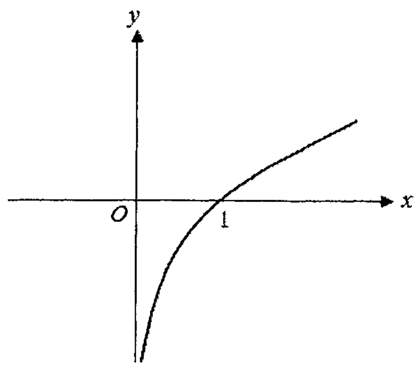
Answer Key

1	$x = 2$ or $x = 4$
2	When $t = 2$, $T = 92.6°C$ Since the temperature of the water would be within the ideal range of 90 – 95 °C, the standards for brewing black tea **would** be adhered to.
3a	$f(x) = 2\sin 3x + 4\cos 3x + 4$
3b	$-27\sqrt{2}$
4a(ii)	grad=0.0006-0.00062 and a=1600 or 1700 Y-intercept= 0.01, 0.015 or 0.02 and $b = 16 - 33$
4a(iii)	Accept $t = 6.06, 6.25$ or 6.45
4a(iv)	xt against t **OR** t against xt
4b	4
5a	$-2 < k < 6$
5b	Since the **discriminant < 0 when $k = -1$** and the **coefficient of x^2**, i.e. 1, is **positive**, $-(x^2 + x) + 2(x^2 + x + 1)$ is always positive
6a	$x = 0.550, 6$
6b	
6c(i)	$\frac{dy}{dx} = 2e^{2x+1} + 2e^{x+1}$ Since $e^{2x+1} > 0$ and $e^{x+1} > 0$ for all real values of x, $2e^{2x+1} + 2e^{x+1} > 0$, $\frac{dy}{dx} \neq 0$ Therefore the curve has no stationary points
6c(ii)	$x = 0$
7a	$x^3 - 125 = (x - 5)(x^2 + 5x + 25)$
7b	$a = 16, b = -5$

8b	$x=\frac{\pi}{4}$
8c	Since $e^{-2x} > 0$ and $(\tan x - 1)^2 \geq 0$ for all real values of x, $\frac{dy}{dx} \geq 0$ therefore y is never decreasing.
8d	The stationary point at $x=\frac{\pi}{4}$ is a **point of inflexion.** Since y is never decreasing, the **gradient of the curve must be positive (or dy/dx > 0) slightly to the left and right** of the stationary point.
9a(i)	$-60°$ or $-\frac{\pi}{3}$
9a(ii)	$-90° < \tan^{-1}x < 90°$
9a(iii)	$0 \leq k \leq 4$
9b(ii)	$x = 80.8°, 170.8°$
10b	$(x-2)^2+(y+3)^2=25$ OR $x^2-4x+y^2+6y-12=0$
10c(i)	(–6, –6)
10c(ii)	Since the y-axis is a tangent to the second circle, the radius of the circle is 6 units. The highest point, H, of the second circle is (–6, 0). Since the circle touches the x-axis at exactly one point at (–6, 0), the x-axis is a tangent to the second circle.
11a	$a=-2n(k-2t)^{n-1}$
11b	$k=16$, $n=1.5$
11c	204.8 m
11d	No since when $t > 8$, v and s are undefined.

SINGAPORE CHINESE GIRLS' SCHOOL
PRELIMINARY EXAMINATION 2025
SECONDARY FOUR
O-LEVEL PROGRAMME

CANDIDATE NAME

CLASS 4

REGISTER NUMBER

CENTRE NUMBER

INDEX NUMBER

ADDITIONAL MATHEMATICS **4049/01**

Paper 1

Thursday **28 August 2025** **2 hours 15 minutes**

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class, register number, centre number and index number at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved electronic scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

FOR EXAMINERS USE

Q1		Q5		Q9			
Q2		Q6		Q10			
Q3		Q7		Q11			
Q4		Q8		Q12		90	

The Question Paper consists of **19** printed pages and **1** blank page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} ab \sin C$$

1. Oil is dripping into an empty inverted right circular cone at the rate of 16 cm^3 per second. The height, h cm, of the cone is three-quarters its radius, r cm.

[Volume of a right circular cone = $\frac{1}{3} \times \pi \times r^2 \times h$]

(a) Show that the volume, V cm^3, of the cone is $V = \frac{1}{4}\pi r^3$. [1]

(b) Calculate, at the instant when the radius is 8 cm, the rate of change of the radius. [3]

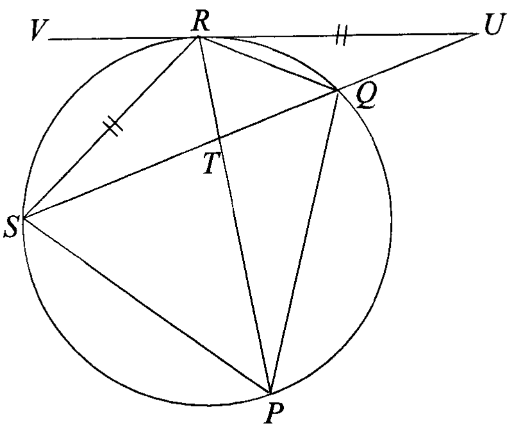
2. The straight line $2x + y = 1$ intersects the curve $x^2 - xy - y^2 + 5 = 0$ at the points A and B.

Show that the length AB is $5\sqrt{5}$ units. [5]

3. **(a)** Find $\frac{\mathrm{d}}{\mathrm{d}x}(2x-3)e^{2x}$. [3]

(b) Hence evaluate $\int_0^1 4xe^{2x}\,\mathrm{d}x$, giving your answers in exact form. [4]

4. The diagram shows a circle passing through the points P, Q, R and S. Chord SQ and RP intersect at T. The straight line URV is a tangent to the circle at R. The tangent meets the line SQ extended at U such that $RU = RS$.



(a) Show that triangle RQU is isosceles. [3]

(b) Show that angle $SPT = 2 \times$ angle QPT. [3]

5. The equation of a curve is $y=(m+1)x^2-8x+3m$, where m is a constant.

(a) In the case where $m = 1$, show that the line $y=1-4x$ is a tangent to the curve. Find the coordinates of the point of contact. [3]

(b) Find the range of values of m for which the curve is above the line $y = 5$. [5]

6. **(a)** By using long division, show that $2x^2+1$ is a factor of $2x^3-4x^2+x-2$. [2]

(b) Express $\dfrac{11x-5x^2-11}{2x^3-4x^2+x-2}$ in partial fractions. [5]

7. **(a)** State the amplitude and period of the graph of $y = 3\sin\frac{x}{2} + 1$. [2]

(b) Sketch the graph of $y = 3\sin\frac{x}{2} + 1$ for $0 \le x \le 4\pi$. [3]

(c) By adding another suitable curve on your sketch, determine the number of solutions of the equation $\sin\frac{x}{2} = -\frac{1}{3} - \frac{2}{3}\cos\frac{x}{2}$. [3]

8. The highest point on a circle C_1 is (2, 8). The equation of the tangent, T, to C_1 at the point (6, 6) is $3y+4x=42$.

(a) Find the equation of C_1. [6]

A second circle, C_2, is the reflection of C_1 in the line T.

(b) Find the equation of C_2. [3]

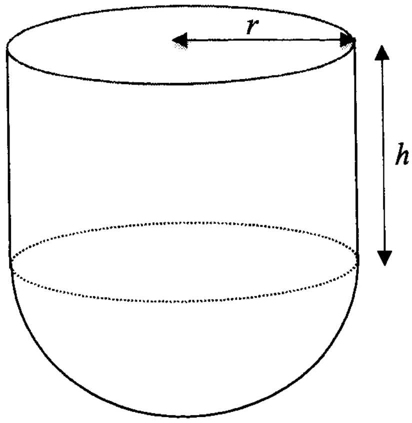
Continuation of working space for question **8(b)**.

9. **(a)** Show that the value of $\sin 105^\circ$ can be expressed as $\frac{1}{4}(p+q)$, where p and q are constants. [3]

(b) Given that $\frac{\cos(A+B)}{\cos(A-B)} = \frac{2}{7}$ and $\sin A \sin B \neq 0$, find the exact value of $\cot A \cot B$. [3]

(c) Given that B is a reflex angle and $\cot B = 3$, find the exact value of $\cos 2B$. [2]

10.



The diagram shows a container which consists of an **open** cylinder of radius r m and height h m joined to a hemispherical bottom of radius r m. The container is to be made of thin metal sheets of negligible thickness. The volume of the cylindrical part of the container is 300 m^3.
[Surface area of a sphere of radius $r = 4\pi r^2$]

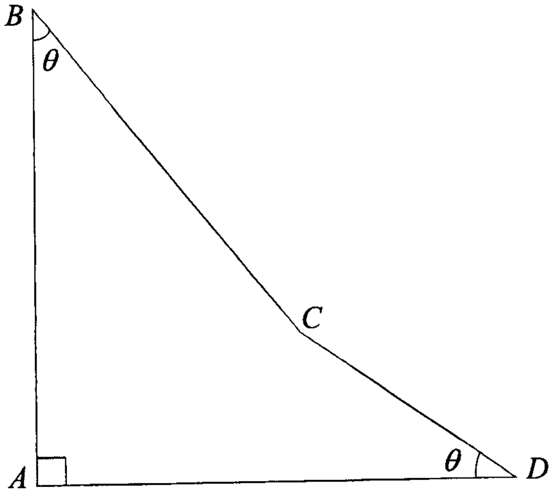
(a) Express h in terms of r. [1]

(b) The manufacturer bought metal sheets at $60 per square metre.
Show that the total cost, $$C$, of the metal sheets required to make the container is given by

$$C = 120\pi r^2 + \frac{36000}{r}.$$ [2]

(c) Given that r can vary, find the stationary value of C and determine its nature. [6]

11.



The diagram shows a figure $ABCD$, in which $BC = 13$ m, $CD = 6$ m, angle $BAD = 90^\circ$ and angle $ADC =$ angle $ABC = \theta$.

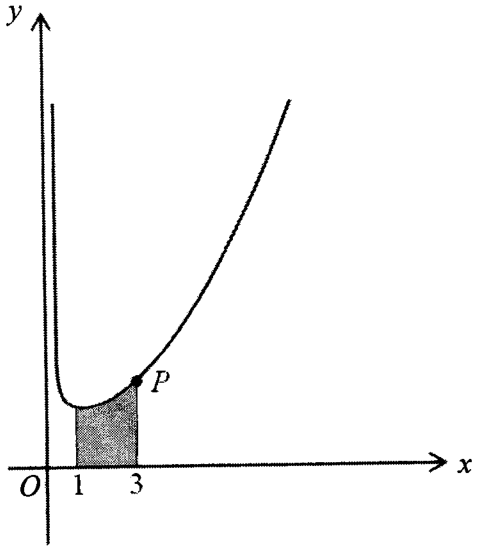
(a) Show that the perimeter, L metres, of $ABCD$ is given by

$$L = 19 + 19\cos\theta + 19\sin\theta.$$ [2]

(b) By expressing L in the form of $19 + R\cos(\theta - \alpha)$, where $R > 0$ and α is acute, find the maximum possible perimeter of $ABCD$. [4]

(c) Find the values of θ for which $L = 45$ m. [3]

12. The diagram shows part of the curve of $y=\frac{1}{4}x^2-\frac{1}{2}\ln\left(\frac{x}{3}\right)$ and $P(3,\ 2.25)$ is a point on the curve.



(a) By using calculus, determine, with full working, whether $y=\frac{4}{3}x-1$ is the equation of the tangent to the curve at P. [5]

(b) Differentiate $x\ln\left(\frac{x}{3}\right)-x$ with respect to x. [2]

(c) Using the result from part **(b)**, find the exact area of the region enclosed by the curve $y=\frac{1}{4}x^2-\frac{1}{2}\ln\left(\frac{x}{3}\right)$, the x-axis, the line $x=1$ and the line $x=3$. [3]

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