Bedok View

Paper 1

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Solve the simultaneous equations

[4]

$$y - x = 4,$$
$$4x^2 - 6xy - 6y = 4.$$

2 Solve the equation $9\sin x = -2\sec x$ for $-\pi \le x \le 0$. [4]

For Examiner's Use

3 **(a)** Explain why the principal value of $\cos^{-1}\left(-\dfrac{\sqrt{2}}{2}\right)$ cannot be $-\dfrac{\pi}{4}$. [1]

[Turn over

For Examiner's Use

(b) Prove that $\cot A + \tan A = \sec A \operatorname{cosec} A$. [4]

For Examiner's Use

4 Express $\dfrac{2x^3+4}{x(x+2)^2}$ in partial fractions. [5]

[Turn over

For Examiner's Use

5 The function f is given by $f(x) = \dfrac{x + a}{x^2 + 3}$.

(a) Find $f'(x)$. [2]

It is given that f increases for $b < x < 1$.

(b) Find the values of a and b. [4]

For Examiner's Use

6 Find the set of values of the constant k for which the curve $y=(k-6)x^2-8x+k$ does not intersect the x-axis and has a minimum point. [6]

[Turn over

For Examiner's Use

7 **(a)** Find $\frac{\mathrm{d}}{\mathrm{d}x}\left[(x-1)\mathrm{e}^{-x}\right]$. [2]

(b) Hence find $\int x\mathrm{e}^{-x}\,\mathrm{d}x$. [4]

For Examiner's Use

8 The radius of a circle, r cm, decreases at a rate of $\frac{2}{(t+1)^2}$ cm per minute.

(a) Given that the initial radius is 4 cm, find an expression for r in terms of t. [3]

(b) Find the rate of change of the area of the circle when the radius is 2.6 cm. [3]

[Turn over

For Examiner's Use

9 **(a)** Find the value of the constant k such that the line $y = kx + 6$ is a tangent to the curve $2x^2 - xy = 3$. [3]

(b) Solve $\left(4 - 2\sqrt{2}\right)x + \sqrt{2} = 3x\sqrt{2} - 1$, giving your answer in the form $a\sqrt{2} + b$, where a and b are rational numbers. [4]

For Examiner's Use

10 An isosceles triangle PQR has vertices $P(1, 0)$, $Q(0, 2)$ and $R(2, 3)$.
$PQ = QR$.

(a) Find the area of the triangle PQR. [2]

(b) If S is a point such that $PQRS$ is a kite and it also lies on the line $x - y = 5$, find the coordinates of S. [5]

11 **(a)** Solve the equation $2^x + 4^{x-1} = 24$. [4]

[Turn over

For Examiner's Use

(b) Solve the equation $\log_{\sqrt{3}}(x-1) = \log_3(x+5)$. [4]

For Examiner's Use

12 The gradient at any point on a curve y is given by $6x - \frac{1}{kx^3}$. The line $4x - 4y = 3$ is a normal to the curve at the point where $x = \frac{1}{2}$.

(a) Show that $k = 2$. [2]

(b) Hence find the x-coordinates of the stationary points. [2]

[Turn over

For Examiner's Use

(c) Find $\frac{d^2y}{dx^2}$ and explain whether the gradient $6x - \frac{1}{kx^3}$ has a turning point. [4]

For Examiner's Use

13 The cost in thousands of dollars, C, of producing n hundred pairs of a certain type of running shoes is given by the formula $C = 1.2n^2 - 14.4n + 53.7$.

(a) Find the initial cost of production. [1]

(b) Express C in the form $a(x+h)^2 + k$ where a, h and k are constants to be determined. [3]

(c) Explain the meaning of the values of h and k in the formula as expressed in **part (b)**. [1]

(d) Find the maximum number of pairs of running shoes for which the cost will be [3]

[Turn over

50 thousand dollars.

For Examiner's Use

14 The line $y = -x + 4$ is a tangent to a circle at the point A and the centre of the circle is at $B(-4, -2)$.

(a) Find coordinates of A. [3]

(b) Find the radius and the equation of the circle. [3]

[Turn over

For Examiner's Use

(c) Find the equation of another tangent to the circle which is parallel to $y=-x+4$. [4]

For Examiner's Use

End of Paper
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[Turn over

For Examiner's Use

Bedok View

Paper 2

Mathematical Formulae

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1 The nutrient absorption rate, N, in mg/hour, of a plant depends on the diameter x, in mm, of its root hairs and is modelled by the function $N(x) = x^2 \ln\left(\frac{2}{x}\right)$, where $0 < x < 2$.

(a) Find $N'(x)$. [3]

(b) Find the optimal root hair diameter that maximises nutrient absorption and show that the nutrient absorption is a maximum. [5]

For Examiner's Use

2 Hot coffee is heated to a temperature of 90 °C. The temperature, T °C, t minutes after removal from the heat source is given by $T = 22 + Ae^{kt}$, where A and k are constants.

(a) Explain why $A = 68$. [1]

(b) After 3.5 minutes, the temperature of coffee is 79 °C. Find k. [3]

[Turn over

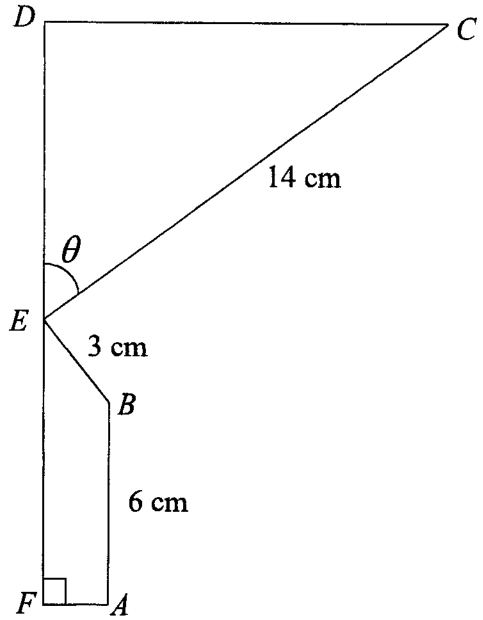
For Examiner's Use

(c) The coffee should only be served when it is between 55 °C to 70 °C. Determine, with working, whether the coffee is suitable to be served 10 minutes after removal from the heat source. [2]

(d) After a long time, T approaches a value. Calculate this value. [1]

For Examiner's Use

3 In the diagram, angle $CED = \theta$. EC is perpendicular to BE. The lengths of AB, BE and EC are 6 cm, 3 cm and 14 cm respectively. AB is parallel to FED and CD is parallel to AF.



(a) Show that $FD = 14\cos\theta + 3\sin\theta + 6$. [3]

For Examiner's Use

(b) Express FD in the form $R\cos(\theta-\alpha)+d$ where $R>0$, $0°<\alpha<90°$, and d is a constant. [3]

(c) Find the value of θ when $FD=12\,\text{cm}$. [2]

(d) Determine if it is possible for FD to be 15 cm. [2]

For Examiner's Use

4 In a biology experiment, a certain type of bacteria multiplies rapidly. The number of bacteria, N, after t hours is modelled by the formula $N = a10^{kt}$, where a and k are constants. The table below shows corresponding values of t and N.

t	2	4	6	8
N	900	2700	8100	24 300

(a) On the grid on the next page, plot $\lg N$ against t and draw a straight line graph to illustrate the information. [4]

[Turn over

For Examiner's Use

(b) Use the graph to estimate the value of each of the constants a and k. [2]

For Examiner's Use

5 **(a)** The cubic polynomial $f(x)$ is such that the coefficient of x^3 is -2 and the roots of $f(x)=0$ are -1, 2 and $\frac{3}{2}$. Find the expression of $f(x)$ in descending powers of x. [3]

(b) Find the quotient when x^3-3x^2-2x+5 is divided by x^2-1. [2]

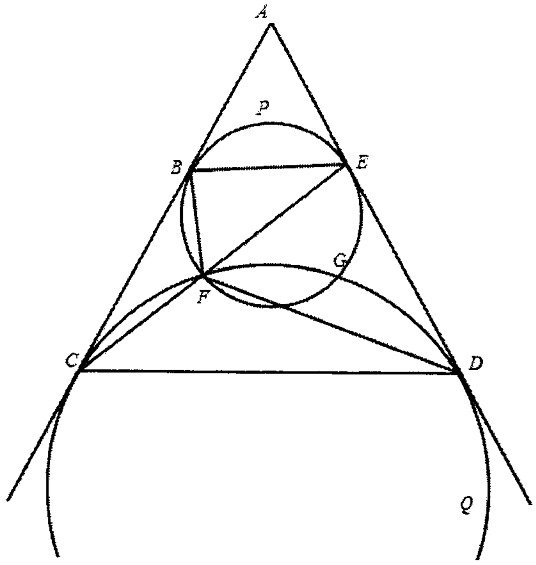
(c) The expression hx^3-7x^2+kx-2, where h and k are constants, has a factor $3x^2-5x-2$. Find the value of h and k. [5]

[Turn over

For Examiner's Use

For Examiner's Use

6 The diagram shows two circles P and Q which intersect each other at F and G. The line ABC is tangent to P and Q at B and C respectively. The line AED is tangent to P and Q at E and D respectively. EFC is a straight line.



(a) Prove that triangle ABE is similar to triangle ACD. [4]

For Examiner's Use

(b) Explain why a circle can be drawn to pass through *BEDC*. [3]

(c) Prove that angle *CBF* = angle *EDF*. [3]

For Examiner's Use

7 **(a)** Find the coefficient of x^3 in the expansion of $(7+2x)\left(1-\dfrac{x}{5}\right)^8$. [4]

For Examiner's Use

(b) In the expansion of $\left(3+\frac{x}{2}\right)^n$, the coefficient of x^2 is three times the coefficient of x^3.

Find n. [4]

For Examiner's Use

8 Do not use a calculator in this question.

(a) Find the exact value of cosec A when $\cos 2A = \frac{1}{3}$ and A is acute. [4]

[Turn over

For Examiner's Use

(b) Hence, find the exact value of $\operatorname{cosec} A - \sqrt{2}\sin 105°$. [5]

For Examiner's Use

9 A particle moves in a straight line so that its acceleration, a m/s^2, is given by $a = -2\cos\frac{1}{2}t$, where t is the time in seconds after leaving a fixed point O.

(a) Sketch the acceleration-time graph of the particle for $0 \le t \le 2\pi$. [2]

(b) The initial velocity of the particle is 3 m/s. Find the earliest time when the particle first comes to rest. [4]

Continuation of working space for question **9(b)**.

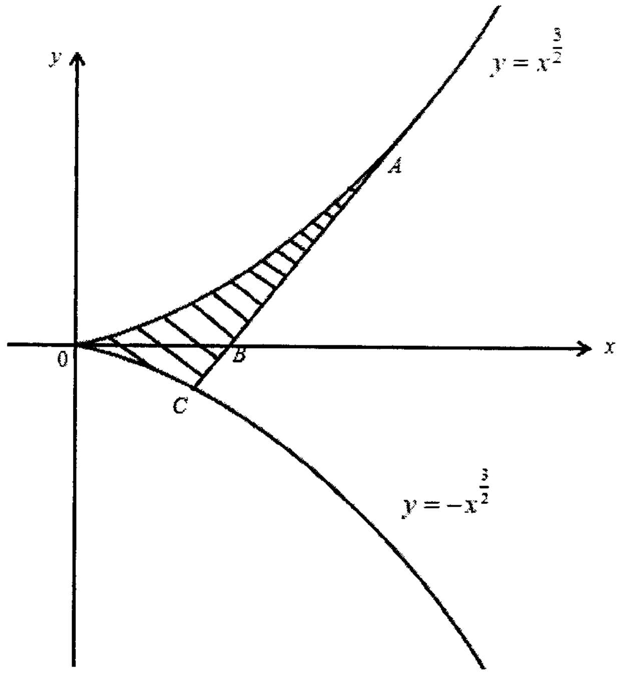
[Turn over

For Examiner's Use

(c) Find the total distance travelled in the first 5 seconds. [5]

For Examiner's Use

10 The diagram shows the graphs of $y = x^{\frac{3}{2}}$ and $y = -x^{\frac{3}{2}}$. The line $y = 2x - 1$ intersects the graph of $y = x^{\frac{3}{2}}$ at $A(1, 1)$, the x-axis at B, and the graph of $y = -x^{\frac{3}{2}}$ at C.



(a) Show that the coordinates of C are $(0.38197, -0.23607)$, correct to 5 significant figures. [5]

[Turn over

For Examiner's Use

Continuation of working space for question **10(a)**.

(b) Find the area of the shaded region. [6]

For Examiner's Use

Continuation of working space for question **10(b)**.

End of Paper

[Turn over

For Examiner's Use