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Qn	Answer Key
1	$k=\frac{1}{4}\ln 2$
2	$\sqrt{8^{-4}}=\frac{1}{64}$
3	$x=8,\ -1\frac{7}{53}$ (or -1.13) $y=4,\ -27\frac{51}{53}$ (or -28.0)
4a	$\underline{\boldsymbol{x>0.5}}$ and $\underline{\boldsymbol{x\neq 1}}$
b	$x=p^{20}$
5	$y=-\frac{1}{2}\sin 4x+\cos 2x+x-\frac{7}{6}$
6a	$2x^4+5x^3-x^2+5x-3$
b	$k=-5$
7i	50 m/s
ii	125 cm
iii	Show that
8a	Show that
bi	$-1<x<2$
bii	Vertical line
9a	$y=\left(\frac{p}{q}\right)\frac{y}{x}+\left(\frac{1}{q}\right)$ $q=\frac{1}{\textbf{Vertical intercept}}$ $p=q(\textbf{Gradient})$
b	21.2
10a	$(x-2)^2+(y+3)^2=20$ OR $x^2+y^2-4x+6y-7=0$
b	$y=-3+\sqrt{20}$ & $y=-3-\sqrt{20}$
c	8 units

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d	Perpendicular bisector of ST will pass through the centre of the circle. Since ST is vertical, y-coord of $B = y$ coord of A $= \frac{1-7}{2} = -3$
e	x-coord of $B = 6$
11a	Proving
B	$x = -180°$ or $-48.2°$ or $48.2°$ or $180°$
12a	3
B	$y = \frac{1}{7}x + \frac{59}{7}$
c	f is a decreasing function
d	f' is a decreasing function
E	$2 + \frac{7}{x-3}$ c = 2

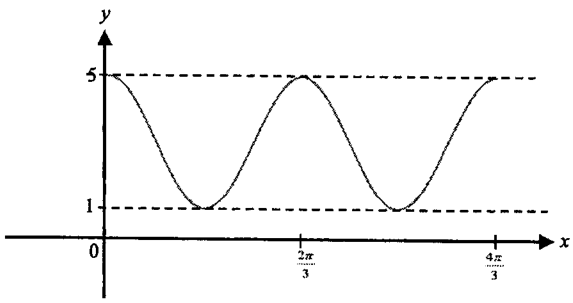
2025 Add Math Prelims Marking Scheme

Qn	Solution	Remarks
1a	$\frac{d}{dx}\left(2e^x+1\right)^{-1}$ $=-\frac{2e^x}{\left(2e^x+1\right)^2}$	B1 chain Rule B1 diff^n of Expo
1b	$\int -\frac{2e^x}{\left(2e^x+1\right)^2}dx=\frac{1}{2e^x+1}\ (+c)$ $\int \frac{e^x}{\left(2e^x+1\right)^2}dx=\frac{-1}{4e^x+2}+c$	M1 integrate their (a) answer to get $\frac{1}{2e^x+1}$ A1 o.e
		4
2	angle TPR = angle TRP ---(1) ($TP=TR$, ΔTPR is an isosceles) angle TPQ = angle TSP ---(2) (alternate segment thm.) angle QPR = angle TPR – angle TPQ angle TRP = angle TSP + angle RPS (exterior angle of Δ) angle RPS = angle TRP – angle TSP = angle TPR – angle TPQ = angle QPR $\therefore$ line PR bisects the angle SPQ. (proven)	B1 (isosceles) B1 (tangent chord or alternate segment theorem) B1 (exterior angle of Δ) B1 correct answer
		4
3a	$V=\frac{1}{12}\pi h^3$ $\frac{dV}{dh}=\frac{1}{4}\pi h^2$ $\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}$ $-10\pi=\frac{1}{4}\pi h^2\times\frac{dh}{dt}$ $\frac{dh}{dt}=-\frac{40}{h^2}$ cm/s	B1 M1 with correct subst^n A1

3b	$A = \frac{1}{4}\pi h^2$ $\frac{dA}{dh} = \frac{1}{2}\pi h$ $\frac{dA}{dt} = \frac{dA}{dh} \times \frac{dh}{dt}$ $\frac{dA}{dt} = \frac{1}{2}\pi(10) \times -\frac{40}{10^2}$ $\frac{dA}{dt} = -2\pi \text{ cm}^2/\text{s}$ Rate of decreasing = 2π cm²/s	M1 with correct subst^n A1 $\frac{dA}{dt}$ must be correct
		5
4a	$-2x^2 + 20x + 12$ $= -2(x^2 - 10x) + 12$ $= -2\left[x^2 - 10x + \left(\frac{10}{2}\right)^2 - \left(\frac{10}{2}\right)^2\right] + 12$ $= -2\left[(x-5)^2 - 25\right] + 12$ $= -2(x-5)^2 + 62$	B2 –1 for each error
b	Height $= -2(x-5)^2 + 62$ $(x-5)^2 \geq 0$ $-2(x-5)^2 \leq 0$ $-2(x-5)^2 + 62 \leq 62$ Height $\leq$ 62m Since height is $\leq$ 62m, the maximum height reach by the ball is 62 metres.	B1 Use of $\geq$ B1 Conclusion
c	Height $= -2(x-5)^2 + 62$ When $x = 10$, $y = 12 > 2$ The pellet will not hit the object.	B1
		5
5	Let h be the height of prism $V = \frac{1}{2}(1+\sqrt{5})^2(\sin 60°)h$ $2\sqrt{3} + 2\sqrt{15} = \frac{1}{2}(1+\sqrt{5})^2(\sin 60°)h$ $2\sqrt{3} + 2\sqrt{15} = \frac{1}{2}(6+2\sqrt{5})\frac{\sqrt{3}}{2}h$	B1 Vol of prism o.e sin 60° or $\sin\frac{\pi}{3}$ B1 correct sq of sides

	$h=\dfrac{2\sqrt{3}+2\sqrt{15}}{\frac{1}{2}\left(6+2\sqrt{5}\right)\frac{\sqrt{3}}{2}}$ $=\dfrac{4\sqrt{3}+4\sqrt{15}}{\left(3+\sqrt{5}\right)\sqrt{3}}$ $=\dfrac{4\sqrt{3}+4\sqrt{15}}{3\sqrt{3}+\sqrt{15}}\times\dfrac{3\sqrt{3}-\sqrt{15}}{3\sqrt{3}-\sqrt{15}}$ $=\dfrac{12(3)-4\sqrt{3}\sqrt{15}+12\sqrt{3}\sqrt{15}-60}{27-15}$ $=\dfrac{8\sqrt{45}-24}{12}$ $=-2+2\sqrt{5}\,\text{cm}$	B1 $h=\dfrac{2\sqrt{3}+2\sqrt{15}}{\text{their area}}$ B1 $\times\dfrac{\text{their conjugate}}{\text{their conjugate}}$ M1 expansion of denominator A1
		6
6a	$\left(2-\dfrac{x}{4}\right)^5$ $=(2)^5+\binom{5}{1}(2)^4\left(-\dfrac{x}{4}\right)+\binom{5}{2}(2)^3\left(-\dfrac{x}{4}\right)^2+\ldots$ $=32-20x+5x^2+\ldots$	B1 B1 B1
b	$\left(4+kx+x^2\right)\left(2-\dfrac{x}{4}\right)^5$ $=\left(4+kx+x^2\right)\left(32-20x+5x^2+\ldots\right)$ Coeff of x $=-80+32k$ Coeff of $x^2=52-20k$ $-80+32k-20k+52=-4$ $-28+12k=-4$ $12k=24$ $k=2$	B1 correct coeff x B1 correct coeff x^2 M1 their sum $=-4$ A1
		7
7a	$\sin(A+B)+\sin(A-B)$ $=\sin A\cos B+\cos A\sin B+\sin A\cos B-\cos A\sin B$ $=2\sin A\cos B$ $k=2$	B1 correct expansion of compound angles B1
b	$P+Q=2A$ $A=\dfrac{P+Q}{2}$ ---(1) $P-Q=2B$ $B=\dfrac{P-Q}{2}$ ---(2) $\sin(A+B)+\sin(A-B)=2\sin A\cos B$	B1 stating of A or B in term of P and Q

	$\sin P + \sin Q = 2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$	B1 correct subst^n of (1) and (2).
c	$\sin 75° + \sin 15°$ $= 2\sin\left(\frac{90°}{2}\right)\cos\left(\frac{60°}{2}\right)$ $= 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{\sqrt{6}}{2}$	M1 use of (b) or compd ∠ B1 Either special angle correct A1
		7
8a	$x - y = 0, x + 3y = 20$ $x + 3x = 20$ $x = 5,$ $y = 5$ Coord $B = (5, 5)$ $5x - y = 4,\ x - y = 0,$ $5x - x = 4$ $x = 1$ $y = 1$ Coord $A = (1, 1)$	M1 Sim Eqn to solve for unknown A1 A1
b	Coord $D = (1 - (5 - 2), 1 + 1)$ $= (-2, 2)$	M1 correct attempt to find x or y A1
c	Area $= 2 \times \frac{1}{2}\begin{vmatrix} 1 & 5 & 2 & 1 \\ 1 & 5 & 6 & 1 \end{vmatrix}$ $= 5 + 30 + 2 - 5 - 10 - 6$ $= 16$ units^2	M1 determinant method A1 √
		7
9a	Let h cm be the height of the casing. Base area $= x^2$ Area of 4 sides $= 4xh$ Total material cost $= 2x^2 + 4xh$ $\$2x^2 + 4(\$1)xh = \$600$ $h = \frac{600 - 2x^2}{4x}$ $= \frac{300 - x^2}{2x}$	M1 find area of materials/ find cost for rect piece A1 correct equation for h unsimplified/ Correct base area

	Vol of casing, V $=x^2h$ $=x^2\left(\dfrac{300-x^2}{2x}\right)$ $=\dfrac{300x-x^3}{2}$ cm^3	B1 AG
b	$\dfrac{dV}{dx}=\dfrac{1}{2}\left(300-3x^2\right)$ At stationary value, $\dfrac{dv}{dx}=0$ $300-3x^2=0$ $x=10$ $\dfrac{d^2V}{dx^2}=-3x$ When $x=10$ $\dfrac{d^2V}{dx^2}<0$ $\therefore$ Maximum Volume Maximum Volume $=\dfrac{300(10)-(10)^3}{2}$ $=1000$ cm^3	B1 M1 set to 0 B1 Test and conclusion of nature √ if min A1 correct volume
		7
10a	$a=2, b=3, c=3$	B1 B1 B1
b		B1 for 2 complete cycles B1 correct maximum and minimum B1 fully correct curve
c	$\cos 3x=d$ $2\cos 3x=2d$ $2\cos 3x+3=2d+3$ $y=2d+3$ When $y=1$ $2d+3=1$ $d=-1$ Or min point $\cos 3x=d$ $d=-1$	M1 find eqn of y A1 B1 B1
		8

11a	area of the triangle *OBX* $= \frac{1}{2}(7.2)(9)\sin\theta$ $= 32.4(\sin\theta)\text{ cm}^2$	B1
b	Area $= 0.5(12)(9)$ $= 54\text{ cm}^2$ $54 = 32.4\sin\theta + \frac{1}{2}(7.2)(12)\sin(90° - \theta)$ $108 = 64.8\sin\theta + 86.4\cos\theta$ $4\cos\theta + 3\sin\theta = 5$	B1 M1 use of $\frac{1}{2}(7.2)(12)\sin(90° - \theta)$ B1 $\sin(90° - \theta)$ to $\cos\theta$ AG
c	$4\cos\theta + 3\sin\theta = R\cos(\theta - \alpha)$ $R = \sqrt{4^2 + 3^2}$ $= 5$ $\tan\alpha = \frac{3}{4}$ $\alpha = 38.9°$	M1 $R = \sqrt{a^2 + b^2}$ A1 M1 $\tan\alpha = \frac{b}{a}$ A1
d	$5\cos(\theta - 36.87°) = 5$ $\theta - 36.87° = 0$ $\theta = 36.9°$	B1
		9
12a	When $y = 0$ $\frac{1}{2} = \frac{2}{(x-2)^2}$ $x = 0$ (n.a) or 4 Coord $A = (4, 0)$ When $y = 7.5$ $8 = \frac{2}{(x-2)^2}$ $x = 1.5$ or 2.5 Coord $B = (2.5, 7.5)$ Coord $C = (1.5, 7.5)$	M1 subst^n to find unknowns A1 A1 A1
12b	Area $= 7.5 + 2\int_0^{1.5} 2(x-2)^{-2} - \frac{1}{2}\,dy$ $= 7.5 + [-4(x-2)^{-1} - x]_0^{1.5}$ $= 7.5 + \left(-4(1.5-2)^{-1} - 1.5\right) - \left(-4(0-2)^{-1}\right)$ $=12$ units	M1 sum of rectangle and 2 symmetrical parts B1 correct integration to linear *x* M1 -2 or -4 $(x-2)^{-1}$ M1 Substn of correct limits A1
		9

13a	$\frac{3x^2+3x+3}{(x+2)(x-1)^2}=\frac{A}{x+2}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$ $3x^2+3x+3=A(x-1)^2+B(x+2)(x-1)+C(x+2)$ When $x=1$, $C=3$ When $x=-2$, $A=1$ $3=A+B$ $B=2$ $\frac{3x^2+3x+3}{(x+2)(x-1)^2}=\frac{1}{x+2}+\frac{2}{x-1}+\frac{3}{(x-1)^2}$	M1 realising the form (C can be linear) M1 remove denominator A1 A1 A1 –1 mark if no final statement
b	$\int_3^4 \frac{3x^2+3x+3}{(x+2)(x-1)^2}\,dx$ $=\int_3^4\left(\frac{1}{x+2}+\frac{2}{x-1}+\frac{3}{(x-1)^2}\right)dx$ $=\int_3^4\left(\frac{1}{x+2}+2\left(\frac{1}{x-1}\right)+3(x-1)^{-2}\right)dx$ $=\left[\ln(x+2)+2\ln(x-1)-3(x-1)^{-1}\right]_3^4$ $=(\ln 6+2\ln 3-1)-(\ln 5+2\ln 2-1.5)$ $=1.49$	M1 integrate to ln A1 $2\ln(x-1)$ B1 $-3(x-1)^{-1}$ M1 substn of limits A1
c	$\int_{-5}^1 \frac{3x^2+3x+3}{(x+2)(x-1)^2}\,dx$ When $x=1$, the denominator of the integral is 0 and thus $\frac{3x^2+3x+3}{(x+2)(x-1)^2}$ does not exist. As such it is not possible to find the value. or $=(\ln 3+2\ln(0-1)-(\ln(-3)+2\ln(-6)-1.5)$ Since ln 0 and ln(–3) and ln(–6) do not exist, it is not possible to find the value.	B1 B1 B1 B1
		12

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2025
Secondary 4

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS **4049/02**

Paper 2

Tuesday 26 August 2025

2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 In an experiment, it was observed that the number of bacteria in a culture doubles every 4 hours.
It is given that N_0 is the number of bacteria present at the start of the experiment, and N is the number of bacteria t hours later.

Calculate the **exact** value of the constant k, given that $N = N_0 e^{kt}$. [4]

At $t = 4$ hour, $N = 2N_0$

$2N_0 = N_0 e^{4k}$ **M1 (equate using t = 4, N = 2N₀)**

$e^{4k} = 2$ **A1**

$4k = \ln 2$ **M1 (take ln on both sides)**

$k = \frac{1}{4}\ln 2$ **A1**

2 Given that $\sqrt{8^x} = \frac{2^x}{4}$, find the value of $\sqrt{8^x}$. [4]

$2^{\frac{3x}{2}} = 2^{x-2}$ **M2 (change all to base 2 and apply law of indices)**

$\frac{3x}{2} = x - 2$ **M1 (solve for x)**

$x = -4$

$\sqrt{8^{-4}} = \frac{1}{64}$ **A1**

3 Solve the simultaneous equations

$$\sqrt{(y+11)^2+x^2} = 17,$$

$$2y = 7x - 48.$$ [5]

$$y = \frac{7}{2}x - 24$$

$$\sqrt{(\frac{7}{2}x - 24 + 11)^2 + x^2} = 17$$

$$\frac{53}{4}x^2 - 91x - 120 = 0$$ M2 (substitution and simplification)

$$x = \frac{364 \pm \sqrt{(-364)^2 - 4(53)(-480)}}{2 \times 53}$$ M1 (Quadratic formula)

$$x = \frac{364 \pm \sqrt{254256}}{106}$$

$= 8$ or $-1\frac{7}{53}$ (or -1.13)

$y = 4$ or $-27\frac{51}{53}$ (or -28.0) A2 (minus one mark for each wrong answer)

4 **(a)** State the range of values of x for $\log_x(2x-1)$ to be defined. [2]

$2x-1>0$
$x>0.5$ **B1**

& $x \neq 1$ **B1**

(b) Given $\log_9 \sqrt{x} = 5\log_3 p$, express x in terms of p. [4]

$\frac{\log_3 \sqrt{x}}{\log_3 9} = 5\log_3 p$ **M1 (change base)**

$\frac{\log_3 \sqrt{x}}{2\log_3 3} = 5\log_3 p$ **B1** $\log_3 9 = 2$ OR $\log_9 3 = \frac{1}{2}$

$\frac{\log_3 \sqrt{x}}{2} = 5\log_3 p$

$\log_3 \sqrt{x} = 10\log_3 p$

$\log_3 \sqrt{x} = \log_3 p^{10}$ **M1 (applying power law)**

$\sqrt{x} = p^{10}$

$x = p^{20}$ **A1**

5 A curve is such that $\frac{d^2y}{dx^2} = 8\sin 4x - 4\cos 2x$.

The curve passes through the point $P\left(\pi,\ -\frac{1}{6}+\pi\right)$ and has a gradient of -1 at P.

Find the equation of the curve. [7]

$\frac{dy}{dx} = -\frac{8\cos 4x}{4} - \frac{4\sin 2x}{2} + c_1$ **B2 (minus one mark for each error)**

Sub $(\pi, -1)$, $-1 = -\frac{8\cos 4\pi}{4} - \frac{4\sin 2\pi}{2} + c_1$

$c_1 = 1$ **M1** **(equate gradient = -1 at $x = \pi$)**

$y = -\frac{2\sin 4x}{4} + \frac{2\cos 2x}{2} + x + c_2$ **B2 (minus one mark for each error)**

Sub $(\pi, -\frac{1}{6}+\pi)$, $-\frac{1}{6}+\pi = -\frac{2\sin 4\pi}{4} + \frac{2\cos 2\pi}{2} + \pi + c_2$ **M1** **(equate $y = -\frac{1}{6}+\pi$ at $x = \pi$)**

$c_2 = -\frac{7}{6}$

$y = -\frac{1}{2}\sin 4x + \cos 2x + x - \frac{7}{6}$ **A1**

6 **(a)** The term containing the highest power of x in the polynomial $f(x)$ is $2x^4$.
Two of the roots of the equation $f(x) = 0$ are $\frac{1}{2}$ and -3.
Given that $x^2 + 1$ is a quadratic factor of $f(x)$, find an expression for $f(x)$ in descending powers of x. [3]

$f(x) = 2(x - \frac{1}{2})(x + 3)(x^2 + 1)$ **B2 (minus one mark for each wrong factor)**

$= 2x^4 + 5x^3 - x^2 + 5x - 3$ **B1**

(b) Find the value of k for which $x^2 + (k-1)x + k^2 - 16$ is exactly divisible by $x - 3$ but not divisible by $x + 4$. [4]

By factor theorem,
$f(3) = 0$ **M1**
$(3)^2 + 3(k-1) + k^2 - 16 = 0$

$9 + 3k - 3 + k^2 - 16 = 0$
$k^2 + 3k - 10 = 0$
$(k+5)(k-2) = 0$
$k = -5$ **or** $k = 2$ **A1(both answers)**

When $k = -5$, When $k = 2$,
$f(-4) = 49$ $f(-4) = 0$ **M1 (check which is a factor)**

$\therefore k = -5$ **A1**

7 An object starts from rest from a point O.

Its velocity, v cm/s, t seconds after leaving O, is such that $\frac{dv}{dt} = 10$.

After 5 seconds, the object reaches a point A.

(i) Find its velocity at A. [2]

$v = 10t + c$ **B1 (integration or** 5×10**)**

When $t = 0,\ v = 0;$

$\therefore c = 0$

$v = 10t$

When $t = 5,\ v = 50.$ **B1**

Velocity = 50 m/s

(ii) Find the distance OA. [2]

$s = 5t^2 + c$

When $t = 0,\ s = 0;$

$\therefore c = 0$

$s = 5t^2$ **M1 (integration)**

When $t = 5,\ s = 125.$ **A1**

Distance = 125 cm

On reaching A, the object then slows so that its velocity V cm/s, T seconds after leaving A, is such that $\frac{dV}{dT} = 10 - k\sqrt{T}$, where k is a constant.

(iii) Given that the object's velocity at a point B where $T = 4$ is 26 cm/s, show that $k = 12$. [3]

$$V = 10T - k\frac{T^{\frac{3}{2}}}{\frac{3}{2}} + c$$ **M1 (integration)**

When $T = 0,\ V = 50$

$\therefore c = 50$ **M1**

$$V = 10T - \frac{2kT^{\frac{3}{2}}}{3} + 50$$

$$26 = 40 - \frac{2k(8)}{3} + 50$$

$\therefore k = 12$ **A1**

8 **(a)** Show that $(m+1)x^2+(4m+3)x+2m=0$ has real and distinct roots for all real values of m. [4]

$(4m+3)^2-4(m+1)(2m)$ **M1 Use of discriminant**

$=16m^2+24m+9-8m^2-8m$

$=8m^2+16m+9$

$=8(m+1)^2+1$ **A1**

$(m+1)^2 \geq 0$

$8(m+1)^2 \geq 0$

$8(m+1)^2+1 \geq 1$

$8(m+1)^2+1>0$ **M1 (attempt to find min value)**

Since the discriminant is positive, the equation has real and distinct roots. **A1**

(b) The equation of a curve is $y=4x^2-4x+3$.

(i) Find the set of values of x for which the curve lies below the line $y=11$. [3]

$4x^2-4x+3<11$ **M1 (form inequality)**

$(x-2)(x+1)<0$ **M1 (factorise)**

$-1<x<2$ **A1**

The straight line L meets the curve $y=4x^2-4x+3$. at one point only.

(ii) Given that L is not a tangent to the curve, what can be deduced about L? [1]

Vertical **B1**

9 **(a)** The variables x and y are connected by the equation

$$x + py = qxy,$$

where p and q are constants.

Explain how a straight line graph can be drawn and state how the values of p and q could be obtained from the line. [4]

$x + py = qxy$

$1 + \frac{py}{x} = qy$

$qy = 1 + \frac{py}{x}$

$y = \left(\frac{p}{q}\right)\frac{y}{x} + \left(\frac{1}{q}\right)$ **M2 (plotting y against $\frac{y}{x}$)**

Vertical intercept $= \frac{1}{q}$

$q = \frac{1}{\textbf{Vertical intercept}}$ **A1**

Gradient $= \frac{p}{q}$

$p = q$**(Gradient)** **A1**

(b) The variables x and y are such that when values of $\lg(y-5)$ are plotted against $\lg x$, a straight line is obtained.
It is given that when $x=10$, $y=15$ and when $x=100$, $y=1005$.
Estimate the value of x when $y=50$. [6]

$\lg(y-5)=m\lg x+c$

When $x=10$ **&** $y=15$; $\lg x=1$, $\lg(y-5)=1$.

When $x=100$ **&** $y=1005$, $\lg x=2$, $\lg(y-5)=3$.

Gradient, $m=\frac{3-1}{2-1}=2$ **M1 (finding gradient) A1**

$\lg(y-5)=2\lg x+c$ **M1 (finding equation) A1**

$1=2(1)+c$
$c=-1$

$\lg(y-5)=2\lg x-1$

When $y=50$, $\lg 45=2\lg x-1$

$$\lg x=\frac{\lg 45+1}{2}$$

$$x=10^{\frac{\lg 45+1}{2}}=21.2$$ **M1 (attempt to solve logarithmic equation) A1**

10 A circle has centre $A(2, -3)$ and radius $\sqrt{20}$.

(a) Write the equation of this circle. [1]

$(x-2)^2+(y+3)^2=20$ OR $x^2+y^2-4x+6y-7=0$ **B1**

(b) Find the equations of the tangents to the circle that are horizontal. [2]

$y=-3+\sqrt{20}$ & $y=-3-\sqrt{20}$ **B1 & B1**

The circle intersects the y-axis at points S and T.

(c) Find the length of ST. [3]

At y-axis, $x = 0$, **M1 (equate $x = 0$)**

$\therefore y = 1$ or -7 **A1**

$ST = 1 + 7 = 8$ units **B1**

A second circle with centre B also passes through S and T.

(d) Explain why the y-coordinate of B is -3. [2]

Perpendicular bisector of ST will pass through the centre of the circle.
since ST is vertical. **B1**

y-coord of $B = y$ coord of A
$= \dfrac{1-7}{2} = -3$ **B1**

(e) Given that the x-coordinate of B is positive and that the radius of the second circle is $\sqrt{52}$, find the x-coordinate of B. [2]

using $(0,\ -7)$ & $B(x,\ -3)$

$x^2 + (-7+3)^2 = 52$

$x = 6$ or -6 (rej since $x > 0$) **M1 (forming equation on distance)**

x-coord of $B = 6$ **A1**

11 (a) Prove $\dfrac{\cos 2\theta}{\tan\theta}+\sin 2\theta=\cot\theta$.

[4]

LHS

$=\dfrac{\cos 2\theta}{\tan\theta}+\sin 2\theta$

$=\dfrac{\cos\theta}{\sin\theta}(1-2\sin^2\theta)+2\sin\theta\cos\theta$ **M1 (double angle formulae)**

$=\dfrac{\cos\theta}{\sin\theta}-\dfrac{2\sin^2\theta\cos\theta}{\sin\theta}+2\sin\theta\cos\theta$ **M1 ($\tan\theta=\dfrac{\sin\theta}{\cos\theta}$)**

$=\cot\theta-2\sin\theta\cos\theta+2\sin\theta\cos\theta$ **M1 (expansion)**

$=\cot\theta=RHS$ **M1 (simplify + AG)**

(b) Solve $3\sin x\tan x-\sec x=1$ for $-180°\leq x\leq 180°$. [6]

$3\sin x\left(\frac{\sin x}{\cos x}\right)-\frac{1}{\cos x}=1$ **M1 (changing all trigo to $\frac{\sin x}{\cos x}$)**

$3\sin^2 x-1=\cos x$

$3(1-\cos^2 x)-1-\cos x=0$ **M1 (use of identity to change all to cos *x*)**

$3\cos^2 x+\cos x-2=0$

$3\sin^2 x-1=\cos x$

$3(1-\cos^2 x)-1-\cos x=0$ **M1 (factorise)**

$3\cos^2 x+\cos x-2=0$

$\cos x=\frac{2}{3}$ or -1 **A1**

$x=-180°$ or $-48.2°$ or $48.2°$ or $180°$
A2 (one mark for any two correct values)

12 A curve has the equation $y = f(x)$, where $f(x) = \dfrac{2x+1}{x-3}$ for $x \neq k$.

(a) State the value of k. [1]

3 **B1**

(b) Find the equation of the normal to the curve at the point $x = 4$. [4]

$$f'(x) = \frac{(x-3)(2)-(2x+1)(1)}{(x-3)^2}$$
$$= -\frac{7}{(x-3)^2}$$

M2 (quotient rule)

At $x = 4$, gradient of normal $= \dfrac{-1}{\dfrac{-7}{(4-3)^2}} = \dfrac{1}{7}$ A1

At $x = 4$, $y = 9$

$$9 = \frac{1}{7}(4) + c$$
$$c = \frac{59}{7}$$

Equation of normal: $y = \dfrac{1}{7}x + \dfrac{59}{7}$ A1

(c) Explain whether f is an increasing or decreasing function for $x > 3$. [2]

For $x > 3$.

$(x-3)^2 > 0$

$-\dfrac{7}{(x-3)^2} < 0$ B1

f is a decreasing function. B1

(d) Determine whether the **gradient** of the curve is an increasing or decreasing function for $x > 3$. [2]

$f''(x) = 14(x-3)^{-3}$ B1

$x > 3$

$x - 3 > 0$

$(x-3)^3 > 0$

$\dfrac{14}{(x-3)^3} > 0$ B1

Gradient is an increasing function. B1

(e) The line $y = c$ does not intersect the curve.

By expressing $\dfrac{2x+1}{x-3}$ in terms of $a + \dfrac{b}{x-3}$ where *a*, *b* and *c* are constants, explain why $c = 2$. [2]

$\dfrac{2x+1}{x-3} = 2 + \dfrac{7}{x-3}$ B1

$x - 3 \neq 0$

$\dfrac{7}{x-3} \neq 0$

$2 + \dfrac{7}{x-3} \neq 2$ B1

$c = 2$