area of }\Delta ABM}\times\frac{\text{area of }\Delta ABM}{\text{area of }\Delta ABC}=\frac{2}{5}\times\frac{1}{2}$ $=\frac{1}{5}$		

2022 Sec 4E5N/4NA(OOS)
EM Prelim P2
Marking Scheme with Marker's Report

Solution:			
1a	$\frac{9x-2}{x^2-4x+4}+\frac{2}{x-2}$ $=\frac{9x-2}{(x-2)^2}+\frac{2}{x-2}$ --- factorization of denominator $=\frac{9x-2}{(x-2)^2}+\frac{2(x-2)}{(x-2)^2}$ --- correct denom. & numerator $=\frac{11x-6}{(x-2)^2}$		
1b	Any method to solve either substitution or elimination $x=2,\ y=-4$ --- A1 each		
1c	$\frac{(3x+2)(3x-2)}{(3x+2)(x-3)}\times\frac{1}{(2-3x)}$ $=\frac{-(3x+2)(2-3x)}{(3x+2)(x-3)}\times\frac{1}{(2-3x)}$ $=\frac{-1}{x-3}$ or $\frac{1}{3-x}$		
1d	$(2^3)^{1-2x}=(2^5)^{3-x}$ Comparing indices:- $3(1-2x)=5(3-x)$ $3-6x=15-5x$ $x=-12$		
1e	$1-5+2x<\frac{3}{2}x+\frac{1}{2}$; $\frac{3}{2}x+\frac{1}{2}\le\frac{4}{5}x+\frac{2}{5}$ $\frac{1}{2}x<4\frac{1}{2}$; $\frac{7}{10}x\le-\frac{1}{10}$ $x<9$; $x\le-\frac{1}{7}$ $x\le-\frac{1}{7}$		
1f	$x=-\frac{1}{7}$		

Solutions:			Marker's comments
2ai	$QS=\sqrt{420^2+340^2}=\sqrt{292000}$ $=540.37$ $\approx 540\text{ m}$		
2aii	$\angle RSQ=\tan^{-1}\dfrac{420}{340}$ $=51.009°$ $\angle QSP=360°-21.009°-280°$ *(∠s at a point)* $=28.991°$ $PQ=\sqrt{750^2+\left(\sqrt{292000}\right)^2-2(750)(\sqrt{292000})\cos 28.991}$ $=381.571$ $=381\text{ m}$		
2b	$\dfrac{\sin\angle PQS}{750}=\dfrac{\sin 28.991°}{381.46}$ $\angle PQS=72.35°(acute)$ (72.33°) $obtuse\ \angle PQS=180°-72.35°$ $=107.65°$ (107.67°) *or Find using Cosine rule* $\angle RQS=180°-90°-51.009°$ *(∠ sum of Δ)* $=38.991°$ Bearing required $=128.991°+107.65°$ $=236.6°$ (236.7°)		
3a	$\begin{pmatrix}6\\8\\10\end{pmatrix}$		
3bi	$\begin{pmatrix}64\\190\end{pmatrix}$		
3bii	The elements represent the total amount collected from the sales of men's and womens' t-shirts respectively.		
3ci	$\begin{pmatrix}115 & 202.5 & 51.5\end{pmatrix}$		
3cii	$\begin{pmatrix}115 & 202.5 & 51.5\end{pmatrix} = \begin{pmatrix}369\end{pmatrix}$ *or* $\begin{pmatrix}10 & 11.5\end{pmatrix}\begin{pmatrix}7\\26\end{pmatrix} = \begin{pmatrix}369\end{pmatrix}$ **Total amount received = $369**		

Solution:			
4a	b = 4.75		
4b	All points plotted correctly Correct Labelled Axis & Scale *Smooth* curve drawn with curve ruler passing through all points		
4c	Ruled straight line through (0,1) Must show gradient = $\frac{1}{3}$		
4d	$-2.4 \le x_1 \le -2.2,\ x_2 = 0,\ 2.2 \le x_3 \le 2.4$		
4e	$A=3,\ B=16$ $\frac{1}{3}x+1=\frac{x^3}{4}-x+1$ $4x+12=3x^3-12x+12$ $3x^3=16x$		
5ai	Vol of solid $P=\frac{2}{3}\times\pi\times 12^3$ $=1152\pi$ $=3619.114737\text{ cm}^3$ Vol of solid $Q=3619.114737 \div 6$ $\approx 603\text{ cm}^3$		
5aii	Slant height of cone $=\sqrt{2}$ cm Total Curved surface area of solid R $=(\pi\times1\times\sqrt{2})+(2\times\pi\times1\times2)$ $=17.0\text{ cm}^2$		
5bi	Vol of solid R $=(\frac{1}{3}\times\pi\times1^2\times1)+(\pi\times1^2\times2)$ $=7.33038\text{ cm}^3$ $=7.33\text{ cm}^3$		
5bii	Number of complete solid R that can be obtained = 82 (nearest whole number)		
6ai	$\angle ADE=90°+\angle ADG$ $\angle CDG=90°+\angle ADG$		

Solution:			
	$\therefore \angle ADE = \angle CDG$		
6aii	$AD = CD$ (sides of square $ABCD$) $\angle ADE = \angle CDG$ [from 6(a)(i)] $DE = DG$ (sides of square $DEFG$) $\therefore \Delta ADE \equiv \Delta CDG$ (SAS)		
6bi	$\angle ADE = 26°$ ($\angle$s in same segment)		
6bii	$\angle AED = 180 - 78$ (adj. $\angle$s on a st. line) $= 102°$ $\angle ACD = 180 - 102$ ($\angle$s in opp. segments) $= 78°$ $\angle CDA = \dfrac{180° - 78°}{2}$ (base $\angle$ of isos. triangle) $= 51°$		
6biii	$\angle CAE = 180 - 26 - 51$ (angles in opp. segments) $= 103°$		
7a	48.1°		
7b	Arc length, $PR = 6(0.84)$ $= 5.04$ cm $OQ^2 = 6^2 + 6.7^2$ $OQ = 6^2 + 6.7^2$ $= 8.9939$ Answers may vary slightly due to students using either Trigo Ratios, Pythagoras' Theorem or Cosine Rule to find OQ. $RQ = 8.9939 - 6$ $= 2.9939$ Perimeter $= 5.04 + 2.9939 + 6.7$ $= 14.7$ cm		
8a	$\dfrac{2\times1.5 + a\times2.5 + 8\times3.5 + 7\times4.5}{17+a} = 3.5$ $2.5a + 62.5 = 59.5 + 3.5a$ $a = 3$		
8bi	0.949 kg		

Solution:			
8bii	The **mid-values** of the masses are used instead as the **exact** masses were not known. Masses were given in range. (adjusted answer key)		
8c	The masses of durians from shop *B* are **heavier (or have higher mass)** than shop *A* because the mean mass of durians from shop *B* (4.4kg) is greater than shop *A* (3.5kg). The masses of durians from shop *B* have a **greater** spread than shop *A* because the standard deviation of the masses of durians from shop *B* (1.2kg) is greater than shop *A* (0.949kg). Or The masses of durians from shop *B* are less consistent than shop *A* because the standard deviation of the masses of durians from shop *B* (1.2kg) is greater than shop *A* (0.949kg).		
9ai	$\overrightarrow{AD}$ $=\overrightarrow{OD}-\overrightarrow{OA}$ $=\begin{pmatrix}-4\\5\end{pmatrix}-\begin{pmatrix}-7\\-5\end{pmatrix}$ $=\begin{pmatrix}3\\10\end{pmatrix}$		
9aii	$\lvert\overrightarrow{AD}\rvert$ $=\sqrt{3^2+10^2}$ $=10.4$ units		
9bi	$-2\mathbf{a}$		
9bii	$\mathbf{a}-\mathbf{b}$		

Solution:			
9c	$\overrightarrow{DE}=\overrightarrow{DB}+\overrightarrow{BE}$ $=2\mathbf{b}+\frac{1}{3}(\mathbf{a}-\mathbf{b})$ $=2\mathbf{b}+\frac{1}{3}\mathbf{a}-\frac{1}{3}\mathbf{b}$ $=\frac{5}{3}\mathbf{b}+\frac{1}{3}\mathbf{a}$ $=\frac{1}{3}(5\mathbf{b}+\mathbf{a})$ (shown) or Also accepted: $\overrightarrow{DE}=\overrightarrow{DA}+\overrightarrow{AE}$ $=\mathbf{b}+\mathbf{a}+\frac{2}{3}(\mathbf{b}-\mathbf{a})$ $=\frac{1}{3}(5\mathbf{b}+\mathbf{a})$ (shown)		
9d	$\overrightarrow{DF}=\overrightarrow{DO}+\overrightarrow{OF}$ $=\mathbf{b}+\frac{1}{5}\mathbf{a}$ $=\frac{1}{5}(5\mathbf{b}+\mathbf{a})$ Must show same vector as 9c $\overrightarrow{DE}=\frac{1}{3}(5\mathbf{b}+\mathbf{a})$ Or $=\frac{5}{3}\left(\mathbf{b}+\frac{1}{5}\mathbf{a}\right)$ $=\frac{5}{3}\overrightarrow{DF}$ Since $\overrightarrow{DF}=k\overrightarrow{DE}$, $\overrightarrow{DF}$ is parallel to $\overrightarrow{DE}$ and ***D* is a common point**, hence *D*, *E*, and *F* lie on the same straight line.		
9e	$\frac{4}{9}$		
10ai	$\frac{1}{2}$		
10aii	$\frac{\pi}{25}$ or 0.04π		

Solution:			
10aiii	$1-\frac{1}{2}-\frac{1}{4}-\frac{\pi}{25}$ $=\frac{1}{4}-\frac{\pi}{25}$ $=\frac{25-4\pi}{100}$ (shown) or P(Red) = P(Yellow-Blue) $=\frac{1}{4}-\frac{\pi}{25}$ $=\frac{25-4\pi}{100}$ (shown)		
10bi	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$		
10bii	$\left(\frac{1}{2}\times\frac{1}{4}\right)+\left(\frac{1}{4}\times\frac{1}{2}\right)$ $=\frac{1}{4}$		

Solution:			Marker's comments
10biii	P(at least 1 dart hit yellow area) = 1− P(both darts not hitting yellow area) $=1-\left(\frac{3}{4}\right)^2$ $=\frac{7}{16}$		
11a	1.6093 km → 1 mile 1 km → $\frac{1}{1.6093}$ miles 584.6 km → 363 miles		
11b	Time taken for train journey $=\frac{584.6}{94.8}$ $=\frac{37}{6}$ h = 6 hrs 10 mins		

Solution:			Marker's comments
11c	**By Train** Both Glen and Jane can depart only at 09 27 from Berlin in order to reach Munich latest by 4pm. Cost of train ride $= 68 \times 2$ $= \$136$ **By Bus** Both Glen and Jane have to take the bus via Bayreuth from Berlin in order to reach Munich latest by 4pm. Duration of bus ride = 7 hrs 40 mins Distance covered by bus in miles $= 7\frac{2}{3} \times 45$ $= 345$ miles $\times 1.6093$ $= 555.2085$ km Cost of bus ride $= [8 + (555.2085 \times 0.05) \times 2$ $= \$71.52$ Recommend Bus Advantage: Lower cost by bus ($71.52) than train ($136) Disadvantage: Longer travelling time by bus (7 hours 40 min) than train (6 hours 10 min) OR Recommend Train Advantage: Shorter travelling time by train (6 hours 10 min) than bus (7 hours 40 min) Disadvantage: Higher cost by train ($136) than bus ($71.52)		