Name: Solution	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION

MATHEMATICS **4052/01**
Paper 1 **21 August 2025**
2 hours 15 mins

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST
Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use

Question	1	2	3	4	5	6	7	8	9	10	11	12
Marks												
Question	13	14	15	16	17	18	19	20	21	22	23	24
Marks												

Table of Penalties		Qn. No.		
Presentation	–1			
Accuracy/ Units	–1		Parent's/ Guardian's Signature	90

This question paper consists of 21 printed pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

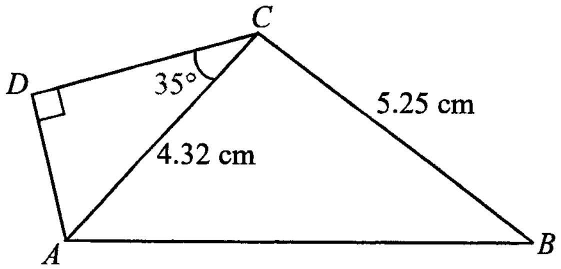
Answer **all** the questions.

1 Alex puts aside \$8000 in an investment fund which offers compounded interest quarterly at an interest rate of 2% per annum.
Calculate the total value of his investment at the end of 3.5 years.

$$\text{Total amount} = 8000\left(1+\frac{\frac{2}{4}}{100}\right)^{4\times 3.5}$$
$$= \$8578.57$$

Answer \$ .... [2]

2 In the quadrilateral *ABCD*, $BC = 5.25$ cm, $AC = 4.32$ cm.
Angle $ADC = 90°$. Angle $ACD = 35°$.



(a) Calculate *AD*.

$$\sin 35° = \frac{AD}{4.32}$$
$$AD = 2.4779$$
$$= 2.48 \text{ cm}$$

Answer .... cm [1]

(b) Given that the area of triangle $ABC = 3.18 \text{ cm}^2$, find the obtuse angle *ACB*.

$$\text{Area} = \frac{1}{2}(AC)(BC)\sin\angle ACB$$
$$3.18 = \frac{1}{2}(4.32)(5.25)\sin\angle ACB$$
$$\sin\angle ACB = 0.28042$$
$$\angle ACB = 163.7°$$

Answer .... [2]

3 The density of gold is 19.32 g/cm³.

The mass of a gold bracelet is 37.51 g.

(a) By rounding these numbers correct to 2 significant figures, find an estimate of the volume of the gold bracelet.
Show the numbers you use.

$$19 \approx \frac{38}{\text{volume}}$$

volume = 2 cm^3

Answer .... cm^3 [2]

(b) Without doing any further calculation, explain why the actual volume of the gold bracelet is less than the answer to **part (a)**.

The mass is rounded up to a larger value
The density is rounded down to a smaller value
Volume is found by dividing a larger mass by a smaller density
The actual volume is less than the estimated volume

....

.... [1]

---

4 A box contains 10 blue water bottles, 7 green water bottles and 5 red water bottles.

(a) Two water bottles are drawn from the box.
In the first draw, a water bottle is randomly drawn from the box with no replacement.
What is the probability that a green bottle will be drawn on the second draw?

P(green in 2^{nd} draw)=P(non-green followed by green) + P(green followed by green

$$= \frac{15}{22} \times \frac{7}{21} + \frac{7}{22} \times \frac{6}{21}$$

$$= \frac{7}{22}$$

Answer .... [2]

(b) How many red water bottles should be added into the bag such that the probability of drawing a red water bottle on the first draw is $\frac{2}{3}$?

Let the number of red water bottles to be added be x.

$$\frac{x+5}{22+x} = \frac{2}{3}$$

$$3x + 15 = 44 + 2x$$

$$x = 29$$

Answer .... [2]

5 Express $\frac{1}{\left(2x^2-3x-2\right)}-\frac{2}{(x-2)}$ as a single fraction in its simplest form.

$$\frac{1}{\left(2x^2-3x-2\right)}-\frac{2}{(x-2)}=\frac{1}{(2x+1)(x-2)}-\frac{2}{(x-2)}$$
$$=\frac{1-2(2x+1)}{(2x+1)(x-2)}$$
$$=\frac{1-4x-2}{(2x+1)(x-2)}$$
$$=\frac{-1-4x}{(2x+1)(x-2)}$$

Answer .... [3]

---

6 Five positive integers have a mean of 18, a median of 14 and a mode of 33.
Given that the two smallest integers are prime numbers, find the five numbers.

Let the 2 smallest numbers be a and b.

$a+b+14+33+33=18\times5$

$a+b=10$

Therefore, $a=3, b=7$.

The five numbers are 3, 7, 14, 33, 33.

Answer .... , ... , ... , ... , ... [2]

---

7 In 2023, the revenue of a company was 12% more than in 2022.
In 2024, it was 30% less than in 2023.

(a) Given that p represents the revenue of the company in 2022, express the revenue in 2023 in terms of p.

$1.12p$

Answer .... [1]

(b) Ben said that in 2024, the revenue of the company was 33.6% less than in 2022.
Do you agree?
You must show your calculations.

$$\text{Revenue in 2024} = 0.7(1.12p)$$
$$= 0.784p$$
$$\text{Percentage change} = \frac{0.784 - p}{p} \times 100\%$$
$$= -21.6\%$$
$$\text{Percentage decrease} = 21.6\%$$

I do not agree as there is only a drop of 21.6% in revenue between 2024 and 2022 instead of 33.6%.

....

....

.... [2]

---

8 **(a)** Solve $\frac{3}{1-x} - \frac{2}{3x-1} = 0$.

$$\frac{3}{1-x} = \frac{2}{3x-1}$$
$$3(3x-1) = 2(1-x)$$
$$9x - 3 = 2 - 2x$$
$$11x = 5$$
$$x = \frac{5}{11}$$

Answer $x =$ .... [2]

(b) Simplify $\frac{4x^2-25}{6x^2-13x-5}$.

$$\frac{4x^2-25}{6x^2-13x-5} = \frac{(2x+5)(2x-5)}{(2x-5)(3x+1)}$$
$$= \frac{(2x+5)}{(3x+1)}$$

Answer .... [2]

9 The ratio of exterior angle : interior angle of a regular polygon is 1 : 9.
Calculate

(a) the number of sides of the polygon,

$$\text{Exterior } \angle = \frac{180°}{10} = 18°$$

$$\text{No. of sides} = \frac{360°}{18°} = 20$$

Answer .... [2]

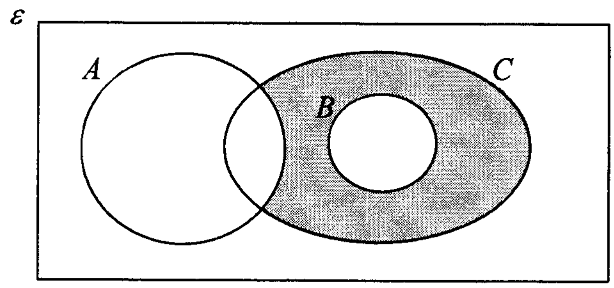
(b) the sum of the interior angles of the polygon.

$$\text{Sum of interior angles} = 180°(20-2) = 3240°$$

Answer .... [1]

---

10 **(a)** Write down the set represented by the following shaded region.



$A' \cap (B' \cap C)$ or $(A \cup B)' \cap C$

Answer .... [1]

(b) Given that $\varepsilon = \{x : x \text{ is an integer}, 1 \le x < 13\}$, and its subsets *A*, *B* and *C* are

$A = \{x : x \text{ is a factor of } 16\}$
$B = \{x : x \text{ is a prime number}\}$
$C = \{x : 9 - x \ge 1\}$

(i) List the elements in the set $A \cap B$.

$A = \{1, 2, 4, 8\}$
$B = \{2, 3, 5, 7, 11\}$
$A \cap B = \{2\}$

Answer .... [1]

(ii) Find $n(B' \cup C)$.

$B' = \{1, 4, 6, 8, 9, 10, 12\}$
$C = \{1, 2 \ldots\ldots 8\}$
$n(B' \cup C) = 11$

Answer .... [1]

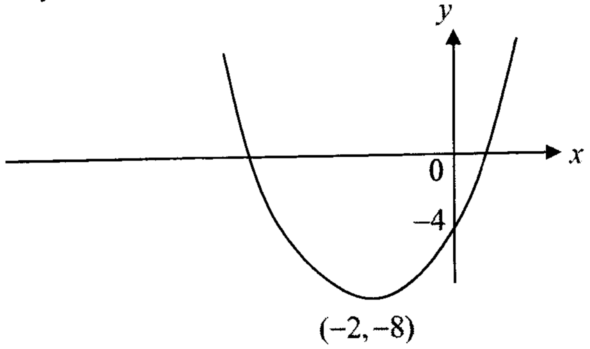
(iii) Explain clearly if $A \subset C$.

$A = \{1, 2, 4, 8\}$
$C = \{1, 2.....8\}$
All elements in Set A are elements in Set C
$A \neq C$
Therefore $A \subset C$

....

.... [1]

11 The figure below shows a graph with y-intercept 4.
It has a minimum point $(-2, -8)$.
Find the equation of the graph in the form $y = x^2 + ax + b$, where a and b are integers.

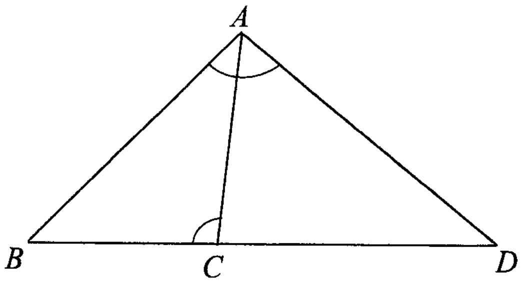


$y = (x+2)^2 - 8$

$y = (x+2)^2 - 8$
$= (x^2 + 4x + 4) - 8$
$= x^2 + 4x - 4$

Answer $y =$ .... [2]

12



In the diagram, angle ACB = angle DAB.

(a) Show that the triangles ABC and DBA are similar.

Answer

$\angle ABC = \angle DBA$ (common angle)

$\angle ACB = \angle DAB$ (given)

Therefore ΔABC is similar to ΔDBA. (AA)

[2]

(b) Given that $AB = 5.2$ cm and $BC = 3.4$ cm, find the length of CD.

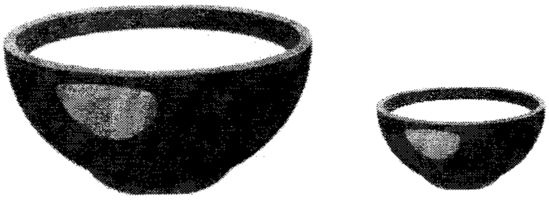
$$\frac{BD}{BA} = \frac{BA}{BC}$$

$$\frac{3.4 + CD}{5.2} = \frac{5.2}{3.4}$$

$CD = 4.55$ cm (nearest 3 sig.fig)

Answer .... cm [2]

13 (a)



The diagram shows two geometrically similar bowls.
The larger bowl has a base area of 64 cm^2.
The smaller bowl has a base area of 12.25 cm^2.

(i) The height of the smaller bowl is 7 cm.
Calculate the height of the bigger bowl.

$$\left(\frac{\text{height of large bowl}}{7}\right)^2 = \frac{64}{12.25}$$

height of large bowl = 16 cm

Answer .... cm [2]

(ii) A hawker stall uses the two types of bowls to serve porridge to customers.
The price of the same type of porridge served using the large bowl is eight times as much as the small bowl.
Which size gives a better value for money? Explain your answer.

$$\frac{\text{Vol. of large bowl}}{\text{Vol. of small bowl}} = \left(\frac{16}{7}\right)^3$$

Vol. of large bowl = 11.942(Vol. of small bowl)

As the volume of the large bowl is more than 8 times the volume of the small bowl, the larger bowl gives better value for money.

.... [2]

(b) The illumination, I units, of a bulb is inversely proportional to the square of the distance, d metres.
The distance from a particular bulb is increased by 25% of its original distance.
Calculate the percentage reduction in the illumination of the bulb.

$$I_1(d_1)^2 = I_2(d_2)^2$$
$$d_2 = 1.25d_1$$
$$I_1(d_1)^2 = I_2(1.25d_1)^2$$
$$I_2 = 0.64I_1$$

$\therefore$ reduction of 36% in illumination.

Answer .... % [2]

14 Simplify $\left(\frac{x^4}{100}\right)^{\frac{1}{2}} \div \left(64x^3\right)^{-\frac{2}{3}}$.

$$\left(\frac{x^4}{100}\right)^{\frac{1}{2}} \div \left(64x^3\right)^{-\frac{2}{3}} = \frac{x^2}{10} \times \left(64x^3\right)^{\frac{2}{3}}$$
$$= \frac{x^2}{10} \times 16x^2$$
$$= 1.6x^4$$

Answer .... [2]

---

15 A map is drawn to a scale of 1 : 25 000.

(a) This scale can be expressed as 1 centimetre represents n kilometres.
Find n.

1 cm : 25000 cm

1 cm : 0.25 km

$n = 0.25$

Answer $n =$ .... [1]

(b) The distance between two points on the map is 41.5 centimetres.
Find the actual distance, in kilometres, between the two points.

1 cm : 0.25 km

41.5 cm : 10.375 km

distance = 10.375 km

Answer .... km [1]

(c) A farm has an area of 51.2 square centimetres on this map.
Find the area of the farm, in square centimetres, on a second map which is drawn to a scale of 1 : 80 000.

Map 1

1 cm : 0.25 km

1 cm^2 : 0.0625 km^2

51.2 cm^2 : 3.2 km^2

Map 2

1 cm : 0.8 km

1 cm^2 : 0.64 km^2

5 cm^2 : 3.2 km^2

Therefore, area on the second map is 5 cm^2.

Answer .... cm^2 [2]

---

16 **(a)** Expand and simplify $(3x-5p)^2$.

$(3x-5p)^2 = (3x-5p)(3x-5p)$
$= 9x^2 - 30px + 25p^2$

Answer .... [1]

(b) Given that $(3x-5p)^2 = 9x^2 - 90x + 225$, find p.

Either

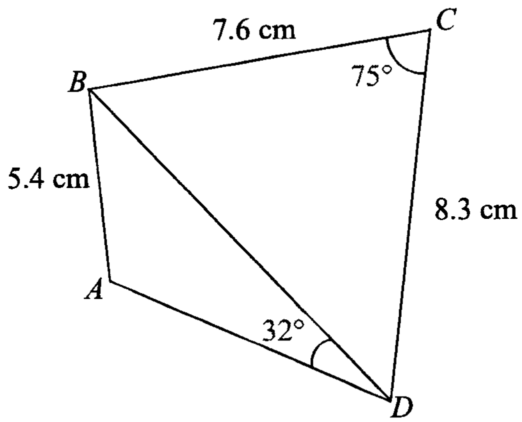
$-30p = -90$ or $25p^2 = 225$

$p = 3$ $\quad$ $p = 3$

Answer $p =$ .... [1]

17 The diagram shows a quadrilateral *ABCD*.

$AB = 5.4$ cm, $BC = 7.6$ cm, $CD = 8.3$ cm, angle $BCD = 75°$ and angle $ADB = 32°$.



(a) Find *BD*.

$BD^2 = (7.6)^2 + (8.3)^2 - 2(7.6)(8.3)\cos 75°$

$BD = 9.6952$

$= 9.70$ cm

Answer .... cm [3]

(b) Find angle *BAD*.

$\dfrac{\sin \angle BAD}{9.6952} = \dfrac{\sin 32°}{5.4}$

$\sin \angle BAD = 0.95142$

$\angle BAD = 107.9°$

Answer .... [2]

18 **(a)** **(i)** Write 3150 as a product of its prime factors.

$3150 = 2 \times 3^2 \times 5^2 \times 7$

Answer .... [1]

(ii) The highest common factor (HCF) of two numbers is 21.
The lower common multiple (LCM) of the two numbers is 3150.
Both numbers are greater than 70. Find the two numbers
Let the two numbers be a and b.

$a = 2 \times 3^2 \times 7 = 126$

$b = 3 \times 5^2 \times 7 = 525$

Answer .... and .... [2]

(b) **(i)** Use prime factors to explain why 14×56 is a perfect square.

$14 \times 56 = 2^4 \times 7^2$

The indices/powers of the prime factors are even integers

14×56 is a perfect square

....

.... [1]

(ii) p and q are both prime numbers. Find the values of p and q so that $14 \times 56 \div \frac{p}{q}$ is a perfect cube.

$$14 \times 56 \div \frac{p}{q} = 2^4 \times 7^2 \times \frac{q}{p}$$

$$= 2^4 \times 7^2 \times \frac{7}{2}$$

$$= 2^3 \times 7^3$$

$\therefore p = 2,\ q = 7.$

Answer $p =$ .... , $q =$ .... [2]

19 **(a)** Factorise completely $49x^2 - (x+3)^2$.

$$49x^2-(x+3)^2=(7x)^2-(x+3)^2$$
$$=(7x+x+3)(7x-x-3)$$
$$=(8x+3)(6x-3)$$
$$=3(8x+3)(2x-1)$$

Answer .... [2]

(b) One solution of the equation $6x^2+kx-14=0$, where k is an integer, is $x=\frac{2}{3}$.

Find

(i) the value of k,

Substitute $x=\frac{2}{3}$ into $6x^2+kx-14=0$,

$$6\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)k-14=0$$
$$k=17$$

Answer $k=$ .... [1]

(ii) the other solution to the equation.

$$6x^2+17x-14=0$$
$$(2x+7)(3x-2)=0$$
$$\therefore x=-\frac{7}{2} \text{ or } \frac{2}{3}$$

The other solution is $x=-\frac{7}{2}$.

Answer $x=$ .... [2]

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20 A bicycle rental station charges by hour for the rental of its bicycles.

On a particular week, the normal bicycles, children bicycles, and family bicycles were rented for 210 hours, 40 hours and x hours respectively on Saturday.
On Sunday, the normal bicycles, children bicycles, and family bicycles were rented for 325 hours, 65 hours and 32 hours, respectively.

(a) Represent the above information in a 2×3 matrix **P**.

$$\mathbf{P} = \begin{pmatrix} 210 & 40 & x \\ 325 & 65 & 32 \end{pmatrix}$$

Answer $\mathbf{P} = \begin{pmatrix} \quad \end{pmatrix}$ [1]

(b) The charges for one hour's rental is \$12 for normal bicycle, \$8 for children bicycle and \$40 for family bicycle.

Find, in terms of x, the matrix $\mathbf{Q} = \mathbf{P}\begin{pmatrix} 12 \\ 8 \\ 40 \end{pmatrix}$.

$$\mathbf{Q} = \begin{pmatrix} 210 & 40 & x \\ 325 & 65 & 32 \end{pmatrix}\begin{pmatrix} 12 \\ 8 \\ 40 \end{pmatrix}$$

$$= \begin{pmatrix} 2840 + 40x \\ 5700 \end{pmatrix}$$

Answer $\mathbf{Q} = \begin{pmatrix} \quad \end{pmatrix}$ [1]

(c) Explain what each element in **Q** represents.

The elements represent the revenue of the bicycle rental station on Saturday and Sunday respectively on that particular week.

....

.... [1]

(d) The revenue from rental of family bicycles on that Saturday is equivalent to the revenue from the rental of children bicycles on that Sunday.
Calculate the value of x.

$\$8 \times 65 = \$40 \times x$

$x = 13$

Answer $x =$ .... [1]

(e) In the following week, a special event was held near the bicycle rental station. The rental revenue increased by 25% and 50% compared to the previous Saturday and Sunday respectively.
Using your answer in **(d)** and matrix multiplication only, calculate the revenue on Saturday and Sunday, respectively.

$$\begin{pmatrix} 1.25 & 0 \\ 0 & 1.5 \end{pmatrix}\begin{pmatrix} 3360 \\ 5700 \end{pmatrix} = \begin{pmatrix} 4200 \\ 8550 \end{pmatrix}$$

Accept the following

$$\begin{pmatrix} 3360 & 0 \\ 0 & 5700 \end{pmatrix}\begin{pmatrix} 1.25 \\ 1.5 \end{pmatrix} = \begin{pmatrix} 4200 \\ 8550 \end{pmatrix}$$

OR

$$\begin{pmatrix} 1.25 & 1.5 \end{pmatrix}\begin{pmatrix} 3360 & 0 \\ 0 & 5700 \end{pmatrix} = \begin{pmatrix} 4200 & 8550 \end{pmatrix}$$

OR

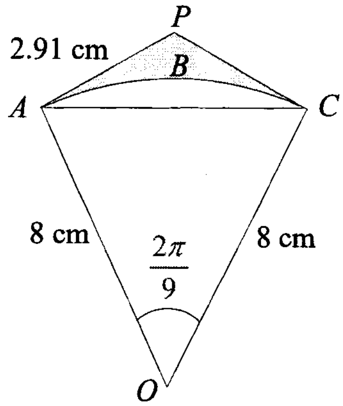
$$\begin{pmatrix} 3360 & 5700 \end{pmatrix}\begin{pmatrix} 1.25 & 0 \\ 0 & 1.5 \end{pmatrix} = \begin{pmatrix} 4200 & 8550 \end{pmatrix}$$

Therefore the revenue of Saturday and Sunday is \$4200 and \$8550 respectively.

Answer \$ .... on Saturday and \$ .... on Sunday [2]

21 ABC is an arc of a circle, centre O and radius 8 cm.

The tangents at A and C meet at P. Angle AOC is $\frac{2\pi}{9}$ radians and $AP = 2.91$ cm.



(a) Show that angle $APC = \frac{7\pi}{9}$ radians.

(Give reasons for each step of your working)

Answer

$\angle PAO = \angle PCO = \frac{1}{2}\pi$ (tangent $\perp$ radius)

$\angle APC = 2\pi - \frac{1}{2}\pi - \frac{1}{2}\pi - \frac{2\pi}{9}$ (sum of $\measuredangle$s in a quad)

$= \frac{7\pi}{9}$ [2]

(b) Find the area of the shaded region.

$$\text{Area of segment } ABC = \frac{1}{2}(8)(8)\left(\frac{2\pi}{9}\right) - \frac{1}{2}(8)(8)\sin\left(\frac{2\pi}{9}\right)$$
$$= 1.7710 \text{ cm}^2 \text{ (5 sig.fig)}$$

$$\text{Area of triangle } APC = \frac{1}{2}(2.91)(2.91)\sin\left(\frac{7\pi}{9}\right)$$
$$= 2.7216 \text{ cm}^2 \text{ (5 sig.fig)}$$

$$\text{Shaded area} = 2.7216 - 1.7710$$
$$= 0.951 \text{ cm}^2$$

Or

$$\text{Area of quadrilateral } APCO = 2\left[\frac{1}{2}(2.91)(8)\right]$$
$$= 23.28 \text{ cm}^2$$

$$\text{Area of sector } AOC = \frac{1}{2}(8)(8)(\frac{2\pi}{9})$$
$$= \frac{64\pi}{9} \text{ cm}^2$$

$$\text{Shaded area} = 23.28 - \frac{64\pi}{9}$$
$$= 0.940 \text{ cm}^2 \text{ (round off to 3 sig.fig)}$$

Answer .... cm^2 [3]

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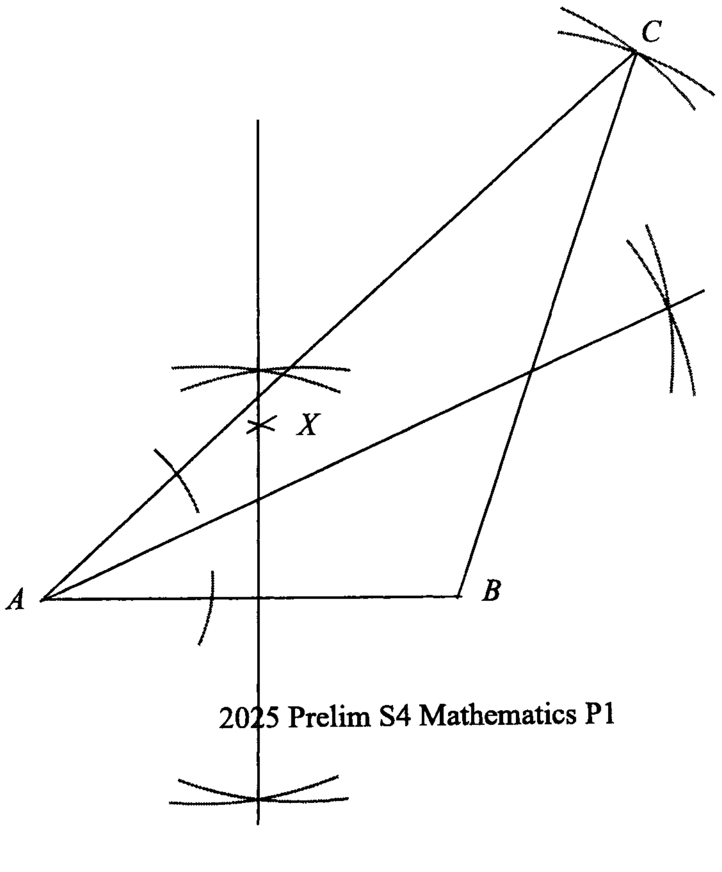
22 Construct triangle ABC where $AB = 5.8$ cm, $BC = 7$ cm and $AC = 9.5$ cm.
Line AB has been drawn.

Answer [1]



(a) On the same diagram, construct

(i) the perpendicular bisector of AB, [1]

(ii) the angle bisector of angle CAB. [1]

(b) Mark clearly a possible point which lies inside the triangle, equidistant from A and B, and is nearer to AC than AB. Label this point X [1]

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23 The figure below shows a series of patterns consisting of dots and lines.
Figure 1 has 4 dots and 7 lines.

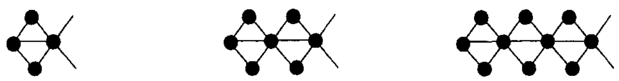


Figure 1 Figure 2 Figure 3

(a) Which figure has 19 dots?
Figure 6

Answer Figure .... [1]

(b) Let L denotes the number of lines.
Write down a formula for L in terms of n, in Figure n.

$L_n = 5n + 2$ or $7 + 5(n - 1)$

Answer $L_n =$ .... [1]

(c) Is there a figure with 83 lines? Explain why.

$L = 83,$

When $5n + 2 = 83$

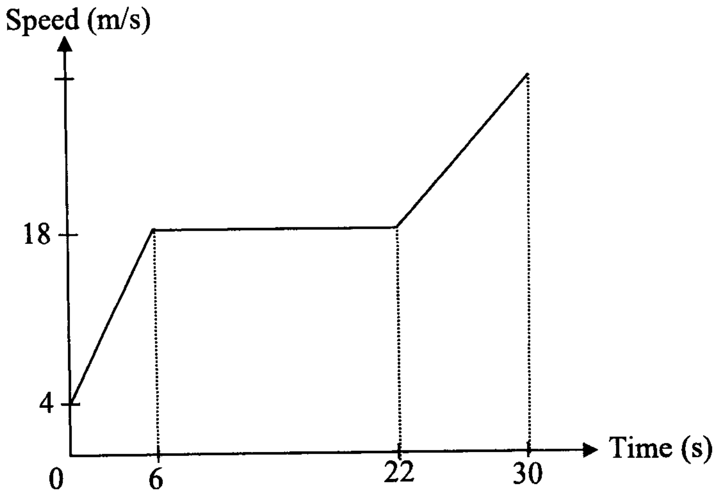
$n = 16.2$

Since n is not a positive integer
Therefore there is no figure with 83 lines.

....

.... [1]

24 The diagram shows a speed time graph of an object during a period of 30 seconds.



(a) Calculate the acceleration of the object when $t = 4$.

$$\text{acceleration} = \frac{18-4}{6}$$

$$= 2\frac{1}{3} \text{ m/s}^2 \quad (\text{accept } 2.33, \ \frac{7}{3})$$

Answer .... m/s^2 [1]

(b) Calculate the distance travelled in the first 15 seconds.

Distance $=\frac{1}{2}(4+18)(6)+9(18)$

$= 228$ m

Answer .... m [2]

(c) Given that the acceleration of the object after $t = 22$ is 1.5 m/s^2, calculate the speed of the object when $t = 30$.

Let the speed be v m/s.

$$\frac{v-18}{30-22}=1.5$$

$$\frac{v-18}{8}=1.5$$

$$v=30$$

OR

Use the point (22,18)

m = 1.5 m/s^2

$18=1.5(22)+c$

$c=-15$

$y=1.5x-15$

At t = 30,

$y=1.5(30)-15$

$=30$

Answer .... m/s [2]

Name: Solution	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION 2025

MATHEMATICS
Paper 2

4052/02
26 August 2025
2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

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Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use

Question	1	2	3	4	5	6	7	8	9	10
Marks	10	8	7	9	9	8	9	10	10	10

Table of Penalties		Question No.		
Presentation	–1			
Accuracy/ Units	–1		Parent's / Guardian's Signature	90

This document consists of **24** printed pages

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{ , where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 **(a)** Solve the inequality $2-x<\frac{3x+4}{2}<5x-3$.

Solution:

$2-x<\frac{3x+4}{2}$ and $\frac{3x+4}{2}<5x-3$

$4-2x<3x+4$ $\quad$ $3x+4<10x-6$

$x>0$ $\quad$ $x>\frac{10}{7}$

$x>\frac{10}{7}$

Answer .... [3]

(b) It is given that $F=\frac{m(7a-2x^2)}{3a}$, where $m\neq 0$ and $a\neq 0$.

(i) Find, in terms of *a*, the value of *F* when $x=-a$ and $m=3$.

Solution:

$$F=\frac{3(7a-2(-a)^2)}{3a}$$
$$=7-2a$$

Answer $F=$ .... [1]

(ii) Express *x* in terms of *F*, *a* and *m*.

Solution:

$$F=\frac{m(7a-2x^2)}{3a}$$
$$7a-2x^2=\frac{3aF}{m}$$
$$x^2=\frac{7a}{2}-\frac{3aF}{2m}$$
$$x=\pm\sqrt{\frac{7a}{2}-\frac{3aF}{2m}}$$
$$=\pm\sqrt{\frac{7am-3aF}{2m}}$$

Answer $x=$ .... [2]

(c) Solve the equation $\frac{x+2}{x+5}+\frac{3x}{x-2}=7$.

Solution:

$$\frac{x+2}{x+5}+\frac{3x}{x-2}=7$$

$$(x+2)(x-2)+3x(x+5)=7(x+5)(x-2)$$

$$x^2-4+3x^2+15x=7(x^2+3x-10)$$

$$4x^2+15x-4=7x^2+21x-70$$

$$3x^2+6x-66=0$$

$$x^2+2x-22=0$$

$$x=\frac{-2\pm\sqrt{2^2-4(1)(-22)}}{2(1)}$$

$$=\frac{-2\pm\sqrt{92}}{2}$$

$$=3.80 \quad \text{or} \quad -5.80 \quad \text{(3sf)}$$

Answer $x=$ .... or .... [4]

2 A company conducted a survey in a girl's school and found that a user's likelihood of logging into a particular app each day depends on whether she logged in the previous day.

If the user logs in on a particular day, the probability that she logs in again the following day is 0.7.
If the user does not log in on a particular day, the probability that she does not log in the next day is 0.8.

It is given that Emily logs in on Monday.

(a) Find the probability that Emily logs in on Tuesday, Wednesday and Thursday.

Solution:

$$\begin{aligned}\text{probability} &= 0.7 \times 0.7 \times 0.7 \\ &= 0.343\end{aligned}$$

Answer .... [2]

(b) Find the probability that Emily does not log in on Wednesday.

Solution:

$$\begin{aligned}\text{probability} &= (0.7 \times 0.3) + (0.3 \times 0.8) \\ &= 0.45\end{aligned}$$

Answer .... [2]

(c) Each time the user logs in, she will earn 10 reward points.
If the user does not log in, no reward points will be given.

(i) Find the probability that Emily earns exactly 20 points by Wednesday.

Solution:

$$\begin{aligned}\text{probability} &= (0.7 \times 0.3) + (0.3 \times 0.2) \\ &= 0.27\end{aligned}$$

Answer .... [2]

(ii) Find the probability that Emily earns more than 10 points by Wednesday.

Solution:

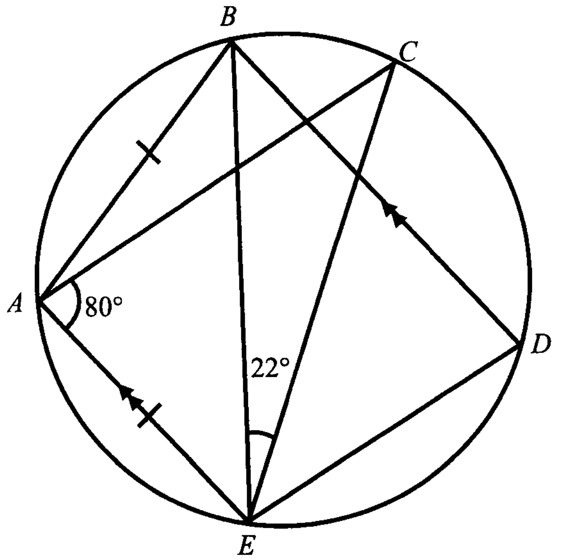
$$\begin{aligned}\text{probability} &= 1 - (0.3 \times 0.8) \\ &= 0.76\end{aligned}$$

or

$$\begin{aligned}\text{probability} &= 0.27 + (0.7 \times 0.7) \\ &= 0.76\end{aligned}$$

Answer .... [2]

3 A, B, C, D and E are points on a circle.
$AE = AB$, AE is parallel to BD, angle $BEC = 22°$ and angle $EAC = 80°$.



(a) Find angle AEB.

Solution:

$\angle BAC = 22°$ (angles in the same segment)

$\angle BAE = 22° + 80°$
$= 102°$

$\angle AEB = \dfrac{180° - 102°}{2}$ (base angles of isos. triangle)
$= 39°$

Answer Angle AEB = ... [2]

(b) Find angle DEC.

Solution:

$\angle BDE = 180° - 102°$ (angles in opp segment)
$= 78°$

$\angle AED = 180° - 78°$ (int angles, $AE // BD$)
$= 102°$

$\angle CED = 102° - 39° - 22°$
$= 41°$

Alternative Solution:

$\angle DBE = \angle BEA$ (alt angles, $AE // BD$)

$= 39°$

$\angle DEC = 180° - 39° \times 3 - 22°$ (angles in opp segment)

$= 41°$

Answer Angle DEC = ... [3]

(c) Angle $AFE = 78°$.

Determine the position of point F.

Justify your answer.

Solution:

$\angle ACE = 180° - 39° - 22° - 80°$ (sum of angles in triangle)

$= 39°$

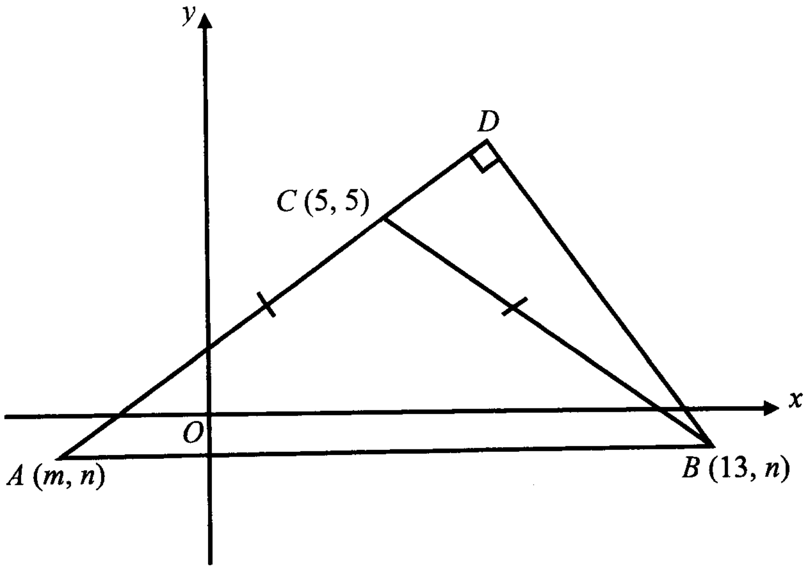
$\angle AFE = 2\angle ACE$

Point F lies on the centre of the circle.

--- [2]

4 In the diagram, BD is perpendicular to AD.

It is given that $AC = BC$ and the gradient of BC is $-\frac{3}{4}$.



(a) Find the equation of line BC.

Solution:

$$y-5=-\frac{3}{4}(x-5)$$

$$y=-\frac{3}{4}x+\frac{35}{4}$$

Answer .... [2]

(b) Find the value of n.

Solution:

$$n=-\frac{3}{4}(13)+\frac{35}{4}$$

$$=-1$$

Alternative Solution:

$$\frac{5-n}{5-13}=-\frac{3}{4}$$

$$n=-1$$

Answer $n =$ .... [1]

(c) Show that $m = -3$.

Solution:

ΔABC is isosceles.

Points A and B are symmetrical about the vertical through C.

$$\frac{m+13}{2} = 5$$

$$m = -3$$

.... [1]

(d) Find the area of triangle ABC.

Solution:

$$\text{area of triangle } ABC = \frac{1}{2} \times 16 \times 6$$
$$= 48 \text{ units}^2$$

Answer .... units^2 [2]

(e) Hence, find the length of BD.

Solution:

$$AC = \sqrt{(5-(-3))^2 + (5-(-1))^2}$$
$$= 10 \text{ units}$$

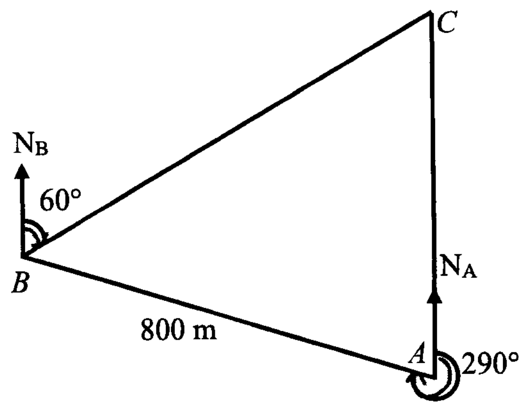
$$\frac{1}{2} \times 10 \times BD = 48$$
$$BD = 9.6 \text{ units}$$

Answer .... units [3]

5 A, B and C are points on the horizontal ground.
B is at a bearing of 290° and 800 m away from A.
C is due north of A and at a bearing of 60° from B.

(a) Find BC.

Solution:



$\angle BCA = 60°$ (alternate angles)

$$\frac{BC}{\sin 70°} = \frac{800}{\sin 60°}$$

$$BC = \frac{800}{\sin 60°} \times \sin 70°$$

$$= 868.05$$

$$= 868 \text{ m (3sf)}$$

Answer .... m [2]

(b) Show that AC is approximately 707.64 m.

Answer
Solution:

$\angle CBA = 180° - 60° - (360° - 290°)$ (sum of angles in a triangle)

$$= 50°$$

$$\frac{AC}{\sin 50°} = \frac{800}{\sin 60°}$$

$$AC = \frac{800}{\sin 60°} \times \sin 50°$$

$$= 707.64 \text{ m (5sf) (shown)}$$

Alternative Solution:

$\angle CBA = 180° - 60° - (360° - 290°)$ (sum of angles in a triangle)

$= 50°$

$AC = \sqrt{800^2 + 868.05^2 - 2(800)(868.05)\cos 50°}$

$= 707.64$ m (5sf) (shown)

[2]

CT is a vertical tower at C.

When Betty is at A, the angle of elevation of the top of the tower, T, is 20°.

(c) Find CT.

Solution:

$\tan 20° = \frac{CT}{707.64}$

$CT = 707.64 \times \tan 20°$

$= 257.56$ m (5sf)

$= 258$ m (3sf)

Answer .... m [2]

(d) Find the largest angle of elevation of T as Betty walks from A to B.

Solution:

Let the perpendicular distance from C to AB be x.

$\sin 70° = \frac{x}{707.64}$

$x = 707.64 \times \sin 70°$

$= 664.96$ m (5sf)

Let the largest angle of elevation of T be θ.

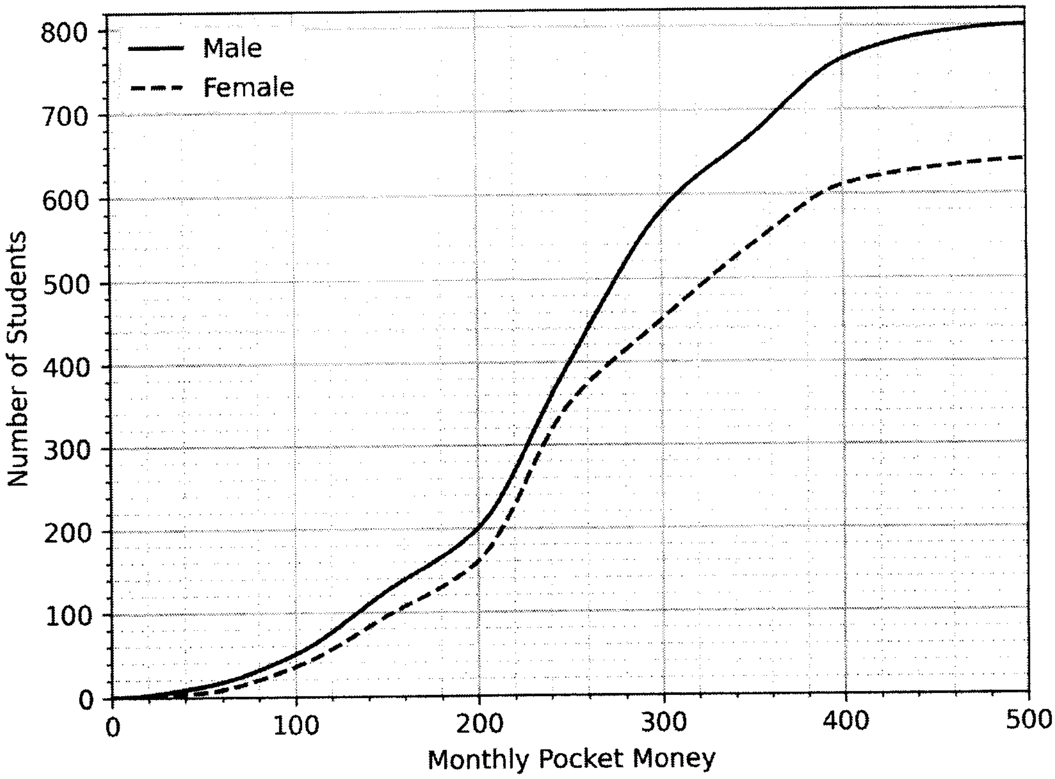
$\tan\theta = \frac{257.56}{664.96}$

$\theta = \tan^{-1}\left(\frac{257.56}{664.96}\right)$

$\theta = 21.2°$

Answer .... [3]

6 A secondary school conducted a survey on the amount of pocket money male and female students received each month.
The cumulative frequency curves below represent the distributions of the amounts received by each group.



(a) Use the curve to estimate

(i) the median monthly pocket money the male students received,

Solution:
Median = \$250

Answer \$ ………………… [1]

(ii) the interquartile range of the monthly pocket money the male students received.

Solution:
Interquartile range = \$310 − \$200
= \$110

Answer \$ ………………… [2]

(b) Find the number of students who received more than \$300 of pocket money each month.

Solution:

Number of male students = (800 – 580) + (640 – 450)
= 410

Answer .... [2]

(c) Amy said, "In this survey, the percentage of male students who received above \$300 of pocket money each month is higher than the percentage of female students who received above \$300 of pocket money each month."
Do you agree with this statement?
Justify your answer.

Answer
Solution:

$$\text{Percentage of male students} = \frac{800-580}{800} \times 100\% = 27.5\%$$

$$\text{Percentage of female students} = \frac{640-450}{640} \times 100\% = 29.6875\%$$

I do not agree with the statement.

[2]

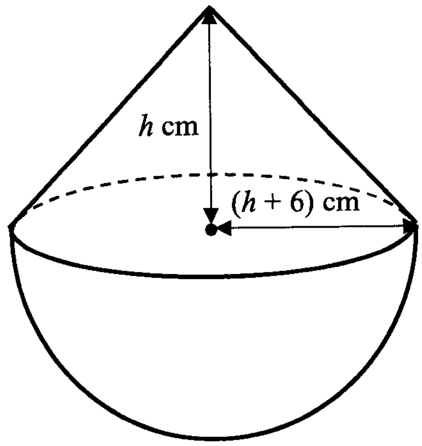
(d) The school realised that there is a mistake in the data.
One of the male student's monthly pocket money should be \$300 instead of \$250.
How does this affect the accuracy of the interquartile range of the male students' monthly pocket money?

Solution:

Upper Quartile = \$310 Lower Quartile = \$200
Both \$250 and \$300 lies within lower and upper quartiles.
This does not affect the interquartile range.

[1]

7 The figure shows a solid consisting of a right circular cone and a hemisphere.
The height and the base radius of the circular cone are h cm and $(h + 6)$ cm respectively.
The volume of the hemisphere is three times the volume of the cone.



(a) Show that $h = 12$.

Solution:

$$\frac{2}{3}\pi(h+6)^3 = 3\times\frac{1}{3}\pi(h+6)^2 h$$

$$2\pi(h+6)^3 - 3\pi(h+6)^2 h = 0$$

$$\pi(h+6)^2\left(2h+12-3h\right) = 0$$

$$\pi(h+6)^2\left(12-h\right) = 0$$

$$h = -6 \text{ (NA)} \quad \text{or} \quad 12 \text{ (shown)}$$

Answer [3]

(b) Find the total surface area of the solid giving your answer correct to the nearest cm^2.

Solution:

$$\text{slant height} = \sqrt{12^2 + (12+6)^2}$$

$$= \sqrt{468}$$

$$\text{total surface area of solid} = \pi(18)\sqrt{468} + 2\pi(18)^2$$

$$= 3259 \text{ cm}^2$$

Answer .... cm^2 [3]

(c) Cathy buys a tin of paint that can cover a surface area of $0.5\ \text{m}^2$.
She cuts the solid into 2 identical parts and plans to paint all surfaces of the two parts with the can of paint.
Does she have enough paint?
Explain your answer.

Answer
Solution:

$$\text{new total surface area of solid} = \pi(18)\sqrt{468} + 2\pi(18)^2 + 2\left(\frac{36\times 12}{2} + \frac{\pi(18)^2}{2}\right)$$
$$= 4709\ \text{cm}^2$$
$$= 0.4709\ \text{m}^2$$

She has enough paint.

[3]

8 A cuboid has a square base of sides x cm.
Its height is h cm and its volume is 150 cm^3.

(a) Show that the total surface area of the cuboid, $A = 2x^2 + \frac{600}{x}$.

Answer
Solution:

$x^2 h = 150$

$h = \frac{150}{x^2}$

$A = 2x^2 + 4xh$

$= 2x^2 + 4x\left(\frac{150}{x^2}\right)$

$= 2x^2 + \frac{600}{x}$

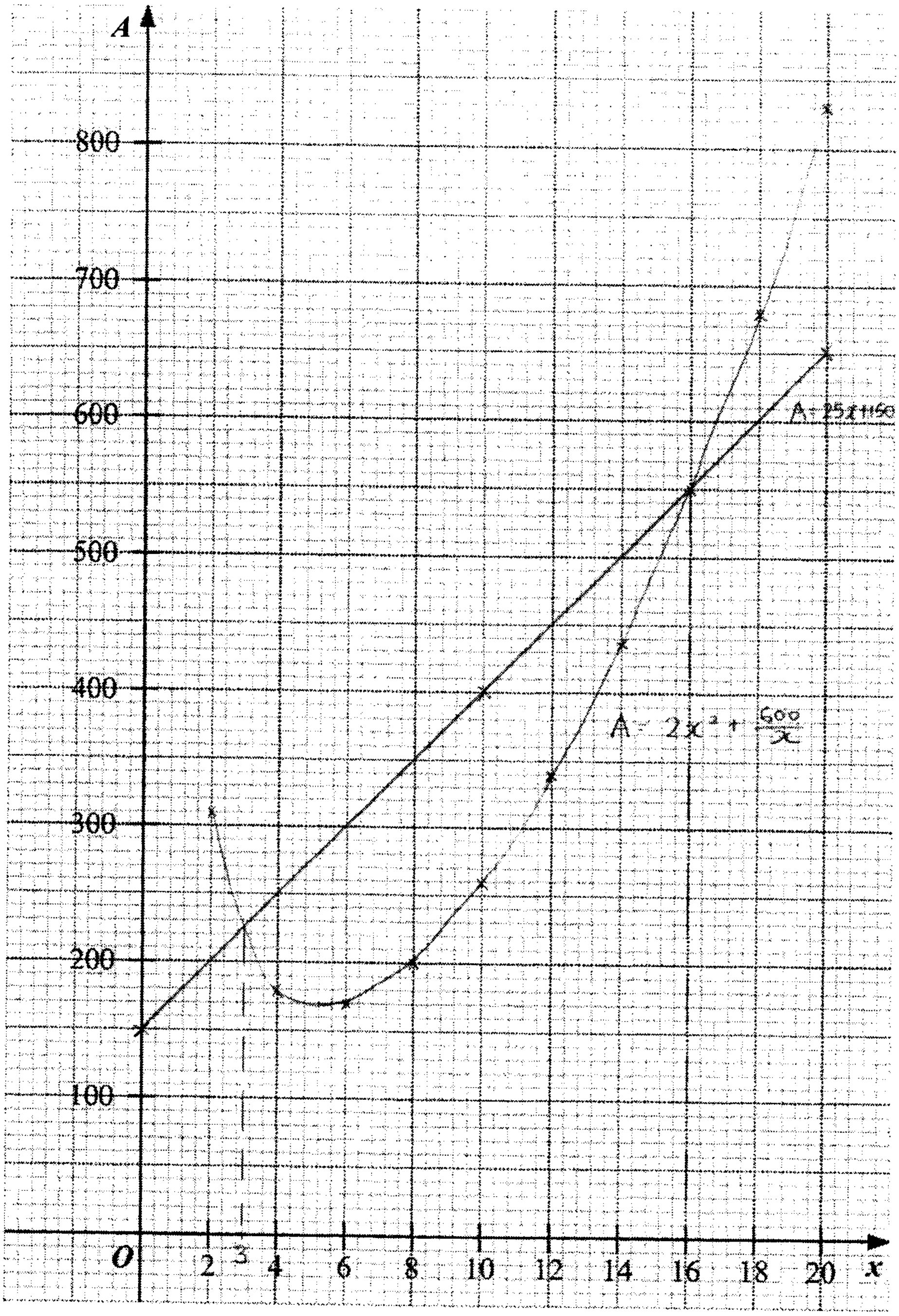
[2]

(b) Complete the table of values for $A = 2x^2 + \frac{600}{x}$.

x	2	4	6	8	10	12	14	16	18	20
A	308	182	172	203	260	338	435	550	681	830

[1]

(c) On the grid, draw the graph of $A = 2x^2 + \frac{600}{x}$ for $2 \le x \le 20$.



[3]

(d) Can the total surface area of the cuboid be 150 cm^2?
Explain your answer.

Solution:
From graph, minimum y is 170 cm^2.

Total surface area of the cuboid cannot be 150 cm^2.

....

.... [1]

(e) A manufacturer designs cuboid packaging boxes with square bases. The total surface area must not exceed the material cost limit which is given by $A = 25x + 150$.

(i) On the same grid, draw the straight line $A = 25x + 150$ for $2 \le x \le 20$.

Answer [1]

(ii) Use the graph to find the possible lengths of the sides of the cuboid where the material cost is maximised.

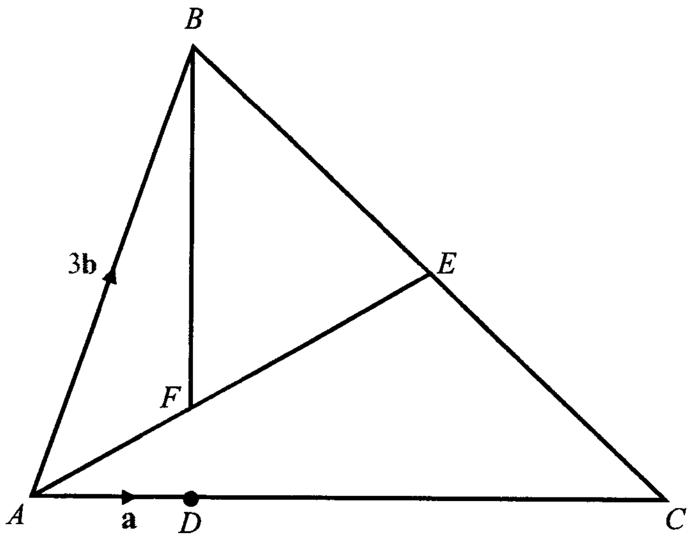
Solution:
3 or 16 [accept 2.6 – 3.4, 15.6 – 16.4]

Answer $x =$ .... cm or .... cm [2]

9 In the diagram, $\overrightarrow{AD} = \mathbf{a}$, $\overrightarrow{AB} = 3\mathbf{b}$ and $AD = \frac{1}{4}AC$.

E is the midpoint of BC.

F is a point on AE such that $AF = \frac{2}{3}FE$.



(a) Express and simplify your answers in terms of **a** and **b**,

(i) $\overrightarrow{BC}$,

Solution:

$\overrightarrow{AC} = 4\overrightarrow{AD}$

$= 4\mathbf{a}$

$\overrightarrow{BC} = \overrightarrow{AC} + \overrightarrow{BA}$

$= 4\mathbf{a} - 3\mathbf{b}$

Answer $\overrightarrow{BC} =$ .... [1]

(ii) $\overrightarrow{AF}$,

Solution:

$\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE}$

$= 3\mathbf{b} + \frac{1}{2}(4\mathbf{a} - 3\mathbf{b})$

$= 2\mathbf{a} + \frac{3}{2}\mathbf{b}$

$\overrightarrow{AF} = \frac{2}{5}\overrightarrow{AE}$

$= \frac{4}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$

Answer $\overrightarrow{AF} =$ .... [2]

(iii) $\overrightarrow{BF}$.

Solution:

$\overrightarrow{BF} = \overrightarrow{BA} + \overrightarrow{AF}$

$= -3\mathbf{b} + \left(\frac{4}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}\right)$

$= \frac{4}{5}\mathbf{a} - \frac{12}{5}\mathbf{b}$

Answer $\overrightarrow{BF} =$ .... [1]

(b) Explain if BF produced will meet AC at D.

Answer
Solution:

$$\overrightarrow{BF} = \frac{4}{5}\mathbf{a} - \frac{12}{5}\mathbf{b} = \frac{4}{5}(\mathbf{a} - 3\mathbf{b})$$

$$\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD}$$
$$= -3\mathbf{b} + \mathbf{a}$$

$$\overrightarrow{BF} = \frac{4}{5}\overrightarrow{BD}$$

The points B, F and D are collinear with common point B.
Hence, BF produced will meet AC at D.

[3]

(c) Find the value of

(i) $\dfrac{\text{area of } \Delta ABF}{\text{area of } \Delta ABE}$,

Solution:

$$\frac{\text{area of } \Delta ABF}{\text{area of } \Delta ABE} = \frac{AF}{AE}$$
$$= \frac{2}{5}$$

Answer .... [1]

(ii) $\dfrac{\text{area of } \Delta BEF}{\text{area of } \Delta ABC}$.

Solution:

$$\frac{\text{area of } \Delta BEF}{\text{area of } \Delta ABC} = \frac{\text{area of } \Delta BEF}{\text{area of } \Delta ABE} \times \frac{\text{area of } \Delta ABE}{\text{area of } \Delta ABC}$$
$$= \frac{FE}{AE} \times \frac{BE}{BC}$$
$$= \frac{3}{5} \times \frac{1}{2}$$
$$= \frac{3}{10}$$

....
Answer ... [2]

10 A secondary school is planning a major upgrade by setting up a solar panel system. The goal is to choose the system that offers the best financial value over its entire lifespan.
The school is evaluating two solar panel providers, SolarSmart and EcoSun.

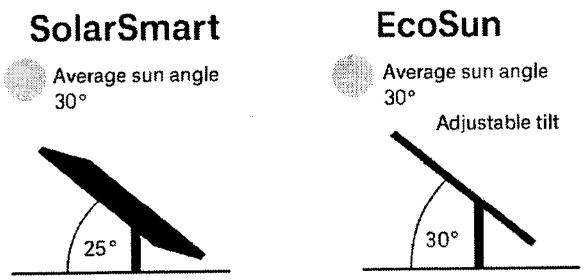
School's current electricity usage:

- School uses an average of 17 000 kilowatt hours (kWh) of electricity per year.
- The cost of electricity is \$0.30 per kWh.

Information on the solar panel providers:

Company	Number of Panels	Type of Panel	Total Installation Cost	Annual Maintenance Cost
SolarSmart	50	Fixed tilt at 25°	\$66 000	\$340
EcoSun	45	Adjustable tilt	\$64 000	\$360

- Each panel's wattage is 320 Watts (W) at 100% efficiency.
- The average sun angle at the school's location is 30°.
- For every 1° difference between the panel tilt and the sun angle, the panel's efficiency drops by 1%.
- Both solar panel systems are guaranteed to last for 25 years.



Additional information:

- Singapore gets an average of 4.2 hours of peak sunlight per day.
- Solar panel output in daily watt hours = solar panel wattage × peak sunlight hours × 75%.
- The school can sell any extra energy it does not use back to the grid for \$0.20 per kWh.

(a) Calculate, in watt hours, the daily power output per panel for SolarSmart.

Solution:

daily power output per panel for SolarSmart $= \frac{95}{100} \times 320 \times 4.2 \times \frac{75}{100}$ [M1]

$= 957.6\text{Wh}$ [A1]

Answer .... Wh [2]

(b) Find the total daily power output from each company, leaving your answer in kilowatt hours (kWh).

Solution:

daily power output for SolarSmart $= 957.6 \times 50$

$= 47880 \text{ Wh}$

$= 47.88 \text{ kWh}$ [B1]

daily power output for EcoSun $= 320 \times 4.2 \times \frac{75}{100} \times 45$

$= 45360 \text{ Wh}$

$= 45.36 \text{ kWh}$ [B1]

Answer Daily power output for SolarSmart = ... kWh

Daily power output for EcoSun = ... kWh [2]

(c) Recommend which company the school should choose.
Justify your decision and show your calculations clearly.

Solution:

SolarSmart:
Annual energy $= 47.88 \text{ kW} \times 365$
$= 17\ 476.2 \text{ kWh}$

net savings per year $= 17\ 000 \times \$0.30 + 476.2 \times \$0.20 - \$340$
$= \$4855.24$

total savings over 25 years $= \$4855.24 \times 25 - \$66\ 000$
$= \$55\ 381$

EcoSun:
Annual energy $= 45.36 \text{ kW} \times 365$
$= 16\ 556.4 \text{ kWh}$

net savings per year $= 16\ 556.4 \times \$0.30 - \$360$
$= \$4606.92$

total savings over 25 years $= \$4606.92 \times 25 - \$64\ 000$
$= \$51\ 173$

The total savings over 25 years for SolarSmart is more than for EcoSun.
The school should choose SolarSmart.

[5]

(d) State one assumption made in your calculation in **(c)**.

Solution:

Solar panel generates the same amount of energy every day. / 365 days in a year. / No degradation in panel performance over time / Electricity prices does not change. / Peak sunlight hours remain constant over 25 years… etc. [1]

END OF PAPER