Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS
Paper 1

4049/01

25 August 2025 Monday 2 hours 15 min

2025 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC) PRELIMINARY EXAMINATIONS

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INSTRUCTIONS TO CANDIDATES
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Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided below the questions.
Give non-exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

	For Examiner's Use													
Qn	1	2	3	4	5	6	7	8	9	10	11	12	13	*Marks Deducted*
Marks														

TOTAL MARKS
/ 90

Category	Accuracy	Units	Notations	Others
Question No.				

Setter: Mr Tan Lip Sing
Vetter: Ms Sabrina Tan

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** Differentiate $\ln\left(\dfrac{3x}{x^2+1}\right)$ with respect to x. [3]

(b) Hence find $\displaystyle\int \frac{2x}{x^2+1}\,dx$. [2]

2 It is given that $\cos A = \frac{1}{2}$ and $\sin B = -\frac{1}{\sqrt{2}}$ where $0° < A < 90°$ and $180° < B < 270°$.

Find, without using a calculator, the exact value of $\cos(A - B)$, leaving your answer in the form $p\sqrt{2} + q\sqrt{6}$, where p and q are real numbers. [4]

3 Baking powder is poured onto a flat surface at a constant rate of 2π cm^{3}s^{-1}, forming a right circular cone. The radius of the cone is always $\frac{1}{18}$ of its height. Find the rate of change of the radius of the cone after 3 seconds of pouring.

$\left[\text{Volume of cone} = \frac{1}{3}\pi r^2 h\right]$ [5]

4 A and B are the points of intersection of the line $4y = 2x + 1$ and the curve $3y - x = 4xy$.

(a) Find the coordinates of A and of B. [4]

(b) Henry says that the line $2y - 4x = 5$ is perpendicular to the line AB.
Is he correct? Justify your answer with workings. [3]

5 **(a)** Write down and simplify the first three terms in the expansion, in descending powers of x, of $\left(2-\frac{3}{x}\right)^8$. [2]

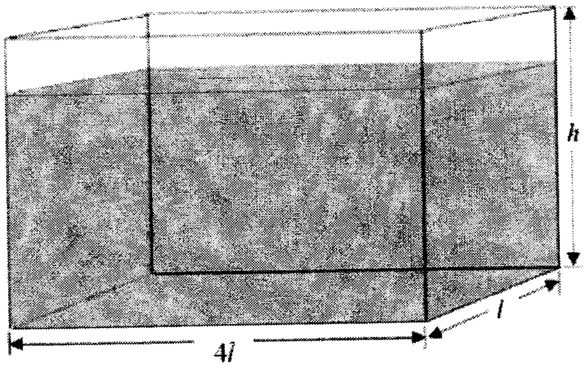
(b) Given that there is no x term in the expansion of $\left(1-2x-kx^2\right)\left(2-\frac{3}{x}\right)^8$, find the constant term in the expansion. [4]

6 **(a)** Express $y = 4x - 4x^2 - 3$ in the form $p(x+q)^2 + r$ where p, q and r are constants. [2]

(b) Hence, explain whether $4x - 4x^2 - 3 = 0$ has any real solutions. [2]

TURN OVER FOR QUESTION 7

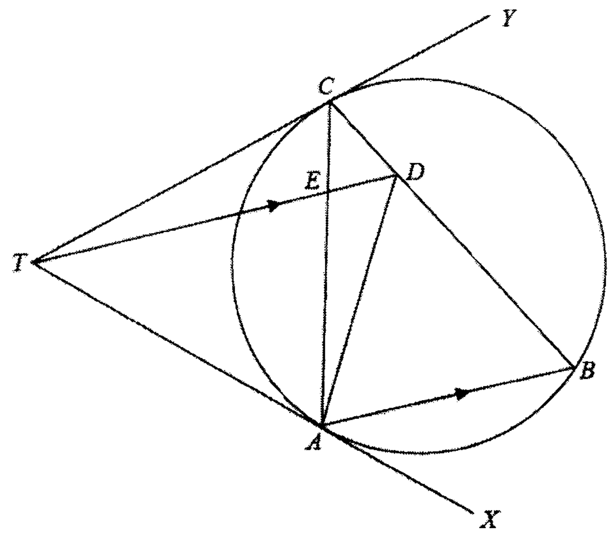
7 Peter constructed an open fish tank with a rectangular base of length $4l$ m, breadth l m, and height h m. He wanted the total outer surface area of the fish tank to be 5 m^2.



(a) Show that the volume of the tank, V m^3, is given by $V = \frac{2}{5}(5l - 4l^3)$. [3]

(b) Determine the area of the rectangular base for the tank to contain the maximum amount of water when filled to the brim. [4]

8 In the diagram below, *TAX* and *TCY* are tangents to the circle at *A* and *C* respectively. *AC* meets *TD* at *E* and *D* is on *BC* such that *TD* is parallel to *AB*.



(a) Prove that angle *ACB* is equal to angle *ATD*. [2]

(b) Using the result from **part (a)**, explain whether a circle can be drawn passing through the points *T*, *A*, *D* and *C*. [2]

(c) Prove that $CE \times EA = DE \times TE$. [3]

9 **(a)** A square has an area $\left(17-8\sqrt{2}\right)$ cm^2. The length of each side of the square can be expressed in the form $\left(a+b\sqrt{2}\right)$ cm, where a and b are integers. Show that $2b^4-17b^2+16=0$. [4]

(b) [The area of a sector is $\frac{1}{2}r^2\theta$ and the arc length of a sector is $r\theta$.]

The sector of a circle with radius, r, has an arc length of $\left(\sqrt{15}-\sqrt{3}\right)$ cm and an area of $\left(3\sqrt{3}-\sqrt{15}\right)$ cm^2. Show that $r=\dfrac{6\sqrt{3}-2\sqrt{15}}{\sqrt{15}-\sqrt{3}}$ and hence express r in the form $\left(p+q\sqrt{5}\right)$ cm, where p and q are integers. [5]

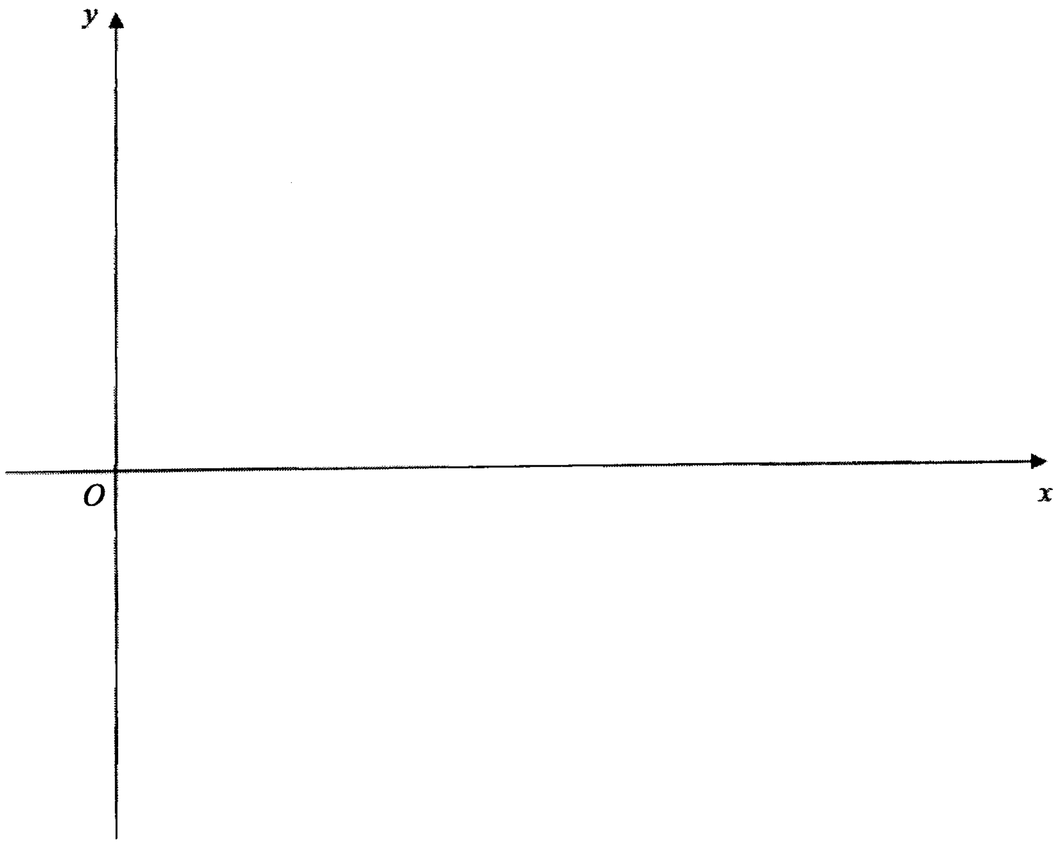
10 It is given that $f(x) = 2\sin\frac{x}{2}$ and $g(x) = 3\cos x + 1$, where $0 \le x \le 2\pi$.

(a) State the period of $f(x)$. [1]

(b) State the smallest value of $f(x)$. [1]

(c) State the largest value of $g(x)$. [1]

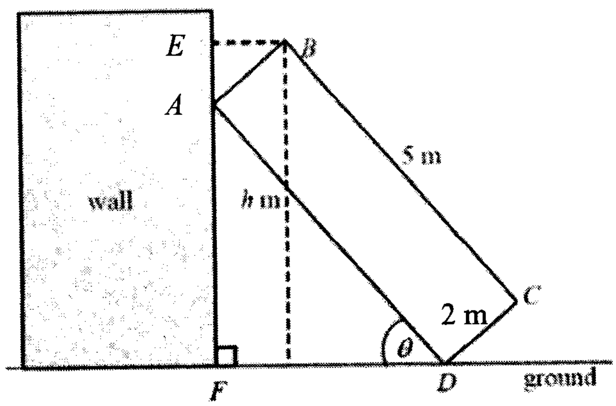
(d) Sketch on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0 \le x \le 2\pi$. Label your graphs clearly. [4]



(e) The solutions to the equation $f(x) = g(x)$ for $0 \le x \le 2\pi$ are a and b, where $a < b$.
State, in terms of a and b, the range of values of x for which $f(x) > g(x)$. [1]

11 The diagram shows a rectangular wooden plank $ABCD$, 5 m by 2 m, leaning against a vertical wall. AD makes an acute angle θ with the horizontal ground.

E is a point on the vertical wall such that EB is parallel to the ground and F is a point on the ground such that AF is perpendicular to FD.
The point B is h m vertically above the ground.



(a) Show that $h = 5\sin\theta + 2\cos\theta$. [2]

(b) Express h in the form $R\sin(\theta+\alpha)$, where $R > 0$, and $0° < \alpha < 90°$. [3]

(c) Find the greatest possible value of h and the value of θ at which it occurs. [2]

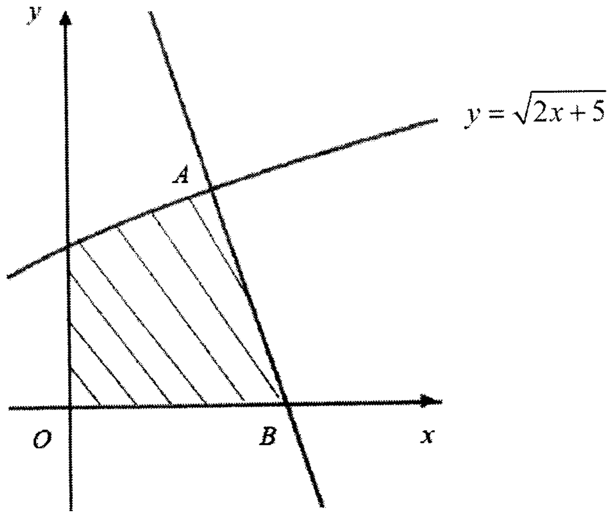
(d) Find the value of θ for which $h = \sqrt{15}$ m. [2]

12 **(a)** By using long division, divide $4x^3+5x^2+x-1$ by $x^2(x+1)$. [1]

(b) Hence, express $\dfrac{4x^3+5x^2+x-1}{x^2(x+1)}$ in partial fractions. [5]

(c) Hence, find $\int \frac{4x^3 + 5x^2 + x - 1}{x^2(x+1)} \mathrm{d}x$. [3]

13 The diagram shows part of the curve $y = \sqrt{2x+5}$. A is a point on the curve and the x-coordinate of A is 2. The normal to the curve at A meets the x-axis at B.



(a) Find the equation of the normal to the curve at A. [5]

(b) Find the area of the shaded region bounded by the normal AB, the curve, the x-axis and the y-axis. [5]

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Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/02**
Paper 2

26 August 2025 Tuesday 2 hours 15 min

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Setter: Ms Sabrina Tan
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This question paper consists of **21** printed pages and **1** blank page.

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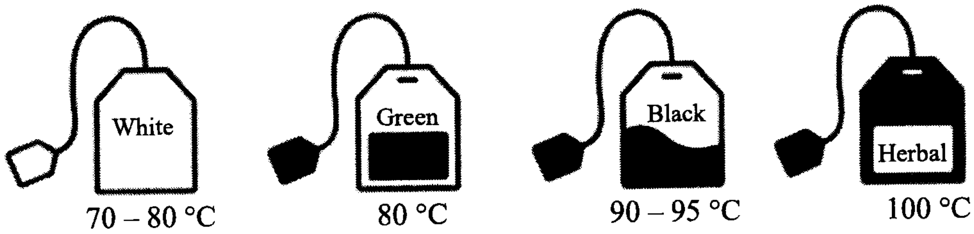
1 The equation of a curve is $y = \dfrac{5x}{6 - 2x}$. Find the x-coordinates of the points at which the gradient of the curve is 7.5. [4]

2 The temperature of water in a tea cup, T °C, t minutes after boiling water is poured into the empty cup can be modelled by the formula $T = 25 + 75\mathrm{e}^{-kt}$.



The diagram above shows the ideal brewing temperatures for various teas at a particular teahouse. At this teahouse, green tea leaves are placed into the tea cup for brewing at the ideal temperature when $t = 6$.

Determine whether the teahouse's standards for brewing tea would be adhered to if black tea leaves are placed into the teacup when $t = 2$. [4]

3 It is given that $f'(x) = 6\cos 3x - 12\sin 3x$ and $f\left(\frac{\pi}{3}\right) = 0$.

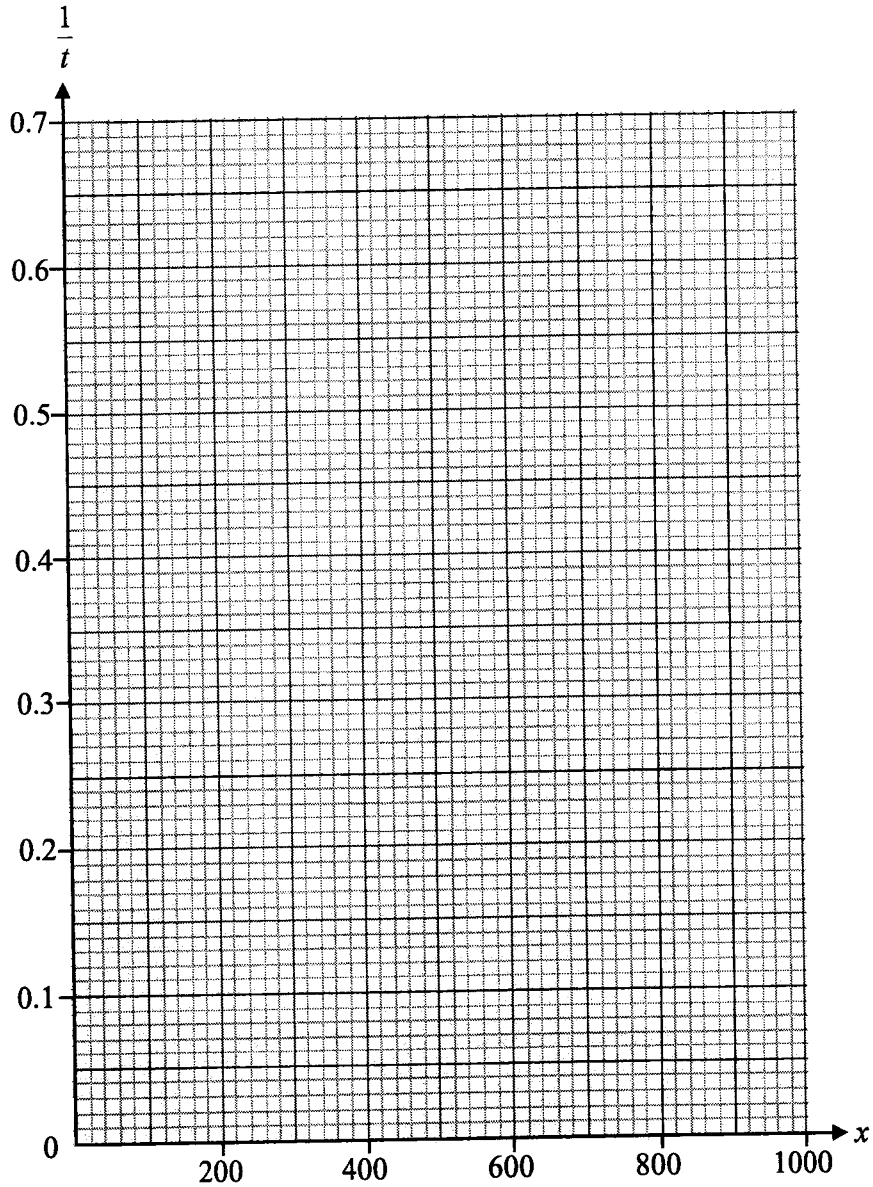
(a) Find f(x). [4]

(b) Find the exact value of $f''\left(\frac{\pi}{12}\right)$. [2]

4 **(a)** The time taken, t in minutes, to download a particular gaming software is related to internet speed, x Mbps. The variables x and t are related by the formula $t = \frac{a}{b + x}$, where a and b are positive constants. The data below shows some measured values of x and t.

x (Mbps)	50	100	200	500	1000
t (minutes)	23	13	7.3	3.2	1.6

(i) Plot $\frac{1}{t}$ against x and draw a straight line graph. [2]



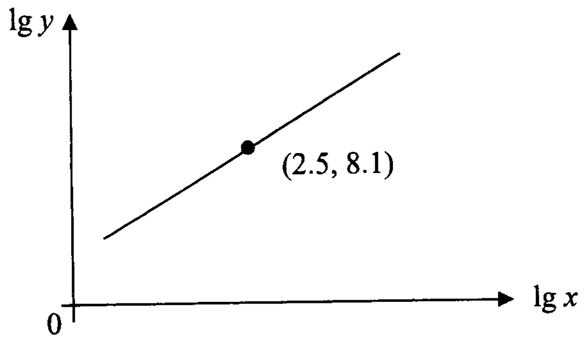
(ii) Use the graph to estimate the value of a and of b, correct to 2 significant figures. [4]

(iii) Use the graph to estimate the time taken for John to download the gaming software if the speed of his internet is 240 Mbps. [2]

(iv) Suggest an alternative set of variables to be plotted on each axis to draw a straight line to represent the formula. [1]

(b) The diagram shows part of a straight line graph drawn to represent the equation $y = px^q$.

Given that the line passes through the point (2.5, 8.1) and has a gradient of 3, find the value of p correct to the nearest integer. [3]



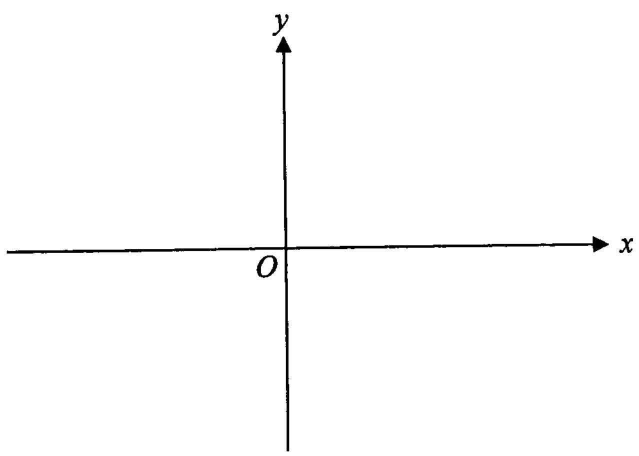
5 **(a)** Find the range of values of the constant k for which $k\left(x^2+x\right)+2\left(x^2+x+1\right)=0$ has no real roots. [4]

(b) Hence, explain why $-\left(x^2+x\right)+2\left(x^2+x+1\right)$ is always positive. [2]

6 **(a)** By using a suitable substitution, or otherwise, solve the equation

$3\log_6 x = \log_x 3 + \log_x 2 + \log_5 25$. [4]

(b) Sketch the graph of $y = \log_6 x$ on the given axes. Label any axial intercepts. [1]



(c) (i) It is given that the equation of a curve is $y = e^{2x+1} + 2e^{x+1} - 3e$.

Explain why the curve has no stationary point. [3]

(ii) Find the x-intercept of the curve $y = e^{2x+1} + 2e^{x+1} - 3e$. [3]

7 **(a)** Factorise $x^3 - 125$. [1]

(b) The expression $-2x^3 + 7x^2 + ax + b$, where a and b are integers, has a factor of $x - 5$ and a remainder of -11 when divided by $2x + 1$. Find the value of a and of b. [4]

(c) Hence, or otherwise, show that the equation $x^3 - 125 = -2x^3 + 7x^2 + ax + b$ has only one real root. [4]

8 The equation of a curve is $y = e^{-2x} \tan x$.

(a) Show that $\frac{dy}{dx} = e^{-2x}(1 - \tan x)^2$. [3]

(b) Find, in terms of π, the x-coordinate of the stationary point for $0 < x < \frac{\pi}{2}$. [2]

(c) Explain why y is never decreasing. [2]

(d) What does your answer to **part (c)** imply about the stationary point found in **part (b)**? Explain your answer. [2]

9 (a) State

(i) the principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, [1]

(ii) the values between which the principal value of $\tan^{-1}x$ must lie, [1]

(iii) the range of values of k for which the equation $4\sin^2 x = k$ has a solution. [1]

(b) **(i)** Prove the identity $\cot x - \cot x \tan^2 x + \tan x = \dfrac{1+\cos 2x}{\sin 2x}$. [4]

(ii) Hence solve the equation $\cot x - \cot x \tan^2 x + \tan x = \operatorname{cosec} 2x - 3$ for $0° \leq x \leq 180°$.

[5]

10 **(a)** Points $P(-2, 0)$, $Q(5, 1)$, $R(6, -6)$ and $S(-1, -7)$ lie on a circle.

PQ and SR are parallel chords of the same length.

Show that the centre of the circle is at $(2, -3)$. [2]

(b) Find the equation of the circle. [3]

(c) A second circle has centre C. It is given that the y-axis is the perpendicular bisector of CR and is also a tangent to the second circle.

(i) State the coordinates of C. [1]

(ii) Hence explain why the x-axis is a tangent to the second circle. [2]

11 A particle moves from point P towards point O. The speed of the particle, t seconds after passing point P, can be modelled by the formula $v = (k - 2t)^n$, where k and n are constants.

(a) Find an expression for the acceleration of the particle in terms of k, t and n. [1]

The particle has a speed of 27 m/s when $t = 3.5$ and comes to instantaneous rest 4.8 metres to the right of point O when $t = 8$.

(b) Find the value of k and of n. [3]

(c) Using the values of k and n found in **part (b)**, find the total distance travelled by the particle in the first 8 seconds. [4]

(d) Explain whether the given model can continue to describe the motion of the particle after 8 seconds. [1]

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