ST JOSEPH'S INSTITUTION
PRELIMINARY EXAMINATION 2025
(YEAR 4)

CANDIDATE NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS **4049/01**

Paper 1 **25 August 2025**

Candidates answer on the Question Paper. **2 hours 15 minutes**
(10:50 – 13:05)

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen in the space provided.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

This document consists of **23** printed pages and **1** blank page.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

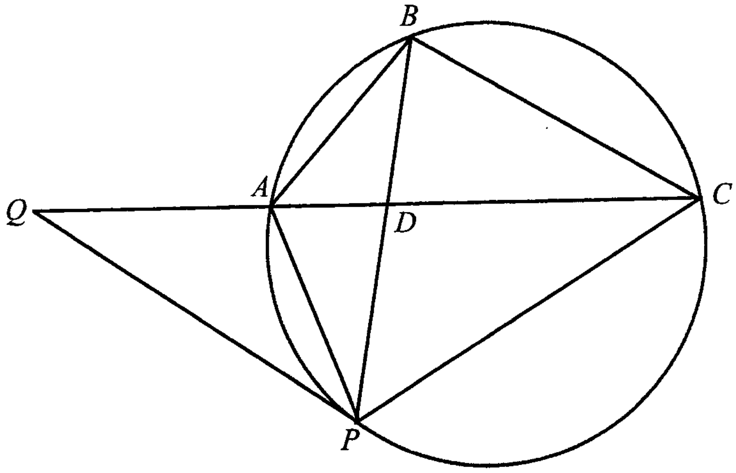
1 Without using a calculator, find the values of the integers a and b for which the solution of the equation $x\sqrt{18} - \sqrt{72} = x\sqrt{3}$ is $\frac{a + 2\sqrt{b}}{5}$. [5]

2 Integrate $\left(x - \frac{5}{2x}\right)^2$ with respect to x. [3]

3 The curve $x^2 + 3xy = x + 6$ and the line $2y - x = 2$ intersect at two points. Find the x-coordinates of the points of intersection. [3]

4 Express $-9+6x-2x^2$ in the form $a(x+b)^2+c$ and hence explain why $-9+6x-2x^2$ is negative for all real values of x. [4]

5



In the diagram, A, B, C and P lie on a circle. The line PQ is the tangent to the circle at P. The tangent meets CA produced at Q. The lines AC and PB intersect at D. AP bisects angle QPB.

(a) Show that triangle ABP is isosceles. [3]

(b) Prove that

(i) triangle ACP is similar to triangle APD, [2]

(ii) $AB^2 = AD \times AC$. [1]

[Turn over

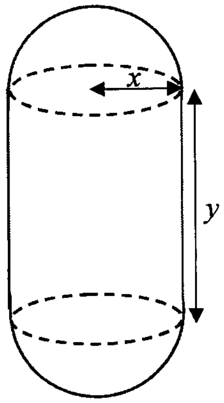
6 Express $\dfrac{2x^4 + x^3 + 9x^2 + 7x + 3}{(x+1)(x^2+5)}$ in partial fractions. [6]

7 It is given that $f(x)=2x^3+ax^2+bx-10$, where a and b are constants, has a factor of $(x-1)^2$.

(a) Find the value of a and of b. [4]

(b) It is given that $g(x)=f(x)+k$, where k is a constant. Using the values of a and b found in **(a)**, find the value of k such that $g(x)$ is divisible by $2x-1$. [2]

8



The diagram shows a solid object made from joining two hemispheres of radius x cm to a cylinder of height y cm. The volume of the object is 750 cm^3.

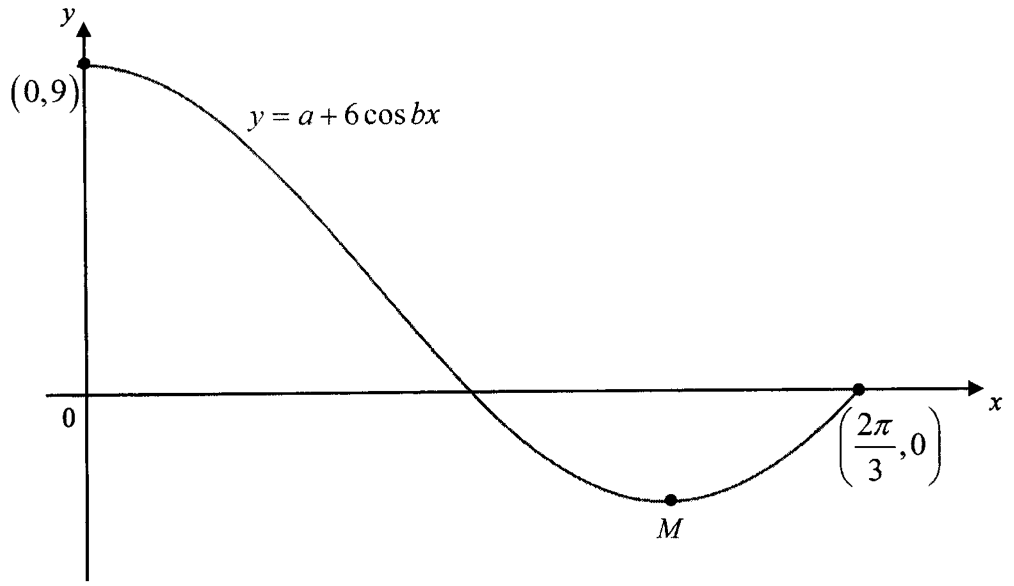
(a) Express y in terms of x. [2]

(b) Show that the surface area, S cm^2, of the object is given by $S = \frac{4}{3}\pi x^2 + \frac{1500}{x}$. [2]

(c) Given that x can vary, find the minimum value of S. [4]

(You are not required to show that S is a minimum.)

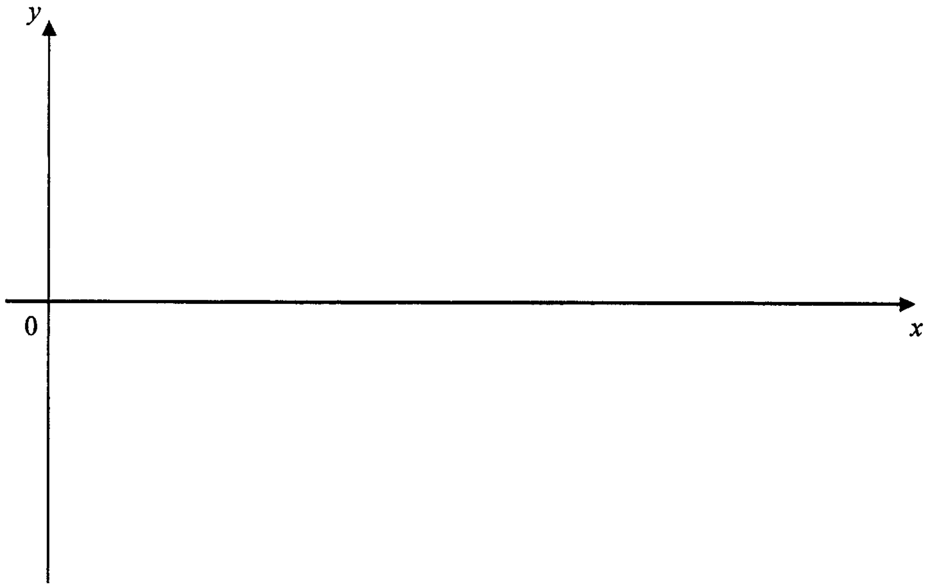
9 **(a)**



The diagram shows the curve $y = a + 6\cos bx$ for $0 \leq x \leq \frac{2\pi}{3}$, where a and b are positive constants. The curve meets the x-axis at the point $\left(\frac{2\pi}{3}, 0\right)$. The curve has a maximum point at $(0, 9)$ and a minimum point at M.

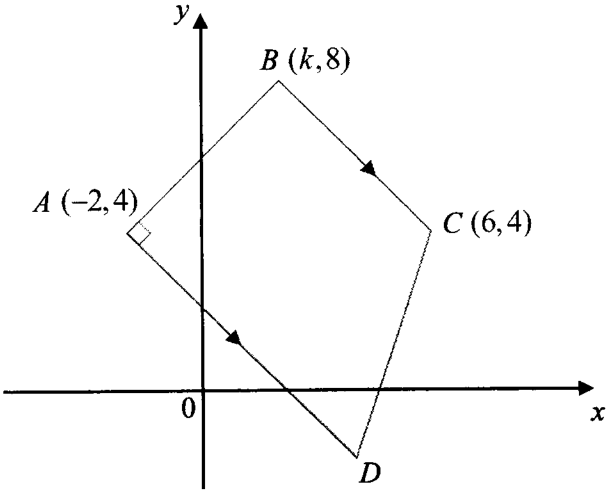
Find the value of a and of b. [3]

(b) **(i)** Sketch, on the axes below, the graphs of $y = -3\sin x$ and $y = 1 - \frac{2x}{\pi}$ for $0 \le x \le 2\pi$. [3]



(ii) Find the number of solution(s) of the equation $2x = \pi + 3\pi \sin x$ for $0 \le x \le 2\pi$. [2]

10



The diagram shows a trapezium with vertices $A(-2,4)$, $B(k,8)$, $C(6,4)$ and D.
The sides AD and BC are parallel and angle $BAD = 90°$.

(a) Show that $k = 2$. [3]

(b) *BDEF* is a quadrilateral. Point *A* is the midpoint of both *BE* and *DF*. Given that the area of *BDEF* is 96 square units, find the coordinates of *D*. [5]

11 **(a)** Prove the identity $\cot\theta - 1 = \dfrac{\cos 2\theta}{\sin\theta(\cos\theta + \sin\theta)}$. [3]

(b) Hence solve the equation $\dfrac{\cos 4\theta}{\sin 2\theta\left(\cos 2\theta + \sin 2\theta\right)} = -\dfrac{1}{2}$ for $-180° \leq \theta \leq 180°$.

[5]

12 **(a)** A certain radioactive substance is known to decay with time such that its mass, M g, after t hours is given by $M = 100\left(4^{-kt}\right)$, where k is a positive constant. The original mass of a block of substance is 100 g. After 40 hours, its mass decreased to 90 g.

(i) Find the value of k. [3]

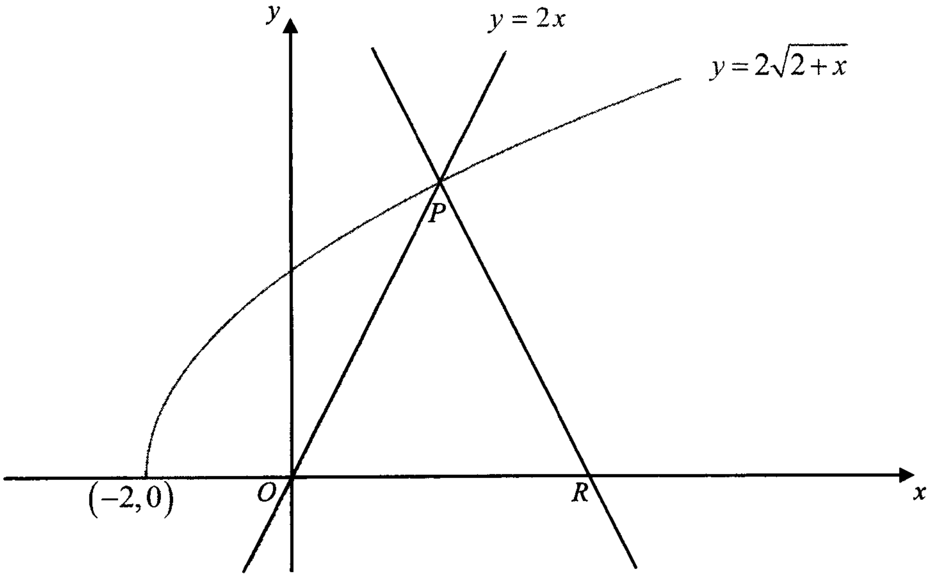
(ii) Calculate the percentage of the original mass of the block remaining when $t = 263$. [2]

(b) Express the equation $\log_5 x + \log_{25}(x-2) = 1$ as a cubic equation in x. [3]

13 **(a)** In the triangle ABC, $AB = x$ cm, $AC = 3x$ cm and angle $BAC = \frac{2\pi}{3}$. If x is increasing at the rate of 0.5 cm/s, find the exact rate at which the area is increasing when $x = 4$. [4]

(b) The variables F and r are such that F is inversely proportional to the square root of r. Given that when $F = 0.5$ and $r = 9$, find the rate at which r is changing with respect to F when $F = 0.5$. [4]

14



The diagram shows part of the graph of $y = 2\sqrt{2+x}$. It cuts the x-axis at $(-2, 0)$. The line $y = 2x$ meets the curve at P. The normal to the curve at P meets the x-axis at R.

(a) Find the coordinates of R. [6]

(b) Show that *OPR* is an isosceles triangle. [3]

BLANK PAGE

ST JOSEPH'S INSTITUTION
PRELIMINARY EXAMINATION 2025
(YEAR 4)

CANDIDATE NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS **4049/02**

Paper 2 **27 August 2025**

Candidates answer on the Question Paper. **2 hours 15 minutes**
(09:00 – 11:15)

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen in the space provided.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 **(a)** Given that $u = 2^x$, express $2^{2x+1} = 5\left(2^x\right)+3$ as an equation in u. [1]

(b) Hence find the value of x for which $2^{2x+1} = 5\left(2^x\right)+3$, giving your answer correct to 2 significant figures. [4]

2 **(a)** Factorise $(2x+1)^3-(2x-1)^3$ completely. [2]

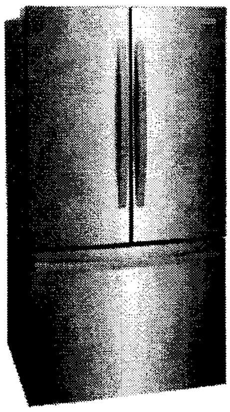
(b) Hence show that the equation $(2x+1)\left[(2x+1)^3-(2x-1)^3\right]=0$ has only one real root.

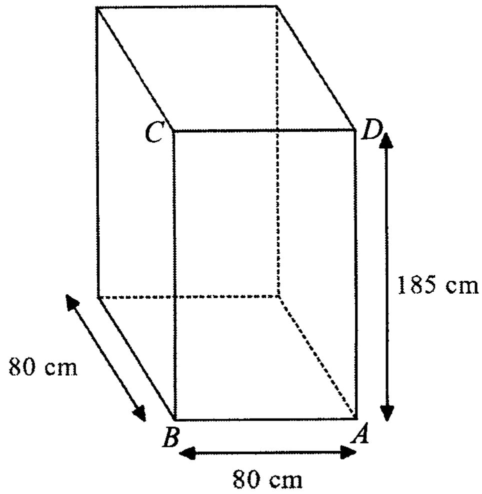
[2]

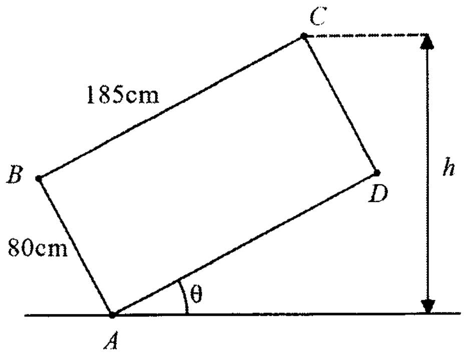
3 **(a)** It is given that $y=(x-3)\sqrt{x+1}$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in the form $\frac{ax-b}{2\sqrt{x+1}}$, where a and b are integers. [4]

(b) Hence find the value of $\int_0^3 \frac{9x}{\sqrt{x+1}}\,\mathrm{d}x$. [5]

4 A mover needs to deliver a refrigerator to a customer. The refrigerator can be modelled by a cuboid of dimensions 80 cm by 80 cm by 185 cm.







To move the refrigerator into the kitchen through a door, the mover has to tilt the refrigerator such that AD makes an acute angle θ with the horizontal ground.

(a) Show that the height, h cm, of C above ground is given by $h = 185\sin\theta + 80\cos\theta$. [2]

(b) Express h in the form $R\sin(\theta+\alpha)$, where $R>0$ and $0° < \alpha < 90°$. [3]

(c) Find the value of θ when $h = 180$. [3]

5 The equation of a curve is $y = 2x^2 + 5$.

(a) Find the set of values of x for which $y > 3x^2$ and represent this set on a number line. [4]

The line $y - kx = 3$ is a tangent to the curve at the point P.

(b) Find the possible values of k. [4]

(c) Given that $k > 0$, find the coordinates of P. [3]

6 **(a)** Explain why the binomial expansion of $\left(ax+\frac{2}{x}\right)^6$ contains only even powers of x. [2]

(b) Write down, and simplify, the first three terms in the expansion of $\left(1-\frac{x}{a}\right)^3$ in ascending powers of x. [2]

(c) Given that the coefficient of $\frac{1}{x^4}$ in the expansion of $\left(ax+\frac{2}{x}\right)^6$ is 1728 times the coefficient of x^2 in the expansion of $\left(1-\frac{x}{a}\right)^3$, find the value of a. [4]

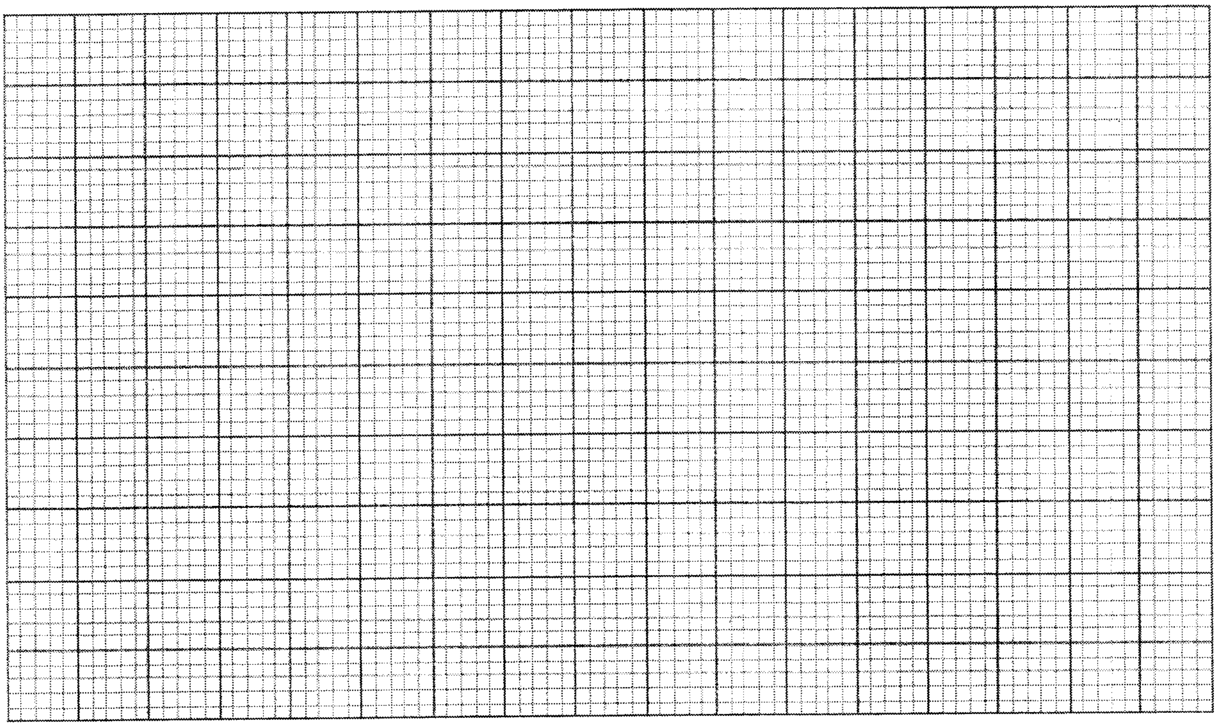
(d) Using the value of a found in **(c)**, show that there is no term independent of x for $\left(1-\frac{x}{a}\right)^3\left(1+\frac{1}{x}\right)$. [2]

7 In an experiment to investigate the relationship between light intensity I lux on a surface and distance d metres from a light source, a student obtained the following results. One of the values of I is incorrect.

d (metres)	2	5	10	20	30
I (lux)	875	141	35	4.22	3.89

It is known that I and d are related by the equation $I = Ad^n$, where A and n are constants.

(a) Plot $\lg I$ against $\lg d$ and draw a straight line graph. [3]



(b) Use your graph to

(i) identify the incorrect reading of I and estimate its correct value, [3]

(ii) estimate the value of A and of n. [4]

(c) Using the table of values, explain, with working, how the light intensity I changes when the distance d from the light source is doubled. [1]

8 A particle starts from a point O and moves in a straight line so that its velocity, v m/s, is given by $v = 5 - e^{3-2t}$ where t is the time in seconds after leaving O.

(a) Find the value of t at which the particle is instantaneously at rest. [3]

(b) Calculate the distance travelled by the particle in the first second. [4]

(c) Explain why the velocity of the particle cannot exceed a certain value. [2]

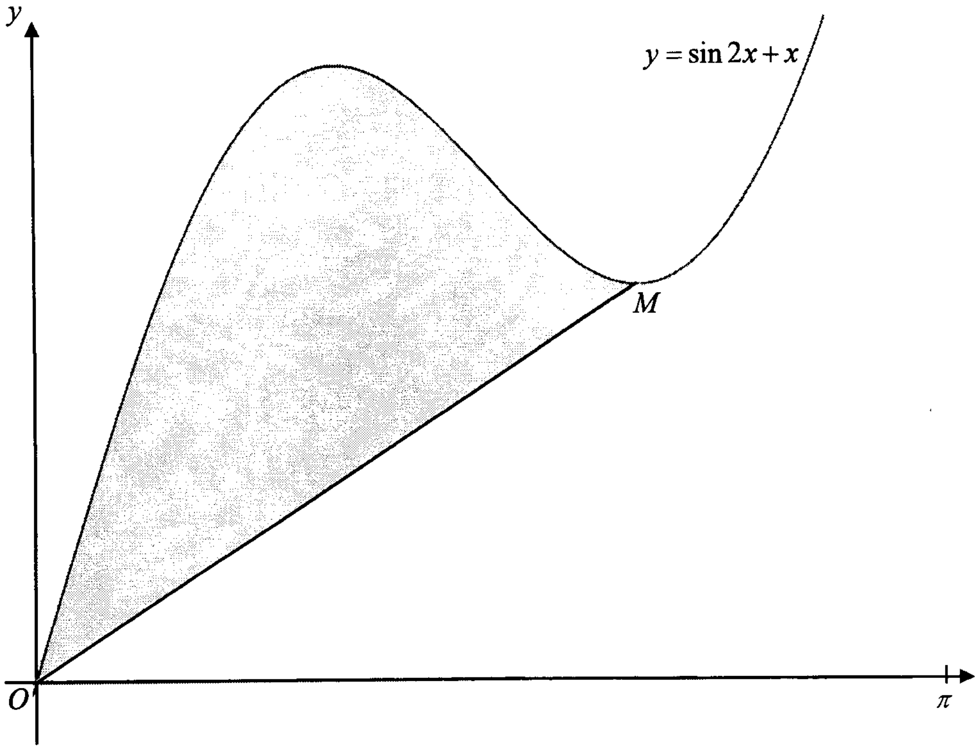
9 A tangent to a circle at the point $(-3,3)$ passes through the origin.

(a) Find the equation of the normal to the circle at the point $(-3,3)$. [3]

(b) Another normal to the circle passes through the point $(0,-1)$ and is parallel to the line $5y=-2x+7$. Find the equation of the circle. [6]

(c) Find the coordinates of the point on the circle which is farthest from the y-axis. Leave your answer in the form $(m+n\sqrt{2},k)$, where m, n and k are constants. [2]

10



The diagram shows the curve $y = \sin 2x + x$ for $0 \leq x \leq \pi$ radians. The point M is the minimum point of the curve and OM is a straight line.

Show that the area of the shaded region is $\left(\frac{\sqrt{3}\pi}{6} + \frac{3}{4}\right)$ square units. [12]

Continuation of working space for question 10.

BLANK PAGE