ST JOSEPH'S INSTITUTION
PRELIMINARY EXAMINATION 2025
(YEAR 4) ADDITIONAL MATHEMATICS 4049/01

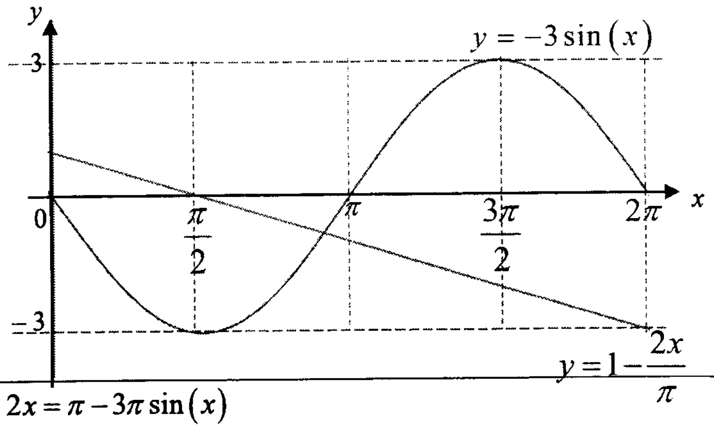
Qn	Answer
1	$x\sqrt{18}-\sqrt{72}=x\sqrt{3}$ $x\sqrt{18}-x\sqrt{3}=\sqrt{72}$ $x\left(\sqrt{18}-\sqrt{3}\right)=\sqrt{72}$ $x=\dfrac{\sqrt{72}}{\left(\sqrt{18}-\sqrt{3}\right)}$ $x=\dfrac{\sqrt{72}}{\left(\sqrt{18}-\sqrt{3}\right)}\times\dfrac{\left(\sqrt{18}+\sqrt{3}\right)}{\left(\sqrt{18}+\sqrt{3}\right)}$ $=\dfrac{\sqrt{1296}+\sqrt{216}}{18-3}$ $=\dfrac{36+6\sqrt{6}}{15}$ $\underline{a=12,}\ \ \underline{b=6}$
2	$\int\left(x-\dfrac{5}{2x}\right)^2\mathrm{d}x=\int\left(x^2-5+\dfrac{25}{4}x^{-2}\right)\mathrm{d}x$ $=\underline{\dfrac{x^3}{3}-5x-\dfrac{25}{4x}+c}$
3	$x^2+3xy=x+6$ ----- (1) $2y-x=2$ $y=\dfrac{1}{2}x+1$ ----- (2) Sub (2) into (1); $x^2+3x\left(\dfrac{1}{2}x+1\right)=x+6$ $x^2+\dfrac{3}{2}x^2+3x-x-6=0$ $2x^2+3x^2+4x-12=0$ $(5x-6)(x+2)=0$ $\underline{x=\dfrac{6}{5}\text{ or }x=-2}$

4	$-2x^2+6x-9=-2\left(x^2-3x+\frac{9}{2}\right)$ $=-2\left(x^2-3x+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+\frac{9}{2}\right)$ $=-2\left(\left(x-\frac{3}{2}\right)^2+\frac{9}{4}\right)$ $=-2\left(x-\frac{3}{2}\right)^2-\left(\frac{9}{2}\right)$ $-2\left(x-\frac{3}{2}\right)^2-\left(\frac{9}{2}\right)$ Since $\left(x-\frac{3}{2}\right)^2\geq 0$ $-2\left(x-\frac{3}{2}\right)^2\leq 0$ **and** $-2\left(x-\frac{3}{2}\right)^2-\left(\frac{9}{2}\right)\leq-\left(\frac{9}{2}\right)$ for all real values of x. Therefore $-9+6x-2x^2$ **is negative** for all real values of x.
5(a)	Let x = Angle QPA *angle APB* $= x$ (*AP* bisects angle *QPB*) *Angle ABP* $= x$ (alt. seg thm) **Since angle *APB* = angle *ABP* = *x*** **Triangle ABP is isosceles**
5(b)(i)	Angle $ACP = = x$ ($\angle$ in the same seg) Angle ACP = angle $APD = x$ Angle CAP = angle PAD (common $\angle$) Triangle ACP and Triangle APD (AA Similarity Test)

5(b)(ii)	$\frac{AD}{AP}=\frac{AP}{AC}$ $AP^2=AC\times AD\quad (\because AP=AB)$ $AB^2=AC\times AD$
6	$\frac{2x^4+x^3+9x^2+7x+3}{(x+1)(x^2+5)}$ Long division to get the quotient and remainder $\frac{2x^4+x^3+9x^2+7x+3}{(x+1)(x^2+5)}$ $=2x-1+\frac{2x+8}{(x+1)(x^2+5)}$ $\frac{2x+8}{(x+1)(x^2+5)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+5}$ $2x+8=A(x^2+5)+(Bx+C)(x+1)$ Let $x=-1$ $2(-1)+8=A((-1)^2+5)$ $6=6A$ $A=1$ Let $x=0$ $8=1(5)+(C)(1)$ $C=3$ Let $x=2$ $2(2)+8=1(2^2+5)+(2B+3)(2+1)$ $2B+3=1$ $B=-1$ $2x-1+\frac{2x+8}{(x+1)(x^2+5)}$ $=2x-1+\frac{1}{(x+1)}+\frac{3-x}{(x^2+5)}$

7(a)	$2x^3+ax^2+bx-10=(x-1)^2(2x+c)$ Comparing constant, $c=-10$ Comparing x^2, $a=-10-4=\underline{\mathbf{-14}}$ Comparing x, $b=20+2=\underline{\mathbf{22}}$
7(b)	$2\left(\frac{1}{2}\right)^3-14\left(\frac{1}{2}\right)^2+22\left(\frac{1}{2}\right)-10+k=0$ $\underline{k=\frac{9}{4}}$
8(a)	$750=\frac{4}{3}\pi x^3+\pi(x)^2y$ $\pi(x)^2y=750-\frac{4}{3}\pi x^3$ $y=\frac{750-\frac{4}{3}\pi x^3}{\pi(x)^2}$ $y=\frac{750}{\pi x^2}-\frac{4x}{3}$
8(b)	$S=4\pi x^2+2\pi x\left(\frac{750}{\pi x^2}-\frac{4x}{3}\right)$ $\underline{=4\pi x^2+\frac{1500\pi x}{\pi x^2}-\frac{8\pi x^2}{3}}$ $=\frac{4}{3}\pi x^2+\frac{1500}{x}$

8(c)	$S=\frac{4}{3}\pi x^2+\frac{1500}{x}$ $\frac{dS}{dx}=\frac{8}{3}\pi x-\frac{1500}{x^2}$ $\frac{8}{3}\pi x-\frac{1500}{x^2}=0$ $\frac{8}{3}\pi x=\frac{1500}{x^2}$ $8\pi x^3=4500$ $x=\sqrt[3]{\frac{4500}{8\pi}}$ (=5.6362) Therefore, $S=\frac{4}{3}\pi\left(\sqrt[3]{\frac{4500}{8\pi}}\right)^2+\frac{1500}{\sqrt[3]{\frac{4500}{8\pi}}}$ ≈ 399.201 = **<u>399 cm²</u>**
9(a)	$\underline{a=3}$ Sub $\left(\frac{2\pi}{3},0\right)$ into $y=a+6\cos bx$ $0=3+6\cos b\left(\frac{2\pi}{3}\right)$ $\cos b\left(\frac{2\pi}{3}\right)=-\frac{1}{2}$ $b\left(\frac{2\pi}{3}\right)=\frac{2\pi}{3}\text{(rej)},\frac{4\pi}{3}$ $\underline{b=2}$

9(b)(i)	$y = 1 - \frac{2x}{\pi}$ $\Rightarrow y = -3\sin x$ 
9(b)(ii)	$2x = \pi - 3\pi\sin(x)$ $\frac{2x}{\pi} = 1 - 3\sin x$ $-3\sin(x) = 1 - \frac{2x}{\pi}$ No. of solution(s) = **1**
10(a)	$\frac{8-4}{k+2} \times \frac{8-4}{k-6} = -1$ $\frac{4}{k+2} \times \frac{4}{k-6} = -1$ $(k+2)(k-6) = -16$ $k^2 - 4k - 12 + 16 = 0$ $k^2 - 4k + 4 = 0$ $(k-2)^2 = 0$ $\underline{k=2}$
10(b)	Let the coordinates of E be (E_x, E_y) $\left(\frac{2+E_x}{2}, \frac{8+E_y}{2}\right) = (-2, 4)$ $(E_x, E_y) = (-6, 0)$ $2 \times \frac{1}{2}\begin{vmatrix} 2 & -6 & 2a & 2 \\ 8 & 0 & -a & 8 \end{vmatrix} = 96$

	$\left[6a+16a-\left(-48-2a\right)\right]=96$ $24a=48$ $a=2$ $D\ \underline{(4,-2)}$ **Or** $\frac{-a-4}{2a+2}=\frac{8-4}{2-6}$ $\frac{-a-4}{2a+2}=-1$ $a+4=2a+2$ $a=-2$ $D(4,-2)$
11(a)	$\cot\theta-1=\frac{\cos 2\theta}{\sin\theta(\cos\theta+\sin\theta)}$ RHS$=\frac{\cos 2\theta}{\sin\theta(\cos\theta+\sin\theta)}$ $=\frac{\cos^2\theta-\sin^2\theta}{\sin\theta(\cos\theta+\sin\theta)}$ $=\frac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{\sin\theta(\cos\theta+\sin\theta)}$ $=\frac{(\cos\theta-\sin\theta)}{\sin\theta}$ $=\cot\theta-1$ $=$RHS (proven)
11(b)	$\frac{\cos 4\theta}{\sin 2\theta(\cos 2\theta+\sin 2\theta)}=-\frac{1}{2}$ $\cot 2\theta-1=-\frac{1}{2}$

	$\cot 2\theta = \frac{1}{2}$ $\tan 2\theta = 2$ $-180° \le \theta \le 180°$ $-360° \le 2\theta \le 360°$ Basic angle = 63.4349 $2\theta = -360 + 63.4349$ or $-180 + 63.4349$ or 63.4349 or $180 + 63.4349$ $\underline{\theta = -148.3°, -58.3°, 31.7°, 121.7°}$
12(a) (i)	$90 = 100\left(4^{-k(40)}\right)$ $0.9 = \left(4^{-k(40)}\right)$ $\lg 0.9 = \lg 4^{-k(40)}$ $\lg 0.9 = -40k \lg 4$ $k = 0.00190003866$ $\underline{k = 0.00190\,(3sf)}$
12(a) (ii)	$M = 100\left(4^{-0.00190003866(263)}\right)$ ≈ 50.02 Therefore, % remaining is **<u>50.0%</u>**.
12(b)	$\log_5 x + \log_{25}(x-2) = 1$ $\log_5 x + \frac{\log_5(x-2)}{\log_5 25} = 1$ $\log_5 x + \frac{\log_5(x-2)}{2} = 1$ $\log_5 x^2 + \log_5(x-2) = 2$ $\underline{x^2(x-2) = 25}$ $\underline{x^3 - 2x^2 - 25 = 0}$

13(a)	Area $= \frac{1}{2}(x)(3x)\sin\frac{2\pi}{3}$ $= \frac{3\sqrt{3}x^2}{4}$ $\frac{dA}{dx} = \frac{3\sqrt{3}}{2}x$ $\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$ $= \frac{3\sqrt{3}}{2}(4) \times 0.5$ $= \underline{3\sqrt{3}\ \text{cm}^2/\text{s}}$
13(b)	$F = \frac{k}{\sqrt{r}}$ Sub $F = 0.5$, $r = 9$, $0.5 = \frac{k}{\sqrt{9}}$ $k = 1.5$ $r = \left(\frac{1.5}{F}\right)^2$ $\frac{dr}{dF} = -\frac{2(1.5)^2}{F^3}$, sub $F = 0.5$ $\frac{dr}{dF} = -\frac{2(1.5)^2}{(0.5)^3}$ $= \underline{-36}$ units per unit increase in F

14(a)	Sub $y = 2\sqrt{2+x}$ into $y = 2x$ $2\sqrt{2+x} = 2x$ $4(2+x) = 4x^2$ $x^2 - x - 2 = 0$ $(x-2)(x+1) = 0$ $x = 2$ or $x = -1$ (rej) $\frac{dy}{dx} = 2\left(\frac{1}{2}\right)(2+x)^{-\frac{1}{2}} \times (1)$ $= (2+x)^{-\frac{1}{2}}$ Sub $x = 2$, $\frac{dy}{dx} = [2+2]^{-\frac{1}{2}}$ $= \frac{1}{2}$ $m_{\perp} = -2$ Let $R = (a, 0)$ $-2 = \frac{4-0}{2-a}$ $2 - a = \frac{4}{-2}$ $a = 4$ $\therefore R = (4,0)$
14(b)	y-coord of $P = 4$ Distance $OP = \sqrt{4^2 + 2^2} = \sqrt{20}$ Distance $PR = \sqrt{(2-4)^2 + 2^2} = \sqrt{20}$ **Since $OP = PR = \sqrt{20}$, therefore OPR is an isosceles triangle.**

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Qn	Solutions
1(a)	$2^{2x+1} = 5(2^x) + 3$ Let $u = 2^x$ $\underline{2u^2 - 5u - 3 = 0}$
1(b)	$(2u+1)(u-3) = 0$ $u = -\frac{1}{2}$ or $u = 3$ $2^x > 0$ therefore $2^x \neq -\frac{1}{2}$ $2^x = 3$ $\lg 2^x = \lg 3$ $x = \frac{\lg 3}{\lg 2}$ $= \underline{\mathbf{1.6}}$ (2 sf)
2(a)	$(2x+1)^3 - (2x-1)^3 = \left[(2x+1)-(2x-1)\right]\left[(2x+1)^2 + (2x+1)(2x-1) + (2x-1)^2\right]$ $= (2)\left[4x^2 + 4x + 1 + 4x^2 - 1 + 4x^2 - 4x + 1\right]$ $= 2(12x^2 + 1)$
2(b)	$2(2x+1)(12x^2+1) = 0$ $12x^2 + 1 = 0$ or $2x + 1 = 0$ Discriminant $= 0^2 - 4(12)(1)$ $= -48 < 0$ No real roots $\underline{x = -\frac{1}{2}}$ is the only real root.

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3 (a)	$y=(x-3)\sqrt{x+1}$ $\frac{dy}{dx}=(1)\sqrt{x+1}+(x-3)\left(\frac{1}{2}\right)(x+1)^{-\frac{1}{2}}(1)$ $=\sqrt{x+1}+\frac{x-3}{2\left(\sqrt{x+1}\right)}$ $=\frac{2(x+1)+x-3}{2\sqrt{x+1}}$ $=\frac{2x+2+x-3}{2\sqrt{x+1}}$ $=\underline{\frac{3x-1}{2\sqrt{x+1}}}$
3(b)	$\int_0^3 \frac{3x-1}{2\sqrt{x+1}}\ dx=\left[(x-3)\sqrt{x+1}\right]_0^3$ $6\int_0^3\left(\frac{3x-1}{2\sqrt{x+1}}+\frac{1}{2\sqrt{x+1}}\right)\mathrm{d}x=6\left\{\left[(x-3)\sqrt{x+1}\right]_0^3+\int_0^3\frac{1}{2\sqrt{x+1}}\mathrm{d}x\right\}$ $\int_0^3\left(\frac{9x}{\sqrt{x+1}}\right)\mathrm{d}x=6\left\{\left[(x-3)\sqrt{x+1}\right]_0^3+\int_0^3\frac{1}{2\sqrt{x+1}}\mathrm{d}x\right\}$ $=6\left\{\left[(3-3)\sqrt{3+1}-(0-3)\sqrt{0+1}\right]+\left[\frac{\frac{1}{2}(x+1)^{1/2}}{\frac{1}{2}}\right]_0^3\right\}$ $=6\left\{(3)+\left[(3+1)^{1/2}-(0+1)^{1/2}\right]\right\}$ $=6\left[(3)+(2-1)\right]$ $=\underline{24}$

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4(a)	*Let E be a point below C (and to the left of D)* In ΔCDE, $\cos\theta = \dfrac{CE}{80}$ $CE = 80\cos\theta$ *Let G be a point below D (and to the right of A)* In ΔADG, $\sin\theta = \dfrac{DG}{185}$ $DG = 185\sin\theta$ $h = CE + EF$ $h = 185\sin\theta + 80\cos\theta$ (shown)
4(b)	$h = 185\sin\theta + 80\cos\theta$ $185\sin\theta + 80\cos\theta = R\sin(\theta+\alpha)$ $R\sin\alpha = 80$ $R\cos\alpha = 185$ $R = \sqrt{80^2 + 185^2}$ $= \sqrt{40625}$ $= 25\sqrt{65}$ $\alpha = \tan^{-1}\left(\dfrac{80}{185}\right)$ $= 23.385°$ $\therefore h = 25\sqrt{65}\sin(\theta + 23.4°)$
4(c)	To clear the kitchen door, max h is 180cm, When h = 180cm $25\sqrt{65}\sin(\theta + 23.385°) = 180$ $\sin(\theta + 23.385°) = \dfrac{180}{25\sqrt{65}}$ $\sin(\theta + 23.385°) = \dfrac{36}{5\sqrt{65}}$ Basic angle =63.259° $\theta + 23.385° = 63.259°$ $\theta = 39.9°$

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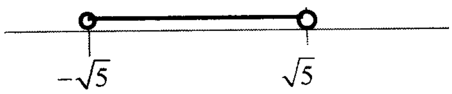
5(a)	$y=2x^2+5$ $2x^2+5>3x^2$ $x^2-5<0$ $(x+\sqrt{5})(x-\sqrt{5})<0$ $\underline{-\sqrt{5}<x<\sqrt{5}}$ 
5(b)	$y=2x^2+5$ ---- (1) $y-kx=3$ $y=kx+3$ ---- (2) $2x^2+5=kx+3$ $2x^2-kx+2=0$ $b^2-4ac=0$ $(-k)^2-4(2)(2)=0$ $k^2=16$ $\underline{k=\pm 4}$
5(c)	$2x^2-4x+2=0$ $x^2-2x+1=0$ $(x-1)^2=0$ $x=1$ $y=4(1)+3$ $\quad=7$ $\underline{P(1,7)}$
6(a)	$\left(ax+\frac{2}{x}\right)^6$ Term $\binom{6}{r}(ax)^{6-r}\left(\frac{2}{x}\right)^r$ $\frac{x^{6-r}}{x^r}=x^{6-2r}$ Powers of x $6-2r=2(3-r)$ Since the power/exponent can be expressed **as a multiple of 2**, therefore all the expansion only contains even powers of x.

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6(b)	$\left(1-\frac{x}{a}\right)^3 = \underline{1-\frac{3x}{a}+\frac{3x^2}{a^2}+\ldots}$
6(c)	$\left(ax+\frac{2}{x}\right)^6$ Term in $\frac{1}{x^4}$ $x^{6-2r}=x^{-4}$ $6-2r=-4$ $r=5$ $\frac{1}{x^4}$ term $=\binom{6}{5}(ax)\left(\frac{2}{x}\right)^5=192a\left(\frac{1}{x^4}\right)$ $\left(1-\frac{x}{a}\right)^3$ Term in $x^2=\frac{3}{a^2}(x^2)$ from (b) $192a=\frac{3}{a^2}\times 1728$ $a^3=\frac{3\times 1728}{192}$ $a^3=27$ $\underline{a=3}$
6(d)	Term independent of x $\left(1-\frac{3x}{a}+\frac{3x^2}{a^2}+\ldots\right)\left(1+\frac{1}{x}\right)$ $=(1)(1)+\left(-\frac{3x}{3}\right)\left(\frac{1}{x}\right)+\ldots$ There is no term independent of x.

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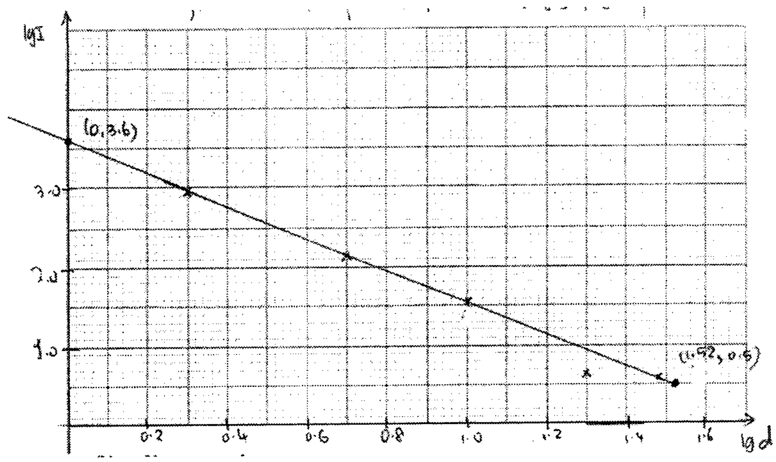
7(a)

d	2	5	10	20	30
I	875	141	35	4.22	3.89
$\lg d$	0.30	0.70	1.00	1.30	1.48
$\lg I$	2.94	2.15	1.54	0.63	0.59



7(b) The incorrect reading of I is 4.22.

From the graph, the correct $\lg I$ reading is 0.95.

$0.95 = \lg I$

$I = 10^{0.95}$

$I = 8.91$ (3 sf)

7(c) $I = Ad^n$

$\lg I = n \lg d + \lg A$

From the graph

$\lg A = 3.6$

$A = 10^{3.6}$

≈ 3981.071

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	$=3980\ (3\text{sf})$ $\text{Gradient} = \dfrac{0.5-3.6}{1.52-0}$ ≈ -2.0394 $= -2.04\ (3\text{sf})$ $n = -2.04$
7(c)	From the table, $\dfrac{I_1}{I_2} = \dfrac{35}{141} = 0.248$ (to 3 sf) When the distance doubled, the intensity decrease to 0.248 (one-quarter) of its previous value
8(a)	$v = 5 - e^{3-2t}$ $5 - e^{3-2t} = 0$ $e^{3-2t} = 5$ $\ln e^{3-2t} = \ln 5$ $3 - 2t = \ln 5$ $t = 0.695$

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8(b)	Distance in the first second $= \left\|\int_{0.69528}^{1}\left(5-e^{3-2t}\right)dt\right\| + \left\|\int_{0}^{0.69528}\left(5-e^{3-2t}\right)dt\right\|$ $= \left\|\left[5t-\left(\frac{e^{3-2t}}{-2}\right)\right]_{0.69528}^{1}\right\| + \left\|\left[5t-\left(\frac{e^{3-2t}}{-2}\right)\right]_{0}^{0.69528}\right\|$ $= \left\|\left[5(1)+\left(\frac{e^{3-2(1)}}{2}\right)\right]-\left[5(0.69528)+\left(\frac{e^{3-2(0.69528)}}{2}\right)\right]\right\|$ $+\left\|\left[5(0.69528)+\left(\frac{e^{3-2(0.69528)}}{2}\right)\right]-\left[5(0)+\left(\frac{e^{3-2(0)}}{2}\right)\right]\right\|$ $= 0.3827356 + \|-4.06636\|$ $= 4.449$ $=$ **4.45 m**
8(c)	$v = 5-e^{3-2t}$ as t approaches ∞, e^{3-2t} approaches 0 $v =$ **5 m/s** **The particle's velocity cannot exceed 5 m/s,** and this is the greatest velocity attained.
9(a)	$m_t = \frac{3-0}{-3-0}$ $= -1$ Gradient of normal = 1 Equation of normal is $y-3=1(x+3)$ $\underline{y=x+6}$
9(b)	Equation of normal is $y=-\frac{2}{5}x-1$ --- (1) $y=x+6$ --- (2)

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	Sub (1) into (2) $-\frac{2}{5}x-1=x+6$ $\frac{7}{5}x=-7$ $x=-5$ Centre = $(-5,1)$ $r^2=(-3+5)^2+(3-1)^2$ $=8$ Therefore equation of circle is $\underline{(x+5)^2+(y-1)^2=8}$
9(c)	$\underline{(-5-2\sqrt{2},1)}$
10	$y=\sin 2x+x$ $\frac{\mathrm{d}y}{\mathrm{d}x}=2\cos 2x+1$ $2\cos 2x+1=0$ $\cos 2x=-\frac{1}{2}$ Basic angle = $\frac{\pi}{3}$ $2x=\pi-\frac{\pi}{3},\pi+\frac{\pi}{3}$ Min point $2x=\frac{4\pi}{3}$ $x=\frac{2\pi}{3}$ $y=\sin\left(\frac{4\pi}{3}\right)+\frac{2\pi}{3}$ $=-\frac{\sqrt{3}}{2}+\frac{2\pi}{3}$

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Area of region below shaded area

$=\frac{1}{2}\left(-\frac{\sqrt{3}}{2}+\frac{2\pi}{3}\right)\left(\frac{2\pi}{3}\right)$

$=\frac{2\pi^2}{9}-\frac{\sqrt{3}\pi}{6}$

Area under graph

$=\int_0^{\frac{2\pi}{3}}\left(\sin(2x)+x\right)\,dx$

$=\left[-\frac{1}{2}\cos 2x+\frac{x^2}{2}\right]_0^{\frac{2\pi}{3}}$

$=\left[-\frac{1}{2}\cos 2\left(\frac{2\pi}{3}\right)+\frac{\left(\frac{2\pi}{3}\right)^2}{2}\right]-[-\frac{1}{2}\cos 2(0)]$

$=\left[\frac{1}{4}+\frac{2\pi^2}{9}\right]+\frac{1}{2}$

$=\frac{2\pi^2}{9}+\frac{3}{4}$

Area of shaded region $=\frac{2\pi^2}{9}+\frac{3}{4}-\left(\frac{2\pi^2}{9}-\frac{\sqrt{3}\pi}{6}\right)$

$=\frac{\sqrt{3}\pi}{6}+\frac{3}{4}$ (shown)

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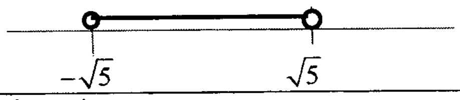
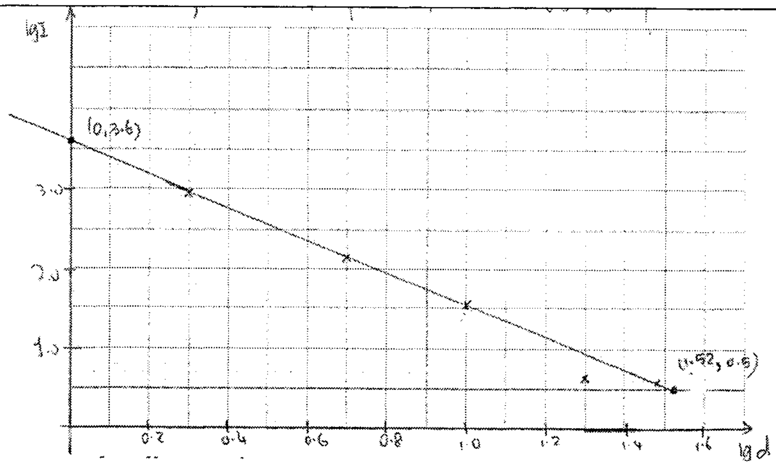
Answer Key

[Turn over

Qn	Solutions	
1(a)	$\underline{2u^2 - 5u - 3 = 0}$	
1(b)	$x = 1.6$	
2(a)	$2\left(12x^2 + 1\right)$	
2(b)	$\underline{x = -\frac{1}{2}}$ is the only real root.	
3 (a)	$\underline{\frac{3x-1}{2\sqrt{x+1}}}$	
3(b)	**24**	
4(a)	$h = 185\sin\theta + 80\cos\theta$ (shown)	
4(b)	$\underline{h = 25\sqrt{65}\sin(\theta + 23.4°)}$	
4(c)	$\underline{\theta = 39.9°}$	
5(a)	$\underline{-\sqrt{5} < x < \sqrt{5}}$ 	
5(b)	$\underline{k = \pm 4}$	
5(c)	$\underline{P(1, 7)}$	
6(a)	x^{6-2r} Powers of x $6 - 2r = 2(3 - r)$ Since the power/exponent can be expressed **as a multiple of 2**,	
6(b)	$\underline{1 - \frac{3x}{a} + \frac{3x^2}{a^2} + \ldots}$	
6(c)	$\underline{a = 3}$	
6(d)	Show is no term independent of x.	
7(a)		

		
7(b)	incorrect reading of I is 4.22. $\underline{I = 8.91\ (3\text{ sf})}$	
7(c)	$A = \underline{3980\ (3\text{sf})}$ $\underline{n = -2.04}$	
7(c)	the intensity decrease to 0.248 (one-quarter) of its previous value	
8(a)	$\underline{t = 0.695}$	
8(b)	Distance in the first second = **4.45 m**	
8(c)	**The particle's velocity cannot exceed 5 m/s.**	
9(a)	$\underline{y = x + 6}$	
9(b)	$\underline{(x+5)^2 + (y-1)^2 = 8}$	
9(c)	$\underline{\left(-5-2\sqrt{2}, 1\right)}$	
10	$\frac{dy}{dx} = 2\cos 2x + 1$	

[Turn over

	$x=\frac{2\pi}{3}$ $y=-\frac{\sqrt{3}}{2}+\frac{2\pi}{3}$ Area of shaded region $=\frac{2\pi^2}{9}+\frac{3}{4}-\left(\frac{2\pi^2}{9}-\frac{\sqrt{3}\pi}{6}\right)$ $=\frac{\sqrt{3}\pi}{6}+\frac{3}{4}$ (shown)	

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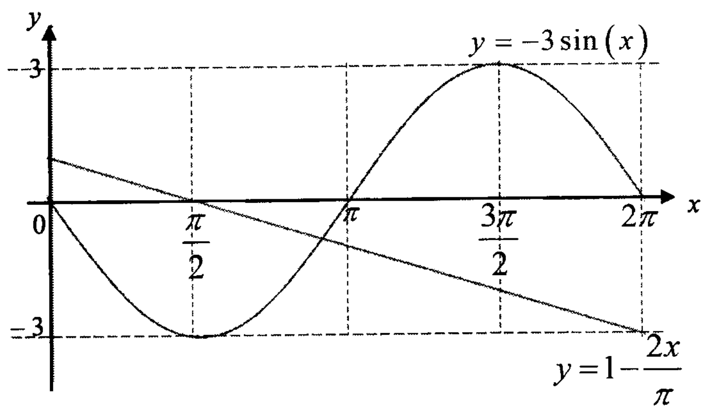
(YEAR 4) ADDITIONAL MATHEMATICS 4049/01
Answer Key

Qn	Answer	
1	$a=12$, $b=6$	
2	$\frac{x^3}{3}-5x-\frac{25}{4x}+c$	
3	$x=\frac{6}{5}$ or $x=-2$	
4	$-2\left(x-\frac{3}{2}\right)^2-\left(\frac{9}{2}\right)$ $-2\left(x-\frac{3}{2}\right)^2-\left(\frac{9}{2}\right)\le-\left(\frac{9}{2}\right)$ for all real values of x. Therefore $-9+6x-2x^2$ **is negative** for all real values of x.	
5(a)	**angle *APB* = angle *ABP*** **Triangle ABP is isosceles**	
5(b)(i)	Triangle *ACP* and Triangle *APD* (AA Similarity Test)	
5(b)(ii)	$AP^2=AC\times AD \quad (\because AP=AB)$ $AB^2=AC\times AD$	
6	$2x-1+\frac{1}{(x+1)}+\frac{3-x}{(x^2+5)}$	
7(a)	$c=-10$ $a=-14$ $b=22$	
7(b)	$k=\frac{9}{4}$	

[Turn over

8(a)	$y=\frac{750}{\pi x^2}-\frac{4x}{3}$	
8(b)	$S=4\pi x^2+2\pi x\left(\frac{750}{\pi x^2}-\frac{4x}{3}\right)$	
8(c)	$x=\sqrt[3]{\frac{4500}{8\pi}}$ (=5.6362) $S=399$ **cm²**	
9(a)	$\underline{a=3}$ $\underline{b=2}$	
9(b)(i)		
9(b)(ii)	No. of solution(s) = **1**	
10(a)	$\underline{k=2}$	
10(b)	D $\underline{(4,-2)}$	
11(a)		
11(b)	$\theta=\underline{-148.3°,-58.3°,31.7°,121.7°}$	
12(a) (i)	$\underline{k=0.00190}$ $\underline{(3sf)}$	
12(a) (ii)	$M=50.02$ Therefore, % remaining is **50.0%**.	
12(b)	$\underline{x^3-2x^2-25=0}$	
13(a)	$\underline{3\sqrt{3}\ \text{cm}^2/\text{s}}$	
13(b)	$\frac{dr}{dF}$ is **−36** units per unit increase in F	

14(a)	$\therefore R=(4,0)$	
14(b)	**$OP=PR=\sqrt{20}$, *OPR* is an isosceles triangle.**	