BP~349

JURONGVILLE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2025
Secondary 4 Express

STUDENT NAME	**MARKING SCHEME**
CLASS	
INDEX NUMBER	

ADDITIONAL MATHEMATICS

4049/01

Paper 1

19 AUGUST 2025

2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **ALL** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total marks for this paper is 90.

DO NOT OPEN THE BOOKLET UNTIL YOU ARE TOLD TO DO SO

For Examiner's Use
90

This document consists of **18** printed pages and **0** blank page.

[Turn Over

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 The curve has the equation $y = x^2 - 4x + 7$.

A line with gradient 1 is a tangent to the curve.

Find the equation of the tangent line. [3]

$\frac{dy}{dx} = 2x - 4$ [M1]

$2x - 4 = 1$

$x = 2.5, y = 3.25$ [M1]

$y - 3.25 = 1(x - 2.5)$

$y = x + 0.75$ [A1]

2 Find the set of values of the constant p for which the curve $y = px^2 + (p-2)x + 5$ lies entirely above the x-axis. [5]

$p > 0$ [B1]

$(p-2)^2 - 4(p)(5) < 0$ [M1]

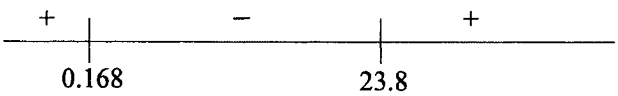
$p^2 - 24p + 4 < 0$

Use of quadratic formula [M1]

Critical Points: $p = 12 \pm 2\sqrt{35}$ [M1]

$12 - 2\sqrt{35} < p < 12 + 2\sqrt{35}$ [A1]

$0.168 < p < 23.8$



3 **(a)** Prove that $\dfrac{\cot^2\theta-\tan^2\theta}{\left(1+\tan^2\theta\right)\left(1+\cot^2\theta\right)}=\cos 2\theta$. [5]

$$LHS=\frac{\dfrac{\cos^2\theta}{\sin^2\theta}-\dfrac{\sin^2\theta}{\cos^2\theta}}{\left(\sec^2\theta\right)\left(\text{cosec}^2\theta\right)}\ \text{[M1]}$$

$$=\frac{\dfrac{\cos^4\theta-\sin^4\theta}{\sin^2\theta\cos^2\theta}}{\dfrac{1}{\cos^2\theta}\times\dfrac{1}{\sin^2\theta}}\ \text{[M1]}$$

$$=\frac{\cos^4\theta-\sin^4\theta}{\sin^2\theta\cos^2\theta}\times\frac{\cos^2\theta\sin^2\theta}{1}\ \text{[M1]}$$

$$=\cos^4\theta-\sin^4\theta$$

$$=\left(\cos^2\theta-\sin^2\theta\right)\left(\cos^2\theta+\sin^2\theta\right)\ \text{[M1]}$$

$$=\left(\cos 2\theta\right)(1)$$

$$=\cos 2\theta\ \text{[A1]}$$

3 **(b)** Hence, solve the equation $\frac{(\cot^2\theta-\tan^2\theta)(2\sin 2\theta)}{(1+\tan^2\theta)(1+\cot^2\theta)}=0.75$ for $-90°\le\theta\le 90°$. [5]

$\frac{(\cot^2\theta-\tan^2\theta)(2\sin 2\theta)}{(1+\tan^2\theta)(1+\cot^2\theta)}=0.75$

$2\sin 2\theta\cos 2\theta=0.75$ [M1]

$\sin 4\theta=0.75$ [M1]

$\alpha=\sin^{-1}(0.75)=48.59037789°$ [M1]

$4\theta=48.5903,-311.4096,131.4096,-228.5903$ [M1]

$\theta=-77.9°,-57.1°,12.1°,32.9°$ [A1]

4 The line $y-x=5$ intersects the curve $x^2+xy+y^2=91$ at two points.
Find the coordinates of these two points. [5]

$y=x+5$

$x^2+x(x+5)+(x+5)^2=91$ [M1]

$x^2+x^2+5x+x^2+10x+25-91=0$

$3x^2+15x-66=0$ [M1]

$x^2+5x-22=0$

Using Quadratic Formula, [B1]

$x=2.82$ or -7.82 [M1]

$(2.82,\ 7.82)$ or $(-7.82,\ -2.82)$ [A1]

5 The function f is defined for $x > 0$ by $f(x) = \dfrac{x^2+4}{x+1}$.

(a) Find $f'(x)$ in the form $\dfrac{ax^2+bx+c}{(x+1)^2}$. [2]

$$f'(x) = \frac{(x+1)(2x)-(x^2+4)(1)}{(x+1)^2} \text{ [M1]}$$

$$= \frac{x^2+2x-4}{(x+1)^2} \text{ [A1]}$$

(b) Hence determine the values of x for which f is increasing. [4]

For f to be increasing, $f'(x)$ must be > 0.

Since, $(x+1)^2 > 0$ for all real values of x, then to solve $x^2+2x-4>0$. [B1]

Show use of quadratic formula [B1]

Critical Points: $x = -3.24$ or 1.24 [M1]

$x < -3.24$ or $x > 1.24$ [A1]

6 A balloon is being inflated such that its volume increases and the radius, r cm of the balloon, changes over time.

The rate of change of the radius at time t seconds is given by $\frac{dr}{dt} = \frac{4}{t+2}$, $t > 0$.

Initially, at $t = 0$, the radius is 3 cm.

(a) Find an expression for r in terms of t. [2]

$r = \int \frac{4}{t+2} dt = 4\ln(t+2) + c$ [M1]

At $t = 0, r = 3, c = 3 - 4\ln 2$ or 0.227

$r = 4\ln(t+2) + 3 - 4\ln 2$ or $4\ln(t+2) + 0.227$ [A1]

(b) The volume V of the balloon is given by $V = \frac{4}{3}\pi r^3$.

Find the rate of increase of the volume of the balloon at $t = 2$. [4]

At $t = 2, r = 4\ln 4 + 3 - 4\ln 2$

$\frac{dv}{dr} = 4\pi r^2$ [M1]

At $t = 2$,

$\frac{dv}{dr} = 418.7464104$, [M1]

$\frac{dr}{dt} = 1$

$\frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}$

$\frac{dv}{dt} = 418.7464104 \times 1$ [M1]

$\frac{dv}{dt} = 419 cm^3 / s$ [A1]

7 **(a)** Express $\dfrac{5x^2+3x+7}{(x+1)(x-2)^2}$ in partial fractions. [6]

$\dfrac{5x^2+3x+7}{(x+1)(x-2)^2}=\dfrac{A}{x+1}+\dfrac{B}{x-2}+\dfrac{C}{(x-2)^2}$ [M1]

$5x^2+3x+7=A(x-2)^2+B(x+1)(x-2)+C(x+1)$ [M1]

When $x=2, C=11$ [M1]

When $x=-1, A=1$ [M1]

When $x=0, B=4$ [M1]

$\dfrac{5x^2+3x+7}{(x+1)(x-2)^2}=\dfrac{1}{x+1}+\dfrac{4}{x-2}+\dfrac{11}{(x-2)^2}$ [A1]

(b) Hence $\displaystyle\int\frac{5x^2+3x+7}{(x+1)(x-2)^2}\,\mathrm{d}x$. [3]

$\displaystyle\int\frac{5x^2+3x+7}{(x+1)(x-2)^2}\,\mathrm{d}x$

$\displaystyle=\int\frac{1}{x+1}+\frac{4}{x-2}+\frac{11}{(x-2)^2}dx$ [M1]

$\displaystyle=\int\frac{1}{x+1}dx+\int\frac{4}{x-2}dx+\int 11(x-2)^{-2}$

$=\ln(x+1)+4\ln(x-2)-\dfrac{11}{x-2}+c$ [A2][Minus 1 for each incorrect part]

8 A curve has equation $y = x\sqrt{x+k} + 3$, where k is a positive constant.

(a) Show that $\frac{\text{d}y}{\text{d}x} = \frac{3x+2k}{2\sqrt{x+k}}$. [3]

$\frac{dy}{dx} = x\left(\frac{1}{2}\right)(x+k)^{-\frac{1}{2}} + \sqrt{x+k}$ [M1]

$\frac{dy}{dx} = \frac{x}{2\sqrt{x+k}} + \sqrt{x+k}$

$\frac{dy}{dx} = \frac{x+2x+2k}{2\sqrt{x+k}}$ [M1]

$\frac{dy}{dx} = \frac{3x+2k}{2\sqrt{x+k}}$ [A1]

(b) Determine the nature of the stationary point on the curve. [6]

$$\frac{d^2y}{dx^2} = \frac{2\sqrt{x+k}(3) - (3x+2k)(2)\left(\frac{1}{2}\right)(x+k)^{-\frac{1}{2}}}{4(x+k)}$$ [M1]

$$\frac{d^2y}{dx^2} = \frac{(x+k)^{-\frac{1}{2}}\left[2(x+k)(3) - (3x+2k)\right]}{4(x+k)}$$ [M1]

$$\frac{d^2y}{dx^2} = \frac{6x+6k-3x-2k}{4(x+k)^{\frac{3}{2}}}$$

$$\frac{d^2y}{dx^2} = \frac{3x+4k}{4(x+k)^{\frac{3}{2}}}$$ [M1]

When $\frac{dy}{dx} = 0, x = -\frac{2}{3}k$ [M1]

At $x = -\frac{2}{3}k, \frac{d^2y}{dx^2} = \frac{2k}{4\left(\frac{1}{3}k\right)^{\frac{3}{2}}}$

Since $k > 0, \frac{d^2y}{dx^2} > 0$. [M1]

$x = -\frac{2}{3}k$ is a minimum point on the curve. [A1]

9 A curve $y = \frac{1+3\ln x}{x^2}$ has a minimum point at P.

(a) Find $\frac{dy}{dx}$. [2]

$$\frac{dy}{dx} = \frac{(x^2)(3)\left(\frac{1}{x}\right) - (1+3\ln x)(2x)}{x^4} \text{ [M1]}$$

$$\frac{dy}{dx} = \frac{3x - 2x - 6x\ln x}{x^4}$$

$$\frac{dy}{dx} = \frac{1-6\ln x}{x^3} \text{ [A1]}$$

(b) Show that the x-coordinate of P is $e^{\frac{1}{6}}$. [2]

At $\frac{dy}{dx} = 0, 1 - 6\ln x = 0$ [M1]

$\ln x = \frac{1}{6}$

$x = e^{\frac{1}{6}}$ [A1]

(c) Find the equation of normal to the curve at the point $x = e$.
Leave your answer in terms of e. [5]

At $x = e, y = \frac{4}{e^2}$ [M1]

At $x = e, \frac{dy}{dx} = -\frac{5}{e^3}$ [M1]

gradient of normal $= \frac{e^3}{5}$ [M1]

$y - \frac{4}{e^2} = \frac{e^3}{5}(x - e)$ [M1]

$y = \frac{e^3}{5}x - \frac{e^4}{5} + \frac{4}{e^2}$ or $y = \frac{e^3}{5}x + \frac{20 - e^6}{5e^2}$ [A1]

10 The table shows experimental values of two variables x and y.

x	1	2	3	4	5	6	7
y	56.2	31.5	25.1	9.96	5.60	3.14	1.76

The variables x and y are related by the equation $y = \frac{h}{k^x}$, where h and k are constants.

It is believed that an error was made in one of the experimental values of y.

(a) On the grid, on the next page, plot $\lg y$ against x and draw a straight line graph to illustrate the information using a scale of 2 cm to represent 1 unit on the x-axis and 8 cm to represent 1 unit on the y-axis. [3]

(b) Use your graph to

(i) identify the abnormal reading of y and estimate the correct value of y, [2]

$y = 25.1$ is an abnormal reading of y. [B1]

correct value of y: $\lg y = 1.25$

$y = 10^{1.25} = 17.8$ [B1] [Accept answers between 13 to 24]

(ii) find the value of h and k. [4]

$y = \frac{h}{k^x}$

$\lg y = \lg h - x \lg k$

$\lg y = (-\lg k)x + \lg h$ [B1]

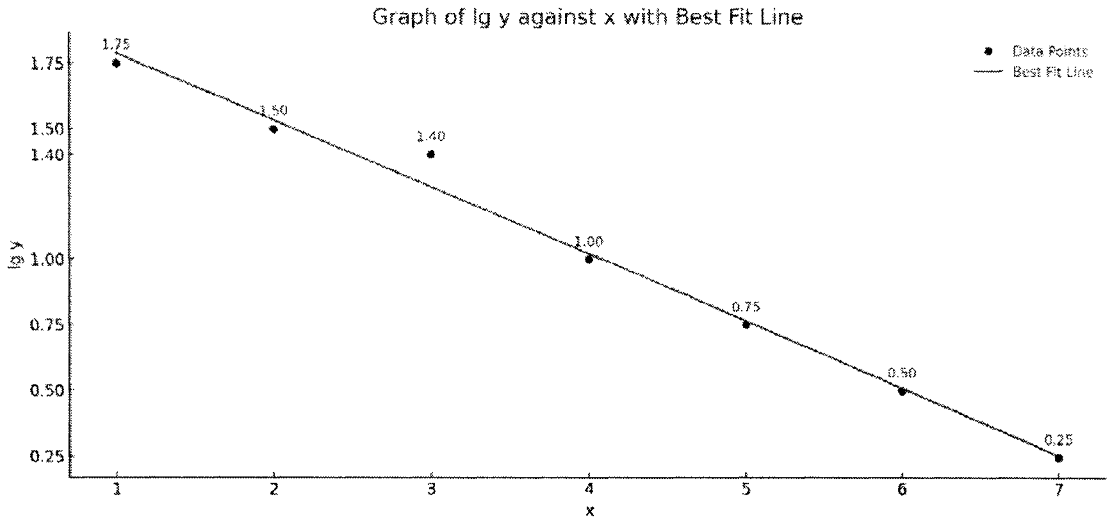
gradient $= -\lg k = \frac{1.75 - 1.50}{1 - 2}$ [M1]

$\lg k = 0.25$

$k = 10^{0.25} = 1.78$ [A1] [Acceptable range ± 0.1]

y-intercept $= \lg h = 2$

$h = 100$ [B1]


Graph of lg y against x with Best Fit Line
Data Points
Best Fit Line
lg y
x
1.75
1.50
1.40
1.00
0.75
0.50
0.25
1
2
3
4
5
6
7

11 A circle, C, has equation $x^2+y^2-10x+6y+9=0$.

(a) Find the coordinates of the centre and radius of C. [3]

$x^2+y^2-10x+6y+9=0$

$x^2-10x+25+y^2+6y+9-25-9+9=0$

$(x-5)^2+(y+3)^2=5^2$

Centre of Circle = (5, −3) [B2][Minus 1 mark for each incorrect coordinate]

Radius = 5 units [B1]

(b) Explain why the y-axis is a tangent to C. [1]

For using the correct method to find the perpendicular distance from the centre of the circle to the y-axis. Distance from centre (5, –3) to y-axis ($x = 0$) = 5
Distance equals radius, so y-axis is a tangent.

(c) Given that the circle C crosses the x-axis at the point $P(1, 0)$, find the equation of the tangent to the circle at P. [3]

Gradient of CP $=\frac{0-(-3)}{1-5}=-\frac{3}{4}$ [M1]

Gradient of tangent $=\frac{4}{3}$ [M1]

$y=\frac{4}{3}x-\frac{4}{3}$ [A1]

(d) Determine whether the point $A(3, -7)$ lies inside, outside or on the circle C? [2]

LHS $=(3-5)^2+(-7+3)^2=20$ [M1]

Since, $20<25$, $(3, -7)$ lies inside the circle. [A1]

12 **(a)** Find the equation of the perpendicular bisector of the points $P(a,\ 4)$ and $Q(6,\ -2)$ in terms of a. [4]

Midpoint of $PQ = \left(\frac{a+6}{2}, \frac{4-2}{2}\right) = \left(\frac{a+6}{2}, 1\right)$ [M1]

Gradient of $PQ = \frac{4-(-2)}{a-6} = \frac{6}{a-6}$ [M1]

Gradient of Perpendicular Bisector $PQ = -\frac{a-6}{6} = \frac{6-a}{6}$ [M1]

$y - 1 = \frac{6-a}{6}\left(x - \frac{a+6}{2}\right)$ [A1]

(b) The perpendicular bisector of line segment PQ passes through the point $R(2,\ 1)$.
Find the possible values of a. [2]

$1-1=\frac{6-a}{6}\left(2-\frac{a+6}{2}\right)$

$0=\frac{6-a}{6}\left(\frac{4-a-6}{2}\right)$ [M1]

$0=(6-a)(-2-a)$

$a=6$ or $a=-2$ [A1]

(c) S is the reflection of point Q across the perpendicular bisector of PQ.
By using the positive value of a found in part (b) calculate the area of the quadrilateral $PQRS$. [4]

Midpoint of $PQ=\left(\frac{6+6}{2},1\right)=(6,1)$ [M1]

$\frac{6+x}{2}=6$

$\frac{-2+y}{2}=1$

$S=(6,4)$ [M1]

Use of shoelace method [M1]

$Area=12\text{ units}^2$ [A1]

End of Paper

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ADDITIONAL MATHEMATICS **4049/02**

Paper 2 **20 AUGUST 2025**

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1 **(a)** Find the first 3 terms in the expansion of $(1+3x)^8$ in ascending powers of x. [2]

$$(1+3x)^8 = 1+8(3x)+\binom{8}{2}(3x)^2+\ldots \quad \text{M1}$$
$$= 1+24x+252x^2+\ldots \quad \text{A1}$$

(b) Hence determine the first 3 terms in the expansion of $(2+6x+3x^2)^8$. [3]

$$(2+6x+3x^2)^8 = 2^8\left(1+3x+\frac{3}{2}x^2\right)^8 \quad \text{M1}$$
$$= 256\left(1+3\left(x+\frac{1}{2}x^2\right)\right)^8 \quad \text{M1}$$
$$= 256\left(1+24\left(x+\frac{1}{2}x^2\right)+252\left(x+\frac{1}{2}x^2\right)^2+\ldots\right)$$
$$= 256\left(1+24x+12x^2+252x^2+\ldots\right)$$
$$= 256\left(1+24x+264x^2+\ldots\right)$$
$$= 256+6144x+67584x^2+\ldots \quad \text{A1}$$

(c) Explain why there is no constant term in the expansion of $\left(2-\frac{1}{3x}\right)^2(1+3x)^8$. [3]

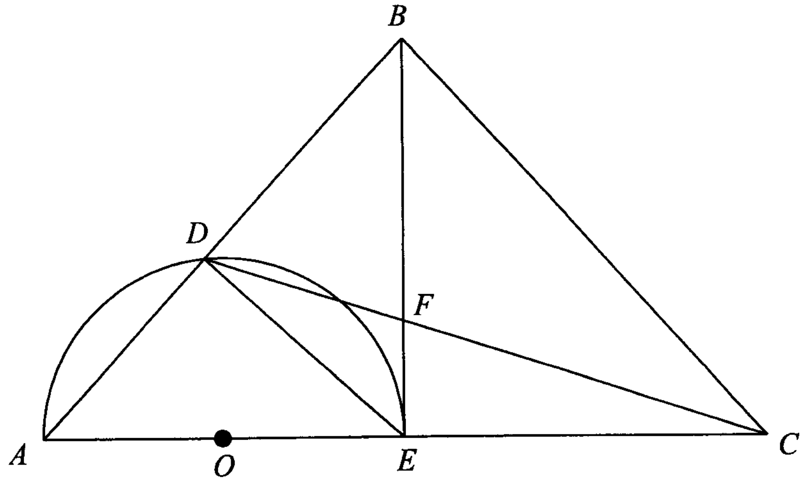
$$\left(2-\frac{1}{3x}\right)^2(1+3x)^8 = \left(2-\frac{1}{3x}\right)^2(1+24x+252x^2+\ldots)$$
$$= \left(4-\frac{4}{3x}+\frac{1}{9x^2}\right)(1+24x+252x^2+\ldots) \quad \text{M1}$$

constant term $= 4(1)-\frac{4}{3}(24)+\frac{1}{9}(252)$ M1

$= 0$

Hence there is no constant term in the expansion of $\left(2-\frac{1}{3x}\right)^2(1+3x)^8$. A1

2



ADE is a semicircle with centre O. It is given that D and E are midpoints of AB and AC respectively and $AB = BC$.

(a) Prove that $\Delta ABE \equiv \Delta CBE$. [3]

In ΔABE and ΔCBE,

$AB = BC$ (given)
$AE = EC$ (E is the midpoint of AC)
BE is a common height

M2 for 3 conditions
M1 for 2 conditions

ΔABE is congruent to ΔCBE. (SSS) A1

(b) Prove that $\angle BEA = 90°$. [1]

Since ΔABE is congruent to ΔCBE, $\angle BEA = \angle BEC$. (corresponding angles of congruent triangles)

$\angle BEA + \angle BEC = 180°$ (angles on a straight line)
$\angle BEA + \angle BEA = 180°$
$\angle BEA = 90°$

B1

(c) Determine, with explanation, whether ΔADE is similar to ΔABC. [3]

Since D and E are midpoints of AB and AC respectively, by midpoint theorem, $DE \,//\, BC$ and $DE = \frac{1}{2}BC$. B1

In ΔADE and ΔABC,
$\angle DAE = \angle BAC$ (common angle) M1
$\angle ADE = \angle ABC$ (corresponding angle, $DE \,//\, BC$) M1
ΔADE is similar to ΔABC. (AA similarity test)

(d) Show that $\frac{DF}{FC} = \frac{1}{2}$. [3]

In ΔDFE and ΔCFB,
$\angle EFD = \angle BFC$ (vertically opposite angle) M1
$\angle FDE = \angle FCB$ (alternate angle, $DE \,//\, BC$) M1
ΔDFE is similar to ΔCFB.

By midpoint theorem, $DE = \frac{1}{2}BC$.

$$\frac{DF}{FC} = \frac{DE}{BC}$$
$$= \frac{1}{2} \quad \text{(proven)}$$ A1

3 A radioactive substance decays according to the equation $A = 15e^{-bt}$, where A is the mass in grams of radioactive substance remaining and t is the time in hours after the radioactive substance starts to decay.

(a) Given that there are 10 grams of radioactive substance remaining after 30 hours, verify that the value of b is 0.013516. [2]

$$A = 15e^{-bt}$$
$$10 = 15e^{-30b}$$
$$e^{-30b} = \frac{10}{15} \qquad \text{M1}$$
$$-30b = \ln\frac{10}{15}$$
$$b = -\frac{1}{30}\ln\frac{10}{15}$$
$$= 0.013516 \qquad \text{A1}$$

(b) Find the mass of radioactive substance left at the end of 1 week. [1]

$$A = 15e^{-0.013516(7\times24)}$$
$$= 1.55\text{g} \quad \text{B1}$$

(c) Find the number of hours for the radioactive substance to decay to half of its original mass. [3]

When $t = 0$, $A = 15$. M1

$$A = 15e^{-0.013516t}$$
$$\frac{15}{2} = 15e^{-0.013516t}$$
$$e^{-0.013516t} = \frac{1}{2} \qquad \text{M1}$$
$$-0.013516t = \ln\frac{1}{2}$$
$$t = 51.3 \text{ hours} \qquad \text{A1}$$

(d) Find the rate at which the substance decays at $t = 50$ hours. [2]

$A = 15e^{-0.013516t}$

$\frac{dA}{dt} = -0.20274e^{-0.013516t}$ M1

When $t = 50$,

$\frac{dA}{dt} = -0.20274e^{-0.013516(50)}$

$= -0.103$

The rate of radioactive decay at $t = 50$ hours is 0.103g/h. A1

4 **(a)** Prove that $\cot(45° - x) = \dfrac{\cot x + 1}{\cot x - 1}$. [3]

$$\cot(45° - x) = \frac{1}{\tan(45° - x)}$$

$$= \frac{1 + \tan 45° \tan x}{\tan 45° - \tan x} \quad \text{M1}$$

$$= \frac{1 + \tan x}{1 - \tan x} \quad \text{M1}$$

$$= \frac{1 + \frac{1}{\cot x}}{1 - \frac{1}{\cot x}}$$

$$= \frac{\cot x + 1}{\cot x - 1} \text{ (proven)} \quad \text{A1}$$

$$\cot(45° - x) = \frac{\cos(45° - x)}{\sin(45° - x)}$$

$$= \frac{\cos 45° \cos x + \sin 45° \sin x}{\sin 45° \cos x - \cos 45° \sin x} \quad \text{M1}$$

$$= \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$= \frac{\frac{\cos x}{\sin x} + 1}{\frac{\cos x}{\sin x} - 1} \quad \text{M1}$$

$$= \frac{\cot x + 1}{\cot x - 1} \text{ (proven)} \quad \text{A1}$$

(b) Hence express $\cot 15°$ in the form $p+q\sqrt{3}$, where p and q are integers. [3]

$$\begin{aligned}
\cot 15° &= \cot(45°-30°) \quad \text{M1 - sub } x = 30° \\
&= \frac{\cot 30° + 1}{\cot 30° - 1} \\
&= \frac{\frac{1}{\tan 30°} + 1}{\frac{1}{\tan 30°} - 1} \\
&= \frac{\sqrt{3}+1}{\sqrt{3}-1} \\
&= \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \quad \text{M1} \\
&= \frac{3+2\sqrt{3}+1}{3-1} \\
&= \frac{4+2\sqrt{3}}{2} \\
&= 2+\sqrt{3} \quad \text{A1}
\end{aligned}$$

5 It is given that $\cos\theta = \frac{5}{13}$, where $0° < \theta < 360°$.

(a) Show that the value of $2-\sin^2\theta$ is $\frac{194}{169}$. [2]

$$2-\sin^2\theta = 2-(1-\cos^2\theta) \quad \text{M1}$$
$$= 1+\cos^2\theta$$
$$= 1+\left(\frac{5}{13}\right)^2$$
$$= \frac{194}{169} \quad \text{A1 – with correct substitution}$$

(b) By finding the value of $\sin^2\theta$, find the exact value of $\tan^2(90°-\theta)$. [4]

$$2-\sin^2\theta = \frac{194}{169}$$
$$\sin^2\theta = \frac{144}{169} \quad \text{B1}$$
$$\tan^2(90°-\theta) = \frac{1}{\tan^2\theta} \quad \text{M1}$$
$$= \frac{\cos^2\theta}{\sin^2\theta}$$
$$= \frac{\left(\frac{5}{13}\right)^2}{\frac{144}{169}} \quad \text{M1}$$
$$= \frac{25}{144} \quad \text{A1}$$

6 The function $f(x)=x^3+px^2+qx-12$, where p and q are constants, has a factor $x+1$ and leaves a remainder of 36 when divided by $x-3$.

(a) Show that $p=5$ and $q=-8$. [4]

$f(x)=x^3+px^2+qx-12$

$f(-1)=0$

$-1+p-q-12=0$

$p-q=13$ ------------------------------------① M1

$f(3)=36$

$27+9p+3q-12=36$

$9p+3q=21$

$3p+q=7$ ------------------------------------② M1

①+②:

$4p=20$ M1

$p=5$

$q=5-13$

$q=-8$

$\therefore p=5$ and $q=-8$ (shown) A1

(b) Using these values of p and q, solve the equation $f(x)=0$. [2]

When $p=5$ and $q=-8$,

$f(x)=0$

$x^3+5x^2-8x-12=0$

$(x+1)(x^2+4x-12)=0$ M1

$(x+1)(x+6)(x-2)=0$

$x=-1$ or $x=-6$ or $x=2$ A1

7 **(a)** Given that $\log_7 x = m$ and $\log_7 y = 2n$, express xy in terms of m and n. [2]

Given that

$\log_7 x = m$ and $\log_7 y = 2n$

$x = 7^m$ $\quad$ $y = 7^{2n}$ $\quad$ M1

$xy = \left(7^m\right)\left(7^{2n}\right)$

$xy = 7^{m+2n}$ $\quad$ A1

(b) Solve the equation $\log_3 3x^2 + 4 = \log_x 27$. [5]

$\log_3 3x^2 + 4 = \log_x 27$

$\log_3 3 + 2\log_3 x + 4 = \log_x 27$ $\quad$ M1

$2\log_3 x + 5 = \dfrac{\log_3 3^3}{\log_3 x}$ $\quad$ M1 – for Change of Base Law

$2\log_3 x + 5 = \dfrac{3}{\log_3 x}$

Let $y = \log_3 x$,

$2y + 5 = \dfrac{3}{y}$ $\quad$ M1

$2y^2 + 5y - 3 = 0$

$(2y - 1)(y + 3) = 0$

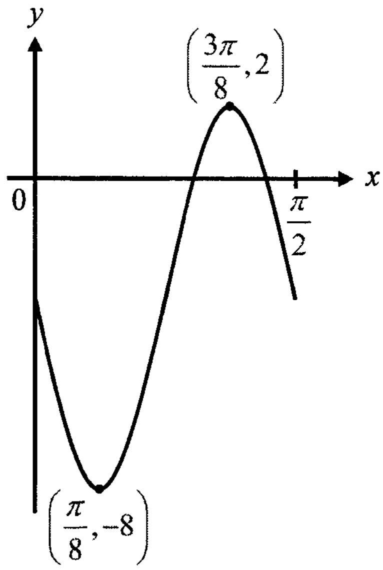
$\therefore y = \dfrac{1}{2}$ *or* $y = -3$

$\log_3 x = \dfrac{1}{2}$ *or* $\log_3 x = -3$ $\quad$ M1

$x = 3^{\frac{1}{2}}$ *or* $x = 3^{-3}$

$= \sqrt{3}$ $\quad$ $= \dfrac{1}{27}$ $\quad$ A1

8



The diagram shows the curve $y = a\sin bx + c$ for $0 \le x \le \frac{\pi}{2}$ radians.

The curve has a minimum point at $\left(\frac{\pi}{8},-8\right)$ and a maximum point at $\left(\frac{3\pi}{8},2\right)$.

(a) Explain why $c = -3$. [2]

$y = c$ is axis of curve

$c = \frac{\text{maximum} + \text{minimum}}{2}$

$= \frac{2+(-8)}{2}$ M1

$= -3$ A1

(b) Explain why $b = 4$. [2]

$\frac{2\pi}{b} = \text{period}$

$\frac{2\pi}{b} = \frac{\pi}{2}$ M1

$b = 4$ A1

(c) Hence find the equation of the curve. [2]

$y = a\sin bx + c$

Since this is a negative sine graph and the amplitude is 5, $a = -5$. M1

The equation of the curve is $y = -5\sin 4x - 3$. A1

9 A boat travelling in a straight line passes a buoy, B, with a speed of v m/s. The velocity of the boat after passing B is given by $v = t^2 - 5t + 4$. The boat comes to rest momentarily, first at point C and then at point D.

(a) Find the distance CD. [5]

$v = t^2 - 5t + 4$

$s = \int t^2 - 5t + 4 \, dt$

$= \frac{t^3}{3} - \frac{5t^2}{2} + 4t + c$

When $t = 0$, $s = 0$, $c = 0$.

$s = \frac{t^3}{3} - \frac{5t^2}{2} + 4t$ M1

At instantaneous rest,

$v = 0$

$t^2 - 5t + 4 = 0$ M1

$(t-1)(t-4) = 0$

$t = 1$ or 4 M1

When $t = 1$,

$s = \frac{1}{3} - \frac{5}{2} + 4$

$= 1\frac{5}{6}$ m

When $t = 4$,

$s = \frac{4^3}{3} - \frac{5(4)^2}{2} + 4(4)$

$= -2\frac{2}{3}$ m

Distance between C and $D = 1\frac{5}{6} - \left(-2\frac{2}{3}\right)$ M1

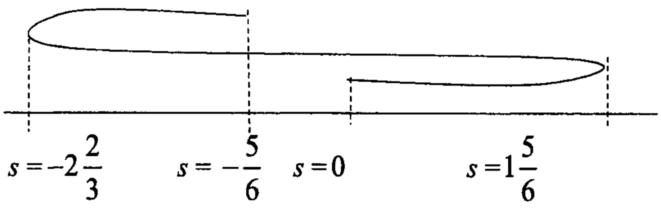
$= 4\frac{1}{2}$ m A1

(b) Find the total distance travelled by the boat in the first 5 seconds after passing B. [3]

When $t = 5$,

$s = \frac{5^3}{3} - \frac{5(5)^2}{2} + 4(5)$ M1

$= -\frac{5}{6}$ m



Total distance travelled by boat $= 1\frac{5}{6} + 4\frac{1}{2} + \left(-\frac{5}{6} + 2\frac{2}{3}\right)$ M1

$= 8\frac{1}{6}$ m A1

(c) Given that A is the point at which the boat has zero acceleration, determine with full working, whether A is nearer to B or to D. [3]

$a = 2t - 5$

When $a = 0$,

$2t - 5 = 0$ M1

$t = 2\frac{1}{2}$

When $t = 2\frac{1}{2}$,

$s = \frac{\left(2\frac{1}{2}\right)^3}{3} - \frac{5\left(2\frac{1}{2}\right)^2}{2} + 4\left(2\frac{1}{2}\right)$ M1

$= -\frac{5}{12}$ m

distance from A to $B = \frac{5}{12}$ m

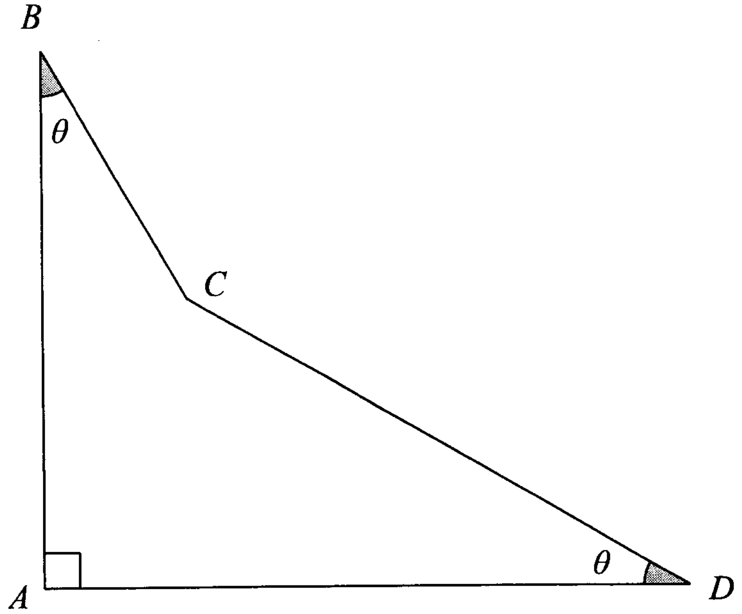
distance from A to $D = \left(2\frac{2}{3} - \frac{5}{12}\right)$ m

$= 2\frac{1}{4}$ m

A is nearer to B.

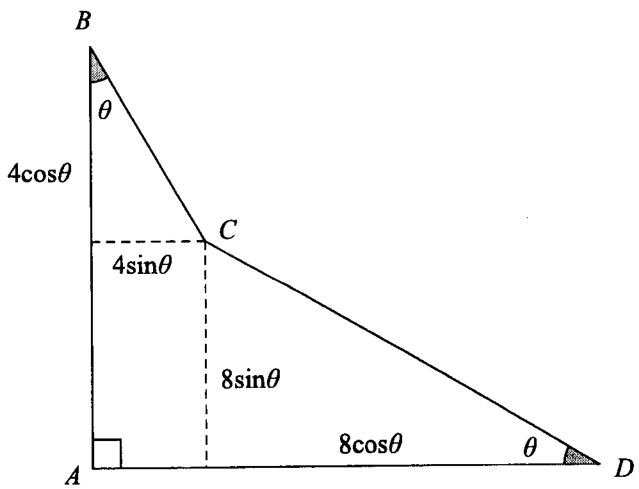
A1 – with conclusion

10



The diagram shows a drawing of a mini garden $ABCD$ in the school. It is given that $BC = 4$ m, $CD = 8$ m and $\angle ABC = \angle ADC = \theta$, where θ is an acute angle measured in degrees.

(a) Show that the area of the mini garden is $A = 20\sin 2\theta - 16\cos 2\theta + 16$. [4]



$A = \frac{1}{2}(4\cos\theta)(4\sin\theta) + \frac{1}{2}(8\cos\theta)(8\sin\theta) + (4\sin\theta)(8\sin\theta)$ M1

$= 8\cos\theta\sin\theta + 32\cos\theta\sin\theta + 32\sin^2\theta$

$= 40\cos\theta\sin\theta + 32\sin^2\theta$ M1

$= 20\sin 2\theta + 32\left(\frac{1-\cos 2\theta}{2}\right)$ M1

$= 20\sin 2\theta - 16\cos 2\theta + 16$ A1

(b) Express A in the form $R\sin(2\theta - \alpha) + 16$, where $R > 0$ and α is an acute angle. [3]

$A = 20\sin 2\theta - 16\cos 2\theta + 16$

$= \sqrt{20^2 + 16^2}\sin\left(2\theta - \tan^{-1}\left(\frac{16}{20}\right)\right) + 16$ M1 for R, M1 for α

$= \sqrt{656}\sin(2\theta - 38.659°) + 16$

$= \sqrt{656}\sin(2\theta - 38.7°) + 16$ A1

(c) Find the value of θ when the area of the garden is 25 m^2. [3]

$\sqrt{656}\sin(2\theta - 38.659°) + 16 = 25$

$\sin(2\theta - 38.659°) = \frac{9}{\sqrt{656}}$ M1

$2\theta - 38.659° = 20.572°$ M1

$\theta = 29.6°$ (1d.p.) A1

(d) Determine the maximum value of A and the corresponding value of θ. [3]

Maximum value of $A = \sqrt{656} + 16$

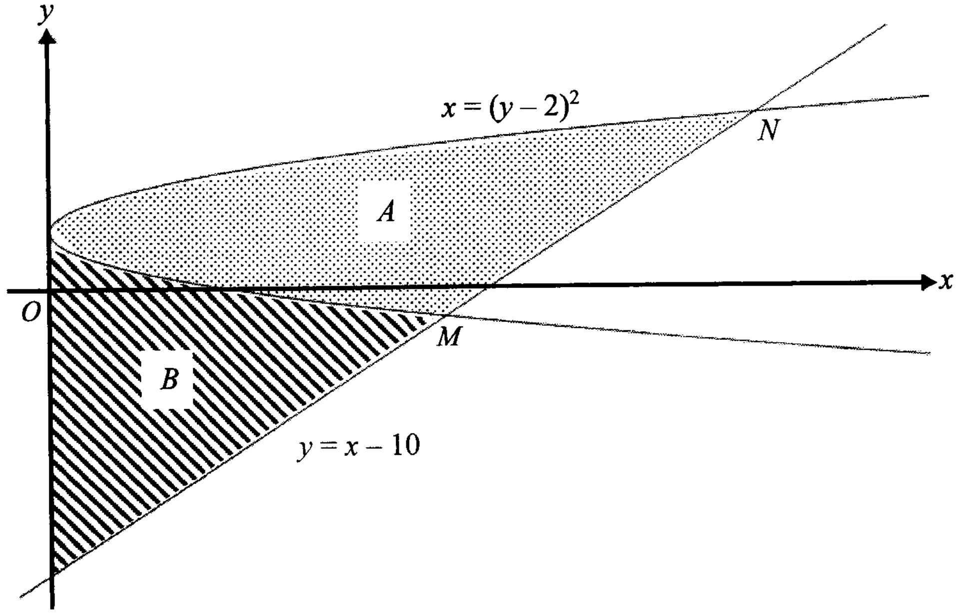
$= 41.6$ m (3s.f.) M1

Corresponding value of θ:

$2\theta - 38.659° = 90°$ M1

$\theta = 64.3°$ (1 d.p.) A1

11



The curve $x=(y-2)^2$ and the line $y=x-10$ intersects at the points M and N.

(a) Find the coordinates of M and N. [3]

$x=(y-2)^2$ --- (1)
$y=x-10$ --- (2)
Sub (1) into (2):
$y=(y-2)^2-10$ M1
$y=(y^2-4y+4)-10$
$y^2-5y-6=0$
$(y+1)(y-6)=0$
$y=-1$ or $y=6$ M1
$x=9$ or $x=16$
Coordinates of M and N are $(9,-1)$ and $(16, 6)$ respectively. A1

(b) Find the area of shaded region A and the area of the shaded region B. [6]

$$\text{area of region } A = \frac{1}{2}(9+16)(6+1) - \int_{-1}^{6}(y-2)^2\,dy \quad \text{M2}$$

$$= 87\frac{1}{2} - \left[\frac{(y-2)^3}{3}\right]_{-1}^{6} \quad \text{M1 – correct integration}$$

$$= 87\frac{1}{2} - \left(\frac{(6-2)^3}{3} - \frac{(-1-2)^3}{3}\right)$$

$$= 57\frac{1}{6} \text{ units}^2 \quad \text{A1}$$

$$\text{area of region } B = \int_{-1}^{2}(y-2)^2\,dy + \frac{1}{2}(10-1)(9) \quad \text{M1}$$

$$= \left[\frac{(y-2)^3}{3}\right]_{-1}^{2} + \frac{81}{2}$$

$$= \frac{(2-2)^3}{3} - \frac{(-1-2)^3}{3} + \frac{81}{2}$$

$$= 49\frac{1}{2} \text{ units}^2 \quad \text{A1}$$

END OF PAPER