TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2025
Secondary 4

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS **4049/01**

Paper 1

Wednesday 20 August 2025

2 hours 15 minutes

Candidates answer on the Question Paper.
No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
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$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

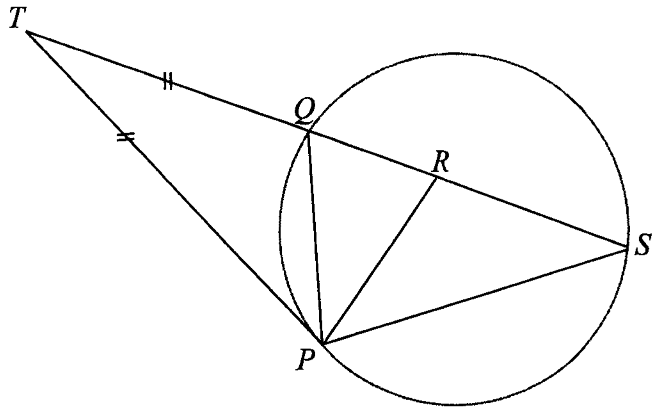
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 (a) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2e^x+1}\right)$. [2]

(b) Hence find $\int \frac{e^x}{\left(2e^x+1\right)^2}\,\mathrm{d}x$ [2]

2



In the diagram, the point P lies on the circle and TP is a tangent to the circle.
TS is a straight line intersecting the circle at Q and S.
Given that point R lies on TS and $TP = TR$, prove that the line PR bisects the angle SPQ. [4]

3 Water is drained from a conical tank at a constant rate of 10π cm^3/s.
The volume of water, V cm^3, when the depth of water is h cm, is given by $V = \frac{1}{12}\pi h^3$ and the surface area of the water, A cm^2, is $A = \frac{1}{4}\pi h^2$.

(a) Find $\frac{dh}{dt}$ in terms of h. [3]

(b) Find the rate at which the surface area of the water is decreasing when the depth of water is 10 cm. [2]

4 The height, y m, above the ground of a pellet shot by a shooting device can be modelled by the equation $y=-2x^2+20x+12$, where x m is the horizontal distance travelled by the pellet.

(a) Express $y=-2x^2+20x+12$ in the form $y=a(x-b)^2+c$, where a, b and c are constants. [2]

(b) Explain why the maximum height reached by the pellet is 62 metres. [2]

(c) A two-metre-tall pole is placed on the ground, at a horizontal distance of 10 metres from the shooting device. Explain if the pellet will hit the pole. [1]

5 The volume of a prism is $\left(2\sqrt{3}+2\sqrt{15}\right)$ cm^3.

The prism has an equilateral triangular base of side $\left(1+\sqrt{5}\right)$ cm.

Without using a calculator, express the height of the prism in the form $\left(a+b\sqrt{5}\right)$ cm, where a and b are integers. [6]

6 **(a)** Write down and simplify the first three terms in the expansion, in ascending powers of x, of $\left(2-\frac{x}{4}\right)^5$. [3]

(b) In the expansion of $\left(4+kx+x^2\right)\left(2-\frac{x}{4}\right)^5$, the sum of the coefficients of x and x^2 is −4. Find the value of the constant k. [4]

7 (a) Find the value of k such that $\sin(A+B)+\sin(A-B)=k\sin A\cos B$. [2]

(b) By letting $P=A+B$ and $Q=A-B$, show that

$$\sin P+\sin Q=2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right).$$ [2]

(c) Find the exact value of $\sin 75° + \sin 15°$. [3]

8 The equations of the lines AB, BC and CA are $x - y = 0$, $x + 3y = 20$ and $5x - y = 4$ respectively. The coordinates of C are (2, 6).

(a) Find the coordinates of A and of B. [3]

(b) Given that $ABCD$ is a parallelogram, find the coordinates of D. [2]

(c) Find the area of the parallelogram *ABCD*. [2]

9 A company manufactures **open** square-based cuboid casings for high-precision instruments, where x cm is the length of the side of the base.
The material for the base of the casing cost twice as much as the material for the sides.
The total material cost for one casing is $600, with the material for the side costing $1/cm².

(a) Show that the volume of the casing, V cm³, is given by $V = \dfrac{300x - x^3}{2}$. [3]

(b) Given that x can vary, find the stationary value of V and determine its nature. [4]

10 It is given that $y = a\cos bx + c$, for $0 \le x \le \frac{4}{3}\pi$, where a, b and c are positive integers.

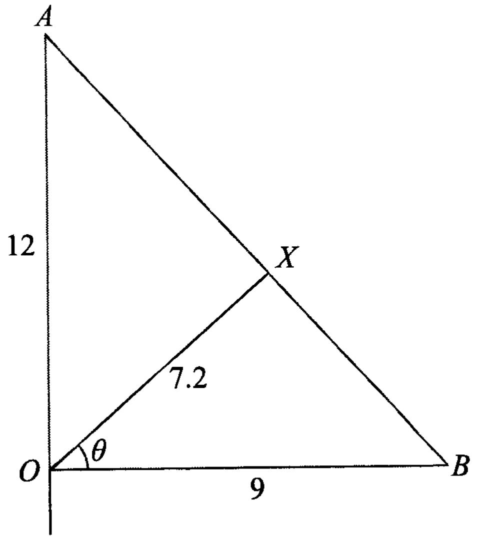
The period of y is $\frac{2}{3}\pi$ and the maximum and minimum value of y is 5 and 1 respectively.

(a) State the values of a, b and c. [3]

(b) Sketch the graph of $y = a\cos bx + c$ for $0 \le x \le \frac{4}{3}\pi$. [3]

(c) Hence, find the value of constant d such that the equation $\cos bx = d$ will have 2 solutions. [2]

11



The diagram shows a frame of a model of a sail.
The main beams of the frame are put together to form a triangle OAB.
The lengths of the main beams are $OA = 12$ cm and $OB = 9$ cm and angle $AOB = 90°$.
A support rod of length 7.2 cm is to be installed from O to X on the diagonal beam AB such that it makes an acute angle θ with the beam OB.
A canvas is to be fixed on points O, A, X and B of the frame to form a sail.

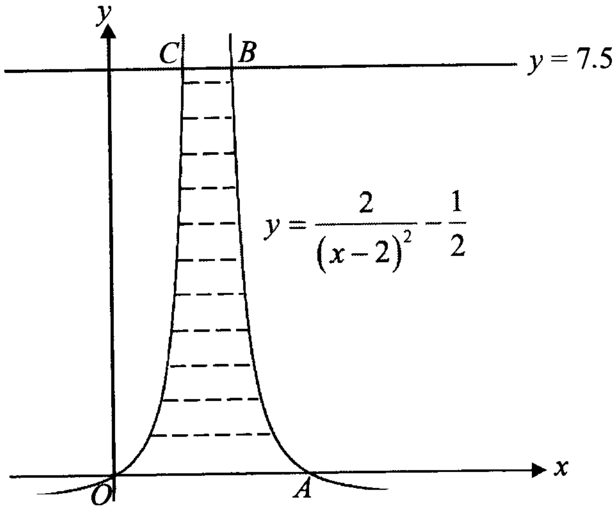
(a) Find the area of the triangle OBX in terms of θ. [1]

(b) By finding the area of the canvas, show that $4\cos\theta + 3\sin\theta = 5$. [3]

(c) Express $4\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$ where $R > 0$ and α is acute. [4]

(d) Find the value of θ. [1]

12



The diagram shows parts of the curve of $y = \frac{2}{(x-2)^2} - \frac{1}{2}$, intersecting the x-axis at O and A and intersecting the line $y = 7.5$ at B and C.

(a) Find the coordinates of A, B and C. [4]

(b) Find the area of shaded region bounded by the curve, the x-axis and the line $y = 7.5$. [5]

13 **(a)** Express $\dfrac{3x^2+3x+3}{(x+2)(x-1)^2}$ in partial fractions. [5]

(b) Hence, evaluate $\int_3^4 \frac{3x^2+3x+3}{(x+2)(x-1)^2}\,\mathrm{d}x$. [5]

(c) Explain why it is not possible to find the value of $\int_{-5}^{1} \frac{3x^2+3x+3}{(x+2)(x-1)^2}\,dx$. [2]

End of Paper

Answer

Qn	Answer
1a	$-\dfrac{2e^x}{\left(2e^x+1\right)^2}$
b	$\dfrac{-1}{4e^x+2}+c$
3a	$-\dfrac{40}{h^2}$ cm/s
b	2π cm²/s
4a	$-2\left(x-5\right)^2+62$
5	$-2+2\sqrt{5}$ cm
6a	$32-20x+5x^2+...$
b	2
7a	2
b	$\sin P+\sin Q=2\sin\left(\dfrac{P+Q}{2}\right)\cos\left(\dfrac{P-Q}{2}\right)$
c	$\dfrac{\sqrt{6}}{2}$
8a	Coord $A=(1,\ 1)$ Coord $B=(5,\ 5)$
B	(–2, 2)
c	16 units²
9b	1000 cm³
10a	$a=2, b=3, c=3$
c	$d=-1$
11a	32.4(sin θ) cm²
c	$\alpha=38.9°$
d	$\theta=36.9°$
12a	Coord $A=(4,\ 0)$ Coord $B=(2.5,\ 7.5)$ Coord $C=(1.5,\ 7.5)$
b	12 units
13a	$\dfrac{3x^2+3x+3}{(x+2)(x-1)^2}=\dfrac{1}{x+2}+\dfrac{2}{x-1}+\dfrac{3}{(x-1)^2}$
b	1.49

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2025
Secondary 4

CANDIDATE NAME

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INDEX NUMBER

ADDITIONAL MATHEMATICS **4049/02**

Paper 2

Tuesday 26 August 2025

2 hours 15 minutes

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1 In an experiment, it was observed that the number of bacteria in a culture doubles every 4 hours.
It is given that N_0 is the number of bacteria present at the start of the experiment, and N is the number of bacteria t hours later.

Calculate the **exact** value of the constant k, given that $N = N_0 e^{kt}$. [4]

2 Given that $\sqrt{8^x} = \frac{2^x}{4}$, find the value of $\sqrt{8^x}$. [4]

3 Solve the simultaneous equations

$$(y+11)^2 + x^2 = 289,$$

$$2y = 7x - 48.$$ [5]

4 **(a)** State the range of values of x for $\log_x(2x-1)$ to be defined. [2]

(b) Given $\log_9 \sqrt{x} = 5\log_3 p$, express x in terms of p. [4]

5 A curve is such that $\frac{d^2y}{dx^2} = 8\sin 4x - 4\cos 2x$.

The curve passes through the point $P\left(\pi,\ -\frac{1}{6}+\pi\right)$ and has a gradient of -1 at P.

Find the equation of the curve. [7]

6 **(a)** The term containing the highest power of x in the polynomial $\mathrm{f}(x)$ is $2x^4$.

Two of the roots of the equation $\mathrm{f}(x) = 0$ are $\frac{1}{2}$ and -3.

Given that $x^2 + 1$ is a quadratic factor of $\mathrm{f}(x)$, find an expression for $\mathrm{f}(x)$ in descending powers of x. [3]

(b) Find the value of k for which $x^2 + (k-1)x + k^2 - 16$ is exactly divisible by $x - 3$ but not divisible by $x + 4$. [4]

7 An object starts from rest from a point O.
Its velocity, v cm/s, t seconds after leaving O, is such that $\frac{dv}{dt} = 10$.
After 5 seconds, the object reaches a point A.

(i) Find its velocity at A. [2]

(ii) Find the distance OA. [2]

On reaching A, the object then slows so that its velocity V cm/s, T seconds after leaving A, is such that $\frac{dV}{dT} = 10 - k\sqrt{T}$, where k is a constant.

(iii) Given that the object's velocity at a point B where $T = 4$ is 26 cm/s, show that $k = 12$. [3]

8 **(a)** Show that $(m+1)x^2+(4m+3)x+2m=0$ has real and distinct roots for all real values of m. [4]

(b) The equation of a curve is $y=4x^2-4x+3$.

(i) Find the set of values of x for which the curve lies below the line $y = 11$. [3]

The straight line L meets the curve $y=4x^2-4x+3$ at one point only.

(ii) Given that L is not a tangent to the curve, what can be deduced about L? [1]

9 **(a)** The variables x and y are connected by the equation

$$x + py = qxy,$$

where p and q are constants.

Explain how a straight line graph can be drawn and state how the values of p and q could be obtained from the line. [4]

(b) The variables x and y are such that when values of $\lg(y-5)$ are plotted against '
a straight line is obtained.
It is given that when $x=10$, $y=15$ and when $x=100$, $y=1005$.
Estimate the value of x when $y=50$. [6]

10 A circle has centre $A(2, -3)$ and radius $\sqrt{20}$.

(a) Write the equation of this circle. [1]

(b) Find the equations of the tangents to the circle that are horizontal. [2]

The circle intersects the y-axis at points S and T.

(c) Find the length of ST. [3]

A second circle with centre B also passes through S and T.

(d) Explain why the y-coordinate of B is -3. [2]

(e) Given that the x-coordinate of B is positive and that the radius of the second circle is $\sqrt{52}$, find the x-coordinate of B. [2]

11 **(a)** Prove $\frac{\cos 2\theta}{\tan\theta}+\sin 2\theta=\cot\theta$. [4]

(b) Solve $3\sin x\tan x - \sec x = 1$ for $-180° \le x \le 180°$. [6]

12 A curve has the equation $y = f(x)$, where $f(x) = \dfrac{2x+1}{x-3}$ for $x \neq k$.

(a) State the value of k. [1]

(b) Find the equation of the normal to the curve at the point $x = 4$. [4]

(c) Explain whether f is an increasing or decreasing function for $x > 3$. [2]

(d) Determine whether the **gradient** of the curve is an increasing or decreasing function for $x > 3$. [3]

(e) The line $y = c$ does not intersect the curve.

By expressing $\frac{2x+1}{x-3}$ as $a+\frac{b}{x-3}$ where a, b and c are constants, explain why $c = 2$. [2]

End of Paper

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