Name : Mark Scheme

METHODIST GIRLS' SCHOOL
Founded in 1887

PRELIMINARY EXAMINATION 2025
Secondary 4

| Wednesday | **MATHEMATICS** | 4052/01 |
| 20 August 2025 | **Paper 1** | 2 h 15 min |

Candidates answer on the Question Paper.

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
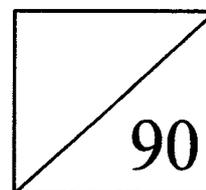Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

90

This question paper consists of 26 printed pages and 2 blank pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle} = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1   Simplify $\dfrac{4a^{-2}}{3r} \div \dfrac{\left(27a^6\right)^{\frac{1}{3}}}{r^3}$, giving your answer in positive index.

$$\dfrac{4a^{-2}}{3r} \div \dfrac{\left(27a^6\right)^{\frac{1}{3}}}{r^3} = \dfrac{4a^{-2}}{3r} \div \dfrac{3a^2}{r^3}$$

M1 for $3a^2$

$$= \dfrac{4a^{-2}}{3r} \times \dfrac{r^3}{3a^2}$$

$$= \dfrac{4r^2}{9a^4}$$ A1

Answer.................................. [2]

2    $4^{m-3} \times 8^{3m-1} = 4$

Use the laws of indices to find the value of m. Show your working.

$$4^{m-3} \times 8^{3m-1} = 4$$

$$2^{2(m-3)} \times 2^{3(3m-1)} = 2^2$$ M1 for expressing terms in base 2

$$2^{2m-6+9m-3} = 2^2$$

$$2^{11m-9} = 2^2$$

$$11m - 9 = 2$$

$$11m = 11$$

$$m = 1$$ A1

Answer **m** = ... [2]

3   Two identical containers are filled with a mixture of rose syrup and evaporated milk in the ratio of 5 : 3 and 4 : 7.
    The contents of the containers are poured into a big bowl and mixed thoroughly.
    Find the ratio of the rose syrup to the evaporated milk in the big bowl.

1st Container	2nd Container
5 : 3	4 : 7
55 : 33	32 : 56

M1 for both ratios

Rose Syrup : Evaporated milk = 87 : 89   A1

Answer .... : .... [2]

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4   Simplify $\dfrac{(3m-1)(2m+3)}{3}+\dfrac{5m-2}{6}$.

$$\frac{(3m-1)(2m+3)}{3}+\frac{5m-2}{6}=\frac{2\left(6m^2+9m-2m-3\right)+5m-2}{6}$$

M1 for expansion of
(3m-1)(2m+3)

$$=\frac{12m^2+14m-6+5m-2}{6}$$

$$=\frac{12m^2+19m-8}{6}$$ A1

Answer.................................... [2]

BP-457

5   A map is drawn to a scale of 1 : 400 000.
     (a)   Find the actual distance, in km, represented by 10.5 cm on the map

1 cm : 4 km
10.5 cm : 42 km   A1

Answer................................. km [1]

(b)   A forest covers an area of 64 km^2.
          Find, in cm^2, the area representing the forest on the map.

1 cm : 4 km
1 cm^2 : 16 km^2
4 cm^2 : 64 km^2 A1

Answer................................. cm^2 [1]

6 **(a)** Written as a product of its prime factors, $240 = 2^4 \times 3 \times 5$ and $2750 = 2 \times 5^3 \times 11$.

(i) Find the smallest positive integer n for which $\sqrt{2750n}$ is a whole number.

$$2750n = 2^2 \times 5^4 \times 11^2 \quad \boxed{\text{M1 for perfect square}}$$
$$2 \times 5^3 \times 11n = 2^2 \times 5^4 \times 11^2$$
$$n = \frac{2^2 \times 5^4 \times 11^2}{2 \times 5^3 \times 11}$$
$$= 2 \times 5 \times 11$$
$$= 110 \quad \boxed{\text{A1}}$$

Answer.................................... [2]

(ii) Find the smallest positive integer k for which $240k$ is a multiple of 2750.

LCM of 240 and 2750 $= 2^4 \times 3 \times 5^3 \times 11$

$240k = 2^4 \times 3 \times 5^3 \times 11$

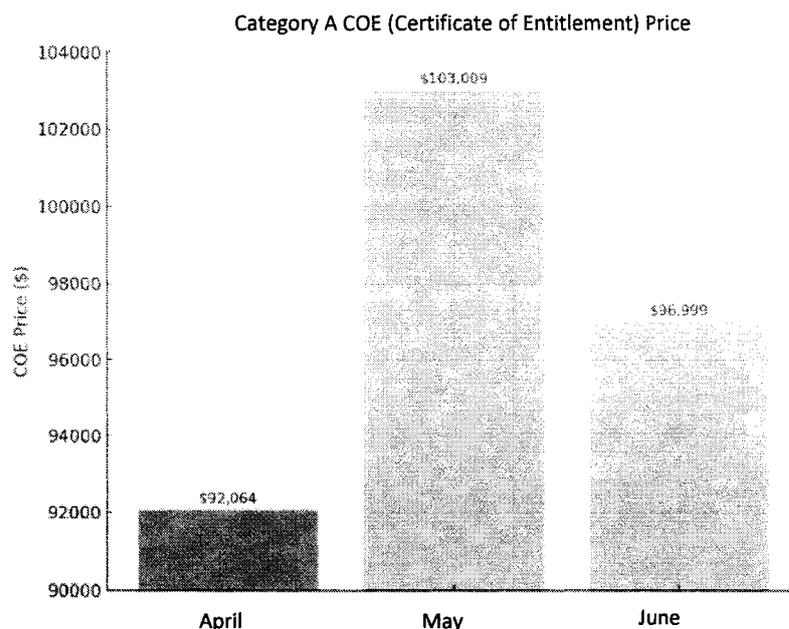$k = \dfrac{2^4 \times 3 \times 5^3 \times 11}{2^4 \times 3 \times 5} = 5^2 \times 11 = 275 \quad \boxed{\text{A1}}$

Answer.................................... [1]

(b) $Y = a^{18} \times b^{12} \times c^{36}$ where a, b and c are prime numbers.
Explain why Y is a perfect cube.

Answer

<u>The indices of the prime factors a, b and c are multiples of 3.</u> [1]

7   The graph shows the Category A Certificate of Entitlement (COE) price for cars up to 1600cc for the months of April, May and June in 2025.

Category A COE (Certificate of Entitlement) Price

State one aspect of the graph that maybe misleading and explain how this may lead to a misinterpretation of the graph.

Answer

By starting the vertical axis at $90000 and not at zero, it exaggerates small differences in COE prices, making the COE price of May look dramatically more expensive than April or June, even though the actual difference is relatively small.   [2]

Category A COE Prices (Correct Representation)
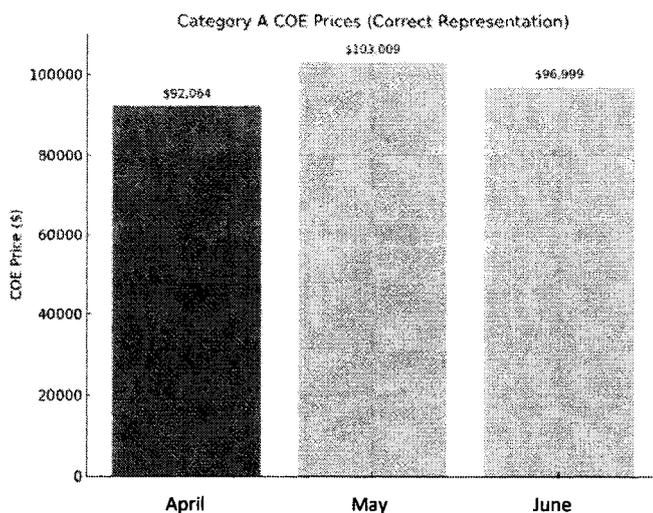
8   The interior angles of a hexagon are in the ratio 3 : 5 : 6 : 7 : 7 : 8.
     Find the smallest exterior angle of the hexagon.

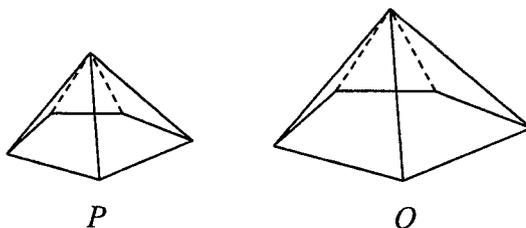$$\text{size of largest interior angle} = 180°(6-2) \times \frac{8}{36} \quad \boxed{M1}$$

$$= 160°$$

$$\text{smallest exterior angle} = 180° - 160°$$

$$= 20° \quad \boxed{A1}$$

Answer..................................° [2]

9   P and Q are similar pentagonal pyramids.
     The ratio of surface areas of the two pyramids, P and Q, is $81 : 121$.

(a) Find the ratio of the height of pyramid P to that of pyramid Q.

     $\text{Height}_P : \text{Height}_Q = \sqrt{81} : \sqrt{121} = 9 : 11$ $\boxed{\text{A1}}$

     Answer...................... : .... [1]

(b) Both pyramids are completely filled with sand. Find the mass of the sand, in kg, in pyramid Q if the mass of sand in the pyramid P is 192 g.

     $\dfrac{0.192}{M_Q} = \dfrac{9^3}{11^3}$ $\boxed{\text{M1}}$

     $M_Q = \dfrac{11^3 \times 0.192}{9^3}$

     $= 0.35055$

     $= 0.351 \,\text{kg} \,(3\text{sf})$ $\boxed{\text{A1}}$

     Answer.............................. kg [2]

10   Factorise completely.
     (a)
     $5 - 5x^2 = 5(1 - x^2)$ $\boxed{\text{M1}}$

     $\quad = 5(1-x)(1+x)$ $\boxed{\text{A1}}$

     Answer.................................... [2]

     (b)
     $2xq - 2yq - yp + xp$

     $= 2q(x-y) + p(x-y)$ $\boxed{\text{M1}}$

     $= (2q+p)(x-y)$ $\boxed{\text{A1}}$

     Answer.................................... [2]

11   Simplify $\dfrac{3}{x+1} - \dfrac{x+4}{x^2+3x+2}$.

$$\dfrac{3}{x+1} - \dfrac{x+4}{x^2+3x+2} = \dfrac{3}{x+1} - \dfrac{x+4}{(x+1)(x+2)}$$

M1 for factorisation of $x^2 + 3x + 2$

$$= \dfrac{3(x+2)-(x+4)}{(x+1)(x+2)}$$

$$= \dfrac{3x+6-x-4}{(x+1)(x+2)}$$

M1 for making single fraction

$$= \dfrac{2x+2}{(x+1)(x+2)}$$

M1 for simplifying numerator

$$= \dfrac{2(x+1)}{(x+1)(x+2)}$$

$$= \dfrac{2}{(x+2)}$$

A1

Answer.................................. [4]

12 **(a)** Express $x^2 - 6x + 8$ in the form of $a(x+b)^2 + c$.

$$x^2 - 6x + 8 = x^2 - 6x + (-3)^2 + 8 - (-3)^2 \quad \boxed{\text{M1}}$$
$$= (x-3)^2 + 8 - 9$$
$$= (x-3)^2 - 1 \quad \boxed{\text{A1}}$$

Answer.................................... [2]

(b) The curve $y = x^2 - 6x + 8$ is drawn.
Using your answer in **(a)**, write down
(i) the coordinates of the turning point,

$(3, -1) \quad \boxed{\text{A1}}$

Answer (.............. , ...). [1]

(ii) the equation of the line of symmetry of the curve.

$x = 3 \quad \boxed{\text{A1}}$

Answer.................................... [1]

13 **(a)** Derek invested a sum of money in an account paying compound interest at 3.4% per annum. After six years, he earned a total interest of $2665.76.
Calculate the sum of money Derek invested in the account to the nearest dollar.

$$P + 2665.76 = P\left(1 + \frac{3.4}{100}\right)^6 \quad \boxed{\text{M1}}$$

$$P\left(1 + \frac{3.4}{100}\right)^6 - P = 2665.76$$

$$P = \frac{2665.76}{\left(1 + \frac{3.4}{100}\right)^6 - 1}$$

$$= 12000.014$$

$$= 12000 \text{ (nearest dollar)} \quad \boxed{\text{A1}}$$

Answer $.................................... [2]

13   (b)   Cathy is going on a holiday to Australia. She changes 3500 Singapore dollars (SGD) into 4172 Australian dollars (AUD) at a bank.

(i)   Calculate the exchange rate between SGD and AUD to the nearest cents.

$$3500 \text{ SGD} = 4172 \text{ AUD}$$
$$1 \text{ SGD} = \frac{4172}{3500} \text{ AUD}$$
$$= 1.192 \text{ AUD}$$
$$= 1.19 \text{ AUD (nearest cents)} \boxed{\text{A1}}$$

Answer   1 SGD = ...AUD [1]

(ii)   At the end of Cathy's holiday in Australia, she has some AUD left. If she converts this amount back to SGD at a money changer.
The exchange rate between AUD and SGD is 1 AUD = 0.91 SGD.

Determine, when changing AUD back to SGD, if the exchange rate offered by the money changer is of better value for Cathy than the bank's exchange rate found in **(b)(i)**.

Answer
From **a(i)**, 1.192 AUD = 1 SGD
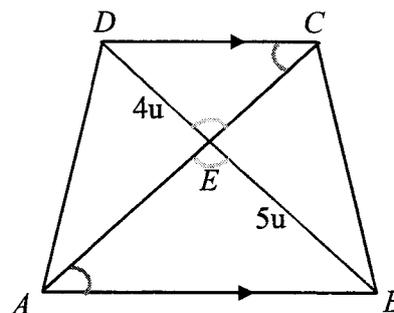
$$1 \text{ AUD} = \frac{1}{1.192} \text{ SGD}$$
$$= 0.83892 \text{ SGD} \boxed{\text{M1}}$$
$$= 0.84 \text{ SGD (nearest cents)}$$

Cathy will receive 0.07 SGD more for every 1 AUD at the exchange rate of 1 AUD = 0.91 SGD, hence it is a better value than the bank's exchange rate found in (a)(i). $\boxed{\text{A1}}$

[2]

14   The diagram shows a trapezium $ABCD$ such that AB is parallel to DC.
      AC and BD are diagonals of the trapezium $ABCD$.
      E is a point on BD such that $5DE = 4EB$.

(a)   Show that triangle ABE and triangle CDE are similar.

Answer

In $\triangle ABE$ and $\triangle CDE$,

$\angle EAB = \angle ECD$ (alt. $\angle$s)

$\angle AEB = \angle CED$ (vert. opp. $\angle$s)

$\therefore \triangle ABE$ is similar to $\triangle CDE$. (AA similarity test)

[2]

(b)   Calculate the value of

(i) $\dfrac{\text{area of } \triangle ABE}{\text{area of } \triangle CDE}$,

       $\dfrac{\text{area of } \triangle ABE}{\text{area of } \triangle CDE} = \left(\dfrac{5}{4}\right)^2 = \dfrac{25}{16}$ $\boxed{\text{A1}}$

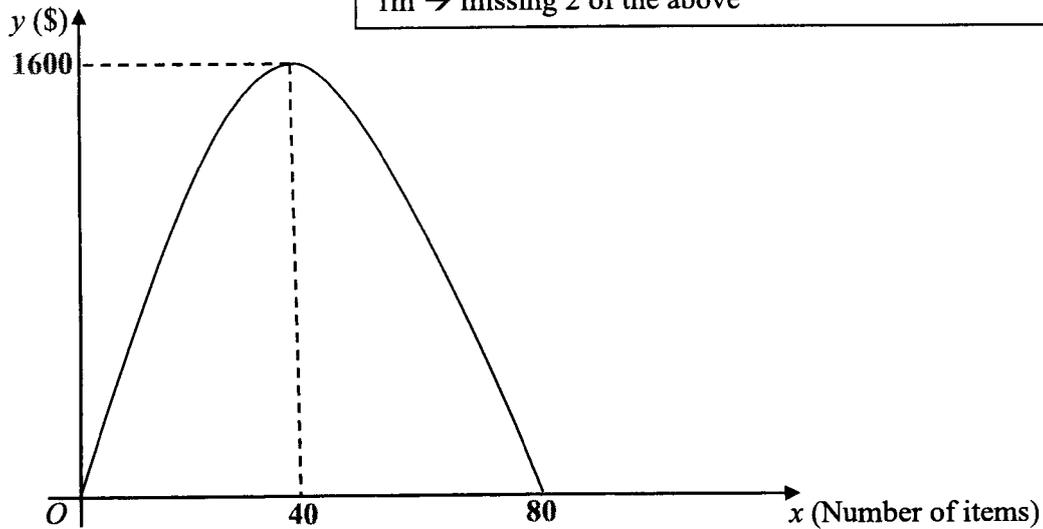   *Answer*................................... [1]

(ii) $\dfrac{\text{area of } \triangle ABE}{\text{area of } \triangle ABC}$.

       $\dfrac{\text{area of } \triangle ABE}{\text{area of } \triangle ABC} = \dfrac{5}{9}$ $\boxed{\text{A1}}$

       Answer................................... [1]

15   The revenue y (in dollars) for selling x items is given by $y = 80x - x^2$ for $0 \le x \le 80$.

(a)   Sketch the graph of $y = 80x - x^2$ for $0 \le x \le 80$, showing its turning point, x and y-intercepts.

> 3m➔ correct shape and **all** 2 intercepts & turning point
> 2m ➔ missing 1 of the above
> 1m ➔ missing 2 of the above

[2]

(b)   "Peter claims that it is possible for him to make a revenue of $2000."
Using your answer in (a), determine if Peter's statement is correct.

Answer

<u>Incorrect</u>
<u>From the graph, Peter can only make a **maximum revenue of $1600**.</u> | A1 |

[1]

16 The pressure, P N/m^2 of a gas contained in an enclosed container, held at a constant temperature is inversely proportional to the volume of the gas, V m^3.

The pressure of a certain mass of gas is 500 N/m^2 when the volume at a constant temperature is 2 m^3.

(a) Find an equation connecting P and V.
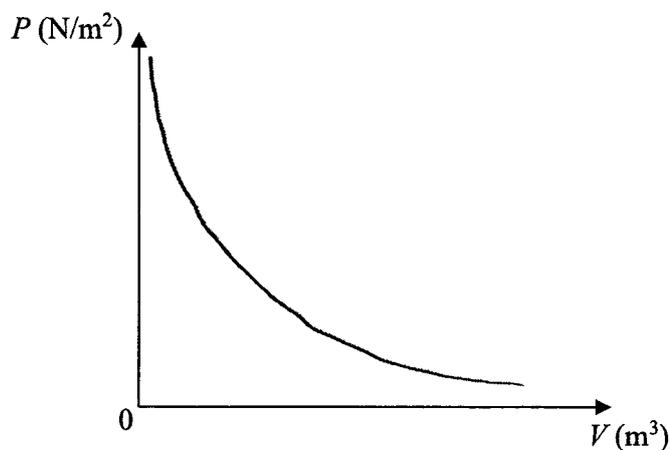
$$P = \frac{k}{V}$$
$$k = 500 \times 2$$
$$= 1000 \quad \boxed{\text{M1}}$$
$$\therefore P = \frac{1000}{V} \quad \boxed{\text{A1}}$$

Answer.................................... [2]

(b) Sketch the graph of P against V.

Answer

[1]

(c) The volume of the gas in the container is increased by 400%.
Calculate the percentage change in the pressure of the gas.

$$P_{new} = \frac{1000}{5V} \quad \boxed{\text{M1}}$$
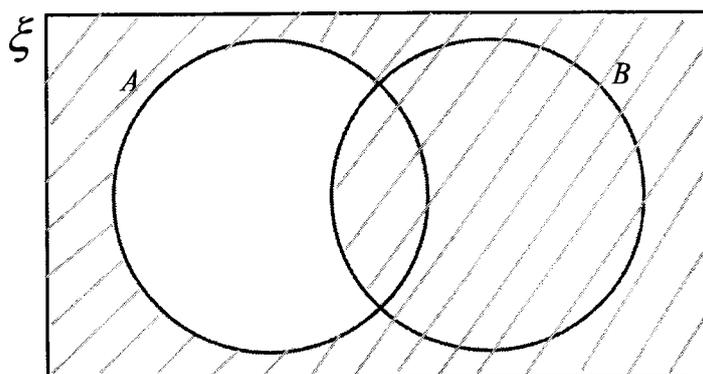$$= \frac{1}{5}\left(\frac{1000}{V}\right)$$
$$= \frac{1}{5}P$$

Percentage change $= \dfrac{1-5}{5} \times 100\%$

$= -80\% \quad \boxed{\text{A1}}$

Answer....................................% [2]

17   On the Venn diagram, shade the region which represents $A' \cup B$.

[1]

(b)

$\varepsilon = \{\text{integers } x : 1 \le x < 10\}$

$P = \{\text{multiples of 3}\}$

$Q = \{\text{perfect squares}\}$

(i)   List the elements in $P' \cap Q'$.

$$\varepsilon = \{1,2,3,4,5,6,7,8,9\}$$
$$P = \{3,6,9\}, \quad P' = \{1,2,4,5,7,8\}$$
$$Q = \{1,4,9\}, \quad Q' = \{2,3,5,6,7,8\}$$
$$P' \cap Q' = \{2,5,7,8\} \boxed{\text{A1}}$$

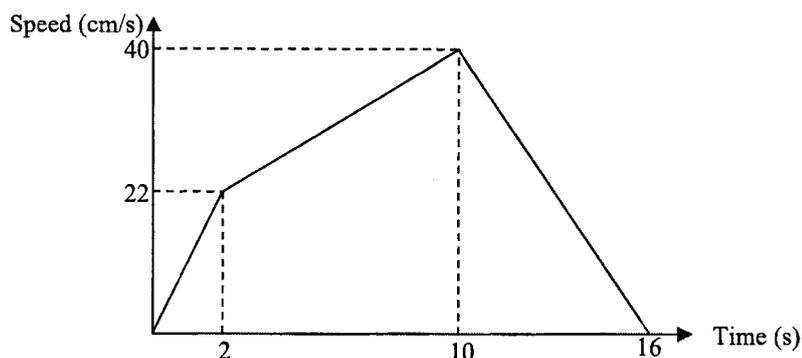*Answer*.. [1]

(ii)   A number, y is chosen at random from the set $P \cup Q$.
Find the probability that $y \notin P$.

$\dfrac{2}{5}$ $\boxed{\text{A1}}$

Answer.................................... [1]

18   The diagram shows the speed-time graph of a particle during a period of 16 seconds.

(a)   Calculate the speed of the car after 6 seconds.

$$\frac{v-22}{4} = \frac{40-22}{8}$$ M1

$$v-22 = \frac{9}{4}(4)$$

$$v = 9 + 22$$

$$= 31$$ A1

OR

$$\frac{40-v}{10-6} = \frac{40-22}{10-2}$$

$$40-v = 9$$

$$v = 40 - 9$$

$$= 31$$

Answer..............................cm/s[2]

(b)   Calculate the average speed of the particle during the 16 seconds.

$$\text{Total distance} = \frac{1}{2}(2)(22) + \frac{1}{2}(22+40)(8) + \frac{1}{2}(40)(6)$$

$$= 22 + 248 + 120$$

$$= 390 \text{ cm}$$ M1 for total distance

$$\text{Average speed} = \frac{390}{16}$$

$$= 24.375 \text{ cm/s}$$ A1

Answer..............................cm/s [2]

19   There are 85 cookies and 25 packets of Ribena in a snack box. After x cookies have

been consumed, the probability that a packet of Ribena is picked to be consumed is $\dfrac{5}{18}$.

Calculate the number of cookies, x, that have been consumed.

P(packet of drink is consumed) $= \dfrac{5}{18}$

$$\dfrac{25}{85+25-x} = \dfrac{5}{18} \quad \boxed{\text{M1}}$$

$$5(110) - 5x = 25(18)$$

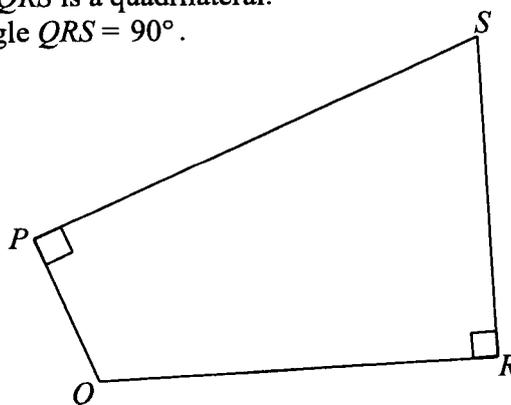$$5x = 550 - 450$$

$$x = \dfrac{100}{5}$$

$$x = 20 \quad \boxed{\text{A1}}$$

Answer.................................... [2]

20   In the diagram, $PQRS$ is a quadrilateral.
      Angle SPQ = Angle QRS = 90°.

Show that $\sin \angle RSP = \sin \angle PQR$.

Answer

sum of angles of a quadrilateral $= 360°$ | M1 for sum of angles of quadrilateral |

$$\angle RSP + \angle PQR = 360° - 90° - 90°$$

$$= 180° \quad \boxed{\text{M1 for supplementary angles}}$$

$\therefore \angle RSP$ and $\angle PQR$ are supplementary angles.

$$\angle PQR = 180° - \angle RSP$$

$$\sin \angle RSP = \sin(180° - \angle RSP) \quad \boxed{\text{M1}}$$

$$\sin \angle PQR = \sin \angle RSP \text{ (shown)}$$

[3]

21   In a fuel economy test conducted for 100 cars, the average distance travelled per litre of fuel (km/l) were recorded and given in the table below.

Distance travelled per litre (d km/l)	Frequency
$10 < d \leq 11$	14
$11 < d \leq 12$	20
$12 < d \leq 13$	38
$13 < d \leq 14$	16
$14 < d \leq 15$	12

(a)   Calculate an estimate for the mean distance travelled per litre of fuel.

12.42 km/l | A1 |

Answer....................................... [1]

(b)   Explain why the value of the mean in part **(a)** is an estimate of the mean distance travelled per litre of fuel.

Answer

The mean of a set of grouped data is an estimate because **grouped data does not give the exact individual values.** Since individual data points are not available, we cannot compute the actual mean, only an estimated mean using **the class midpoints.**

[1]

(c)   Find the standard deviation.

1.18050 = 1.18 km/l (3sf) | A1 |

Answer....................................... [1]

22   S is the point $(1, -3)$ and T is the point $(5, w)$.

$\overrightarrow{PQ} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ and $\overrightarrow{QR} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$.

(a)   Find $\left|\overrightarrow{QR}\right|$.

$\left|\overrightarrow{QR}\right| = \sqrt{8^2 + 6^2}$

$= \sqrt{100}$

$= 10$ units   ☐ A1

Answer.................................... [1]

(b)   Express $\overrightarrow{ST}$ as a column vector, in terms of w.

$\overrightarrow{ST} = \overrightarrow{SO} + \overrightarrow{OT}$

$= \begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ w \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 3+w \end{pmatrix}$ ☐ A1

Answer.................................... [1]

(c)   Find the value of w given that $\overrightarrow{ST}$ is parallel to $\overrightarrow{PQ}$.

$\begin{pmatrix} 4 \\ 3+w \end{pmatrix} = k \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} k \\ 4k \end{pmatrix}$

$k = 4$ ☐ M1

$3 + w = 4(4)$

$w = 16 - 3$

$= 13$ ☐ A1

Answer $w = $ .... [2]

23   The first four terms in a sequence of numbers are given below.

$T_1 = 1 \times 3$	$= 3 \times 1$	$= 3 \times 1$	$= 3$
$T_2 = 1 \times 3 + 3 \times 3$	$= 3 \times (1 + 3)$	$= 3 \times 4$	$= 12$
$T_3 = 1 \times 3 + 3 \times 3 + 5 \times 3$	$= 3 \times (1 + 3 + 5)$	$= 3 \times 9$	$= 27$
$T_4 = 1 \times 3 + 3 \times 3 + 5 \times 3 + 7 \times 3$	$= 3 \times (1 + 3 + 5 + 7)$	$= 3 \times 16$	$= 48$

(a)   Find an expression, in terms of n, for T_n.

$T_n = 3n^2$ $\boxed{\text{A1}}$

Answer $T_n = $ .... [1]

(b) T_p and T_{p+1} are consecutive terms in the sequence.
Find and simplify an expression, in terms, of p, for $T_{p+1} - T_p$.

$$T_{p+1} - T_p = 3(p+1)^2 - 3p^2$$
$$= 3(p^2 + 2p + 1) - 3p^2$$
$$= 3(2p + 1)$$
$$= 6p + 3 \quad \boxed{\text{A1}}$$

Answer.................................. [1]

(c)   Explain why two consecutive terms of the sequence cannot have a difference of 4.

Answer

When $6p + 3 = 4$ [1]

$p = \dfrac{1}{6}$
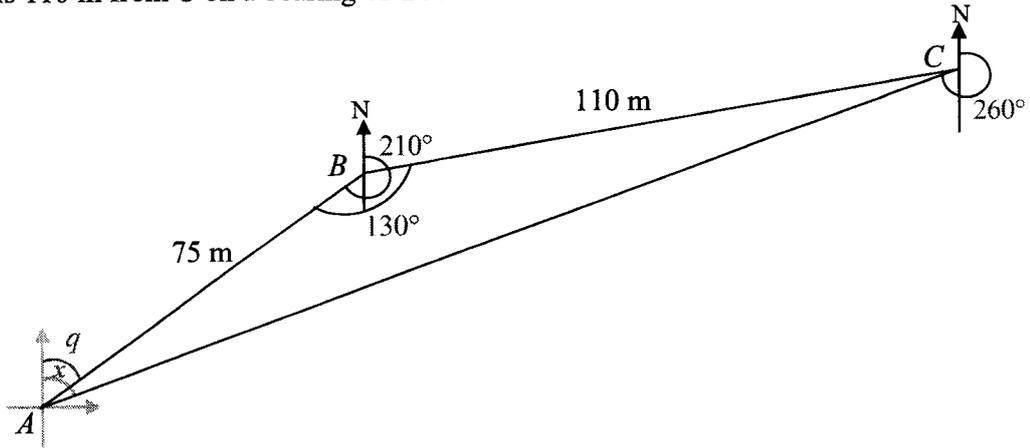
p is not an integer $\boxed{\text{A1}}$

(d)   The first three terms of a second sequence are 12, 27 and 48.
By using **(a)** or otherwise, write down an expression, in terms of n, for the nth term, S_n of this second sequence.

$3(n+1)^2$

$\boxed{\text{A1}}$

Answer.................................. [1]

24   A, B and C are three points on a level ground.
      Angle $ABC = 130°$.
      A is 75 m from B on a bearing of $210°$.
      B is 110 m from C on a bearing of $260°$.

(a)   Calculate the distance, AC,

$$AC = \sqrt{75^2 + 110^2 - 2(75)(110)\cos 130°}, AC > 0 \quad \boxed{M1}$$

$$= 168.3181$$

$$= 168 \text{ m (3sf)} \quad \boxed{A1}$$

Answer.................................. [2]

(b)   Calculate the bearing of C from A..

$$\frac{\sin \angle BAC}{110} = \frac{\sin 130°}{168.3181} \quad \boxed{M1}$$

$$\sin \angle BAC = \frac{110 \sin 130°}{168.3181}$$

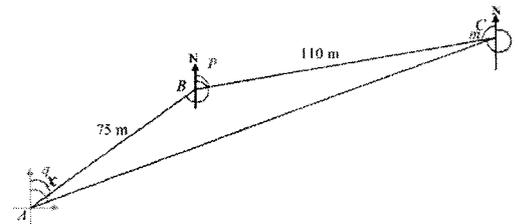$$\angle BAC = \sin^{-1}\left(\frac{110 \sin 130°}{168.3181}\right)$$

$$= 30.0416°$$

$$q = 180 - (360 - 210) \quad \boxed{M1}$$

$$= 30$$

$$x = 30 + 30.0416$$

$$= 60.0416$$

Bearing of C from A=060.0° (1dp) $\boxed{A1}$

Answer.................................. [3]

(c) A kite was seen flying 55 m vertically above point B.
Find the greatest angle of elevation of the kite observed by a person walking in a straight path from A to C.

$$\sin 30.0416° = \frac{BW}{75}$$ M1 for BW

$$BW = 75\sin 30.0416°$$

OR

$$\frac{1}{2}(75)(110)\sin 130° = \frac{1}{2}(168.3181)BW$$

$$BW = \frac{\frac{1}{2}(75)(110)\sin 130° \times 2}{168.3181}$$

$$BW = 37.5471 \text{ m}$$
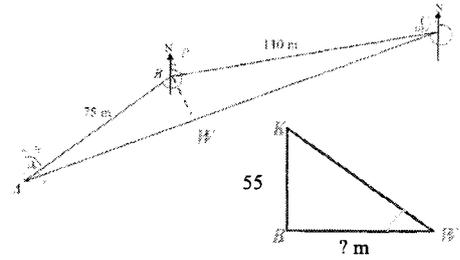
M1 for BW =37.5471m

$$\tan \angle KWB = \frac{55}{37.5471}$$ M1

$$\angle KWB = \tan^{-1}\left(\frac{55}{37.5471}\right)$$

$$= 55.6796°$$

$$= 55.7° \text{ (1dp)}$$ A1

greatest angle of elevation of the kite =55.7°

Answer.................................... [4]

25   An enrichment centre offers Basic and Advanced programming lessons on weekdays and weekends.
Each student attends 4 weekly lessons per month.
The table shows the number of students who attend weekly Basic and Advanced programming lessons in a particular month

	Basic	Advanced
Weekday	18	9
Weekend	15	13

This information can be represented by the matrix T.

$$T = \begin{pmatrix} 18 & 9 \\ 15 & 13 \end{pmatrix}$$

(a)   Evaluate the matrix $W = 4T$.

$$W = \begin{pmatrix} 72 & 36 \\ 60 & 52 \end{pmatrix}$$

Answer W =................................. [1]

(b)   The enrichment centre charges $60 per lesson for Basic Programming and $110 per lesson for Advanced Programming.
The lesson charges can be represented by a 2×1 matrix N where $N = \begin{pmatrix} 60 \\ 110 \end{pmatrix}$.

(i)   Find WN.

WN

$$= \begin{pmatrix} 72 & 36 \\ 60 & 52 \end{pmatrix} \begin{pmatrix} 60 \\ 110 \end{pmatrix}$$

$$= \begin{pmatrix} 8280 \\ 9320 \end{pmatrix}$$

Answer WN =................................. [1]

(ii) Explain what the elements in *WN* represent.

Answer
<u>The elements represent the **total amount of money** the enrichment centre collects **monthly for**
<u>lessons on **weekdays and weekends** respectively.</u> [1]

(c) The charges for both Basic Programming and Advanced Programming lessons
are increased by 7%, and the attendance remains the same.
Calculate the total amount of money collected monthly after the increase in
charges.

$$\text{total amount collected} = \frac{107}{100} \times (8280 + 9320) \quad \boxed{\text{M1}}$$

$$= \frac{107}{100} \times 17600$$

$$= \$18832 \quad \boxed{\text{A1}}$$

Answer.................................... [2]

26   In the diagram, D and E are points on BC such that $BD = DC = AC$.
      Triangle ADE is congruent to triangle FCE and angle $ADE = \theta$.
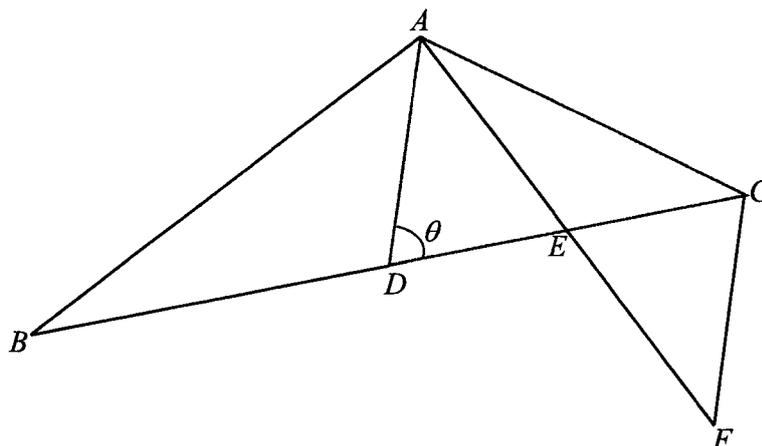
Show that triangle ABD is congruent to triangle FAC.
Give a reason for each statement you make.

Answer

$BD = AC$ (Given)

Since $\triangle ADE \equiv \triangle FCE$, $AD = FC$ and $\angle ADE = \angle FCE = \theta°$ | M1 for $AD = FC$ |

Since $DC = AC$, $\triangle ADC$ is an isosceles triangle.

$\angle CAD = \theta°$ (base $\angle$s of isos.$\triangle$) | M1 for angle CAD |

Since $\angle ADE = \angle FCE = \theta°$, $AD // CF$.

$\angle ACF = (180 - \theta)°$ (int.$\angle$)

$\angle BDA = (180 - \theta)°$ (supp. $\angle$)

$\angle ACF = \angle BDA$

$\therefore \triangle ABD \equiv \triangle FAC$ (SAS)

| For finding angle ACF on the condition that AD//CF is stated.

M1 for finding angle BDA |

[3]

END OF PAPER

Name : _____

MARK SCHEME

Class	Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2025
Secondary 4

Thursday	**MATHEMATICS**	**4052/02**
22 August 2025	**Paper 2**	2 hours 15 minutes

Candidates answer on the Question Paper.

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

/90

This question paper consists of 26 printed pages and 2 blank pages.

2

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r l$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of a triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1   (a)   There are 2 500 sheets of paper of the same thickness in a stack of paper.

    (i)   Given that the height of the stack of paper is 25 cm, write down the thickness of a single sheet of paper in metres in standard form.

$\dfrac{0.25}{2500} = 0.0001$ $= 1 \times 10^{-4}$	B1

Answer .... m [1]

    (ii)   The stack of paper weighs 13.1 kg, correct to one decimal place.
           Find the minimum mass of the stack of paper.

13.05	B1

Answer .... kg [1]

    (iii)   The number 2500 is correct to k significant figures.
            State a possible value of k.

2, 3 or 4	B1

Answer $k =$ .... [1]

4

(b) Jim is climbing a mountain that is 4 500 metres high. The temperature at the base of the mountain is 7 °C, and as he climbs, the temperature drops at a constant rate.

The temperature at the top of the mountain is 15 °C lower than at the base of the mountain.

(i) Find the temperature at the top of the mountain.

| $7°C - 15°C = -8°C$ | B1 |

Answer .... °C [1]

(ii) At what height during his climb does the temperature reach −2 °C?

| $\dfrac{7-(-2)}{15} \times 4500 = 2700$ | B1 |

Answer .... m [1]

(c) In 2023, Sandra earned 20% less than what she earned in 2022.
In 2024, she earned 15% more than what she earned in 2023.
If she earned $49 680 in 2024, how much did she earn in 2022?

| $49680 \div 1.15 \div 0.8$ | M1 |
| $= 54000$ | A1 |

Answer $ .... [2]

2 (a) (i) Solve the inequality $x-4 < 5x+1 \le \dfrac{7}{2}-2x$.

$$x-4 < 5x+1 \le \dfrac{7}{2}-2x$$

$$x-4 < 5x+1 \qquad\qquad 5x+1 \le \dfrac{7}{2}-2x$$

$$4x > -5 \qquad\qquad 7x \le \dfrac{5}{2}$$

$$x > -\dfrac{5}{4} \qquad\qquad x \le \dfrac{5}{14} \qquad \text{M1}$$

$$-\dfrac{5}{4} < x \le \dfrac{5}{14} \qquad\qquad\qquad \text{A1}$$

Answer .... [2]

(ii) List all the integers that satisfy $x-4 < 5x+1 \le \dfrac{7}{2}-2x$.

| −1, 0 | B1 |

Answer .... [1]

6

(b) $a = \dfrac{(b+1)^3}{4} - c^2$

(i)   Find a when $b = 2$ and $c = -1$.

$a = \dfrac{(2+1)^3}{4} - (-1)^2$	
$= \dfrac{27}{4} - 1$	
$= \dfrac{23}{4}$	B1

Answer $a =$ .... [1]

(ii)   Arrange the formula to make b the subject.

$a = \dfrac{(b+1)^3}{4} - c^2$	
$a + c^2 = \dfrac{(b+1)^3}{4}$	M1
$4(a+c^2) = (b+1)^3$	M1
$b = \sqrt[3]{4(a+c^2)} - 1$	A1

Answer $b =$ .... [3]

7

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3   (a)
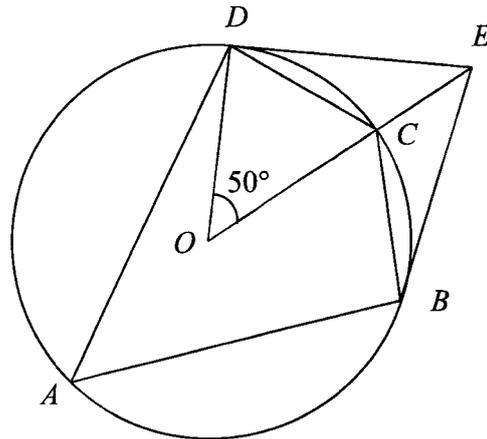
A, B, C and *D* are points on a circle, centre *O*.
EB and *ED* are tangents to the circle.
Angle *DOC* = 50°.

Find, giving reasons for each answer,
(i)   angle *OEB*,

$\angle ODE = 90°$ (tan $\perp$ rad)	M1
$\angle DEO = 180° - 90° - 50°$ ($\angle$ sum in a $\triangle$)	M1
$= 40°$	
$\angle OEB = 40°$ (tangents from ext point)	A1

Answer ....° [3]

(ii)   angle *DAB*,

$\angle COB = 50°$ (tangents from ext. pt)	M1
$\angle DOB = 100°$	
$\angle DAB = 50°$ ($\angle$ at centre $= 2 \times \angle$ at circumference)	A1

Answer ....° **[2]**

(iii)   angle *EBC*.

$\angle OBC = \dfrac{180° - 50°}{2}$ (base $\angle s$ of isos Δ) $= 65°$ $\angle EBC = 90° - 65°$ $= 25°$	B1

Answer ....° **[1]**

(b)
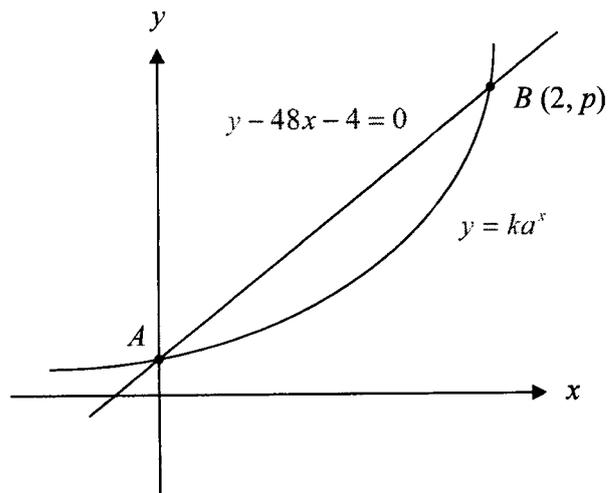
The diagram shows the graphs of $y - 48x - 4 = 0$ and $y = ka^x$.
They intersect at the points A and B. A is a point on the y – axis.
Find the values of k and of a.

$y - 48x - 4 = 0$	
$\qquad x = 0, \ y = 4$	
$\qquad\quad A(0, 4)$	M1
$\qquad x = 2, \ p = 100$	
$\qquad\quad B(2, 100)$	M1
$y = ka^x$	
$\qquad x = 0, \ y = 4 \quad 4 = ka^0$	
$\qquad k = 4$	A1
$\qquad x = 2, \ y = 100 \quad 100 = 4a^2$	
$\qquad a = 5$	A1

Answer k = ...

a = ... [4]

(c) The diagram shows a rhombus *ABCD* such that *AB* = *BD* = 18 cm.
ABFD and *CBED* are sectors of circles with centre *A* and *C* respectively.
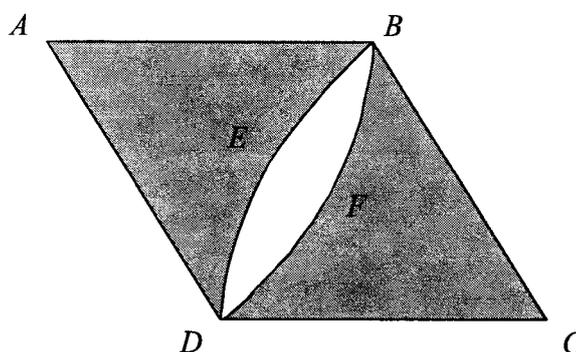
(i) Explain why angle $BCD = \dfrac{\pi}{3}$ radians.

AB = *BC* = *DC* (sides of rhombus)	
AB = *BD* (given)	
BC = *BD*	
triangle *BCD* is an equilateral triangle	B1
$\angle BCD = \pi \div 3$	
$= \dfrac{\pi}{3}$ radians	

[1]

(ii) Calculate the area of the shaded region.

Area of rhombus	
$= 2\left[\dfrac{1}{2}(18)^2 \sin\dfrac{\pi}{3}\right]$	M1
$= 290.59$ (5 s.f.)	
Shaded area	
$= 2 \times \left[290.59 - \dfrac{1}{2}(18)^2 \dfrac{\pi}{3}\right]$	M1
$= 222$ cm^2 (3 s.f.)	A1

Answer .... cm^2 [3]

12

4 **(a)** Complete the table of values for $y = \dfrac{x^2}{3} + \dfrac{3}{x} - 4$.

Values are given to two decimal places where appropriate.

x	0.5	1	1.5	2	3	4	5	6
y	2.08	−0.67	−1.25		0	2.08	4.93	8.5

[1]

−1.17	B1

(b) On the grid opposite, draw the graph of $y = \dfrac{x^2}{3} + \dfrac{3}{x} - 4$ for $0.5 \le x \le 6$.   [3]

(c) By drawing a tangent, find the gradient of the curve at $(3,0)$.

Correct tangent drawn	M1
1.65, accept 1.32 to 1.98	A1

Answer .... [2]

(d) (i) On the same axes, draw the line with gradient $\dfrac{4}{3}$ that passes through $(3,2)$.

[2]

gradient of $\dfrac{4}{3}$,	B1
passes through (3,2)	B1

(ii) Write down the x-coordinate(s) of the point(s) where the line intersects the curve.

1 ± 0.1, 4.85 ± 0.1	B1

Answer .... [1]

(iii) The x-coordinate(s) of the point(s) of intersection satisfy the equation $x^3 + Ax^2 + Bx + C = 0$.
Find the values of A, B and C.

$y = \dfrac{x^2}{3} + \dfrac{3}{x} - 4$	
$y = \dfrac{4}{3}x - 2$	
$\dfrac{x^2}{3} + \dfrac{3}{x} - 4 = \dfrac{4}{3}x - 2$	M1
$x^3 + 9 - 12x = 4x^2 - 6x$	
$x^3 - 4x^2 - 6x + 9 = 0$	M1
$A = -4, B = -6, C = 9$	A1

Answer $A =$ ....

$B =$ ....

$C =$ .... [3]

14
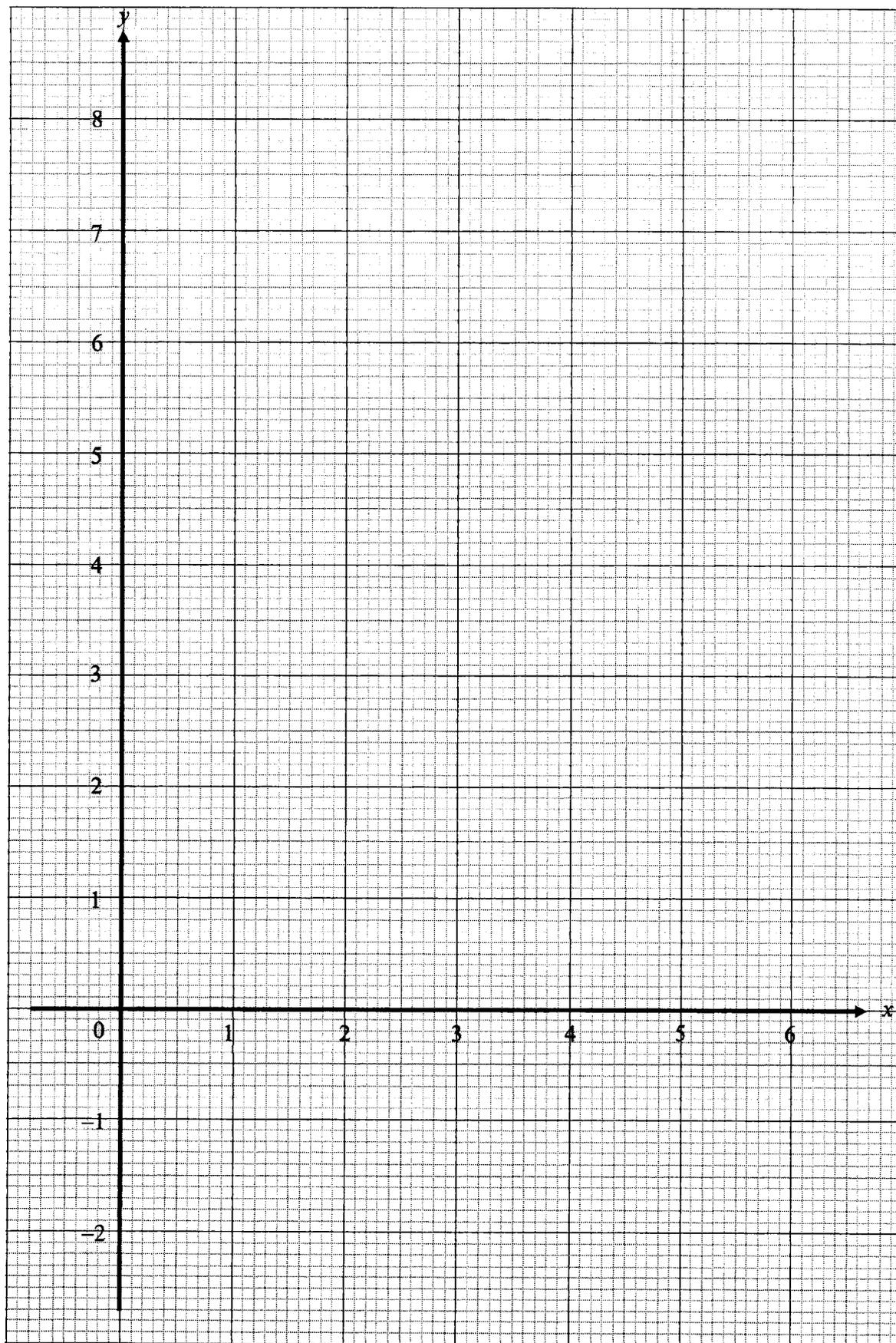
5 **(a)**

In the diagram, ABC is a triangle such that $\overrightarrow{AB} = 5b$ and $\overrightarrow{AC} = 2a$.

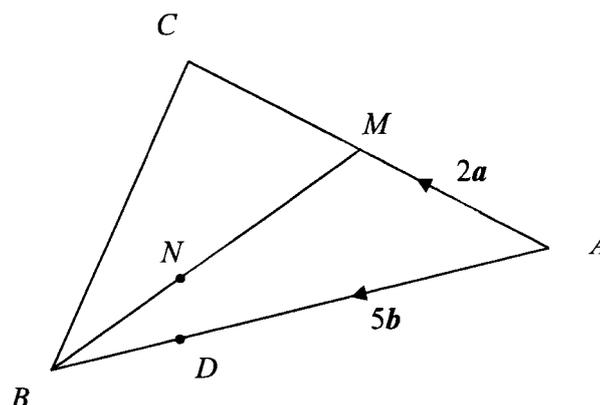It is given that $\dfrac{BD}{DA} = \dfrac{1}{4}$, $\overrightarrow{BN} = \dfrac{1}{3}\overrightarrow{BM}$ and M is the midpoint of AC.

(a) Write each of the following as simply as possible in terms of a and b,

(i) $\overrightarrow{AD}$,

| $\overrightarrow{AD} = 4b$ | B1 |

Answer .... [1]

(ii) $\overrightarrow{BM}$,

| $\overrightarrow{BM} = a - 5b$ | B1 |

Answer .... [1]

(iii) $\overrightarrow{CD}$.

| $\overrightarrow{CD} = -2a + 4b$ | B1 |

Answer .... [1]

(b)   Show that $\overrightarrow{CN} = \frac{5}{3}(2b - a)$. [2]

$\overrightarrow{CN} = \overrightarrow{CM} + \overrightarrow{MN}$	
$= -a + \frac{2}{3}(5b - a)$	M1
$= -a + \frac{10}{3}b - \frac{2}{3}a$	
$= -\frac{5}{3}a + \frac{10}{3}b$	
$= \frac{5}{3}(2b - a)$	A1

(c)   Explain why C, N and D lie in a straight line. [2]

$\overrightarrow{CD} = 2(2b - a)$	
$\overrightarrow{CN} = \frac{5}{3}(2b - a)$	
$\overrightarrow{CN} = \frac{5}{3}\left[\frac{1}{2}(2b - a)\right]$	
$= \frac{5}{6}(2b - a)$	
$= \frac{5}{6}\overrightarrow{CD}$	M1
Since $\overrightarrow{CN} = \frac{5}{6}\overrightarrow{CD}$, and C is a common point,	
$\therefore C$, N and D lie in a straight line.	A1

6 (a) The cumulative frequency curve shows the distribution of the speeds of 300 cars passing a road on a particular morning.

cumulative frequency

Speed (km/h)
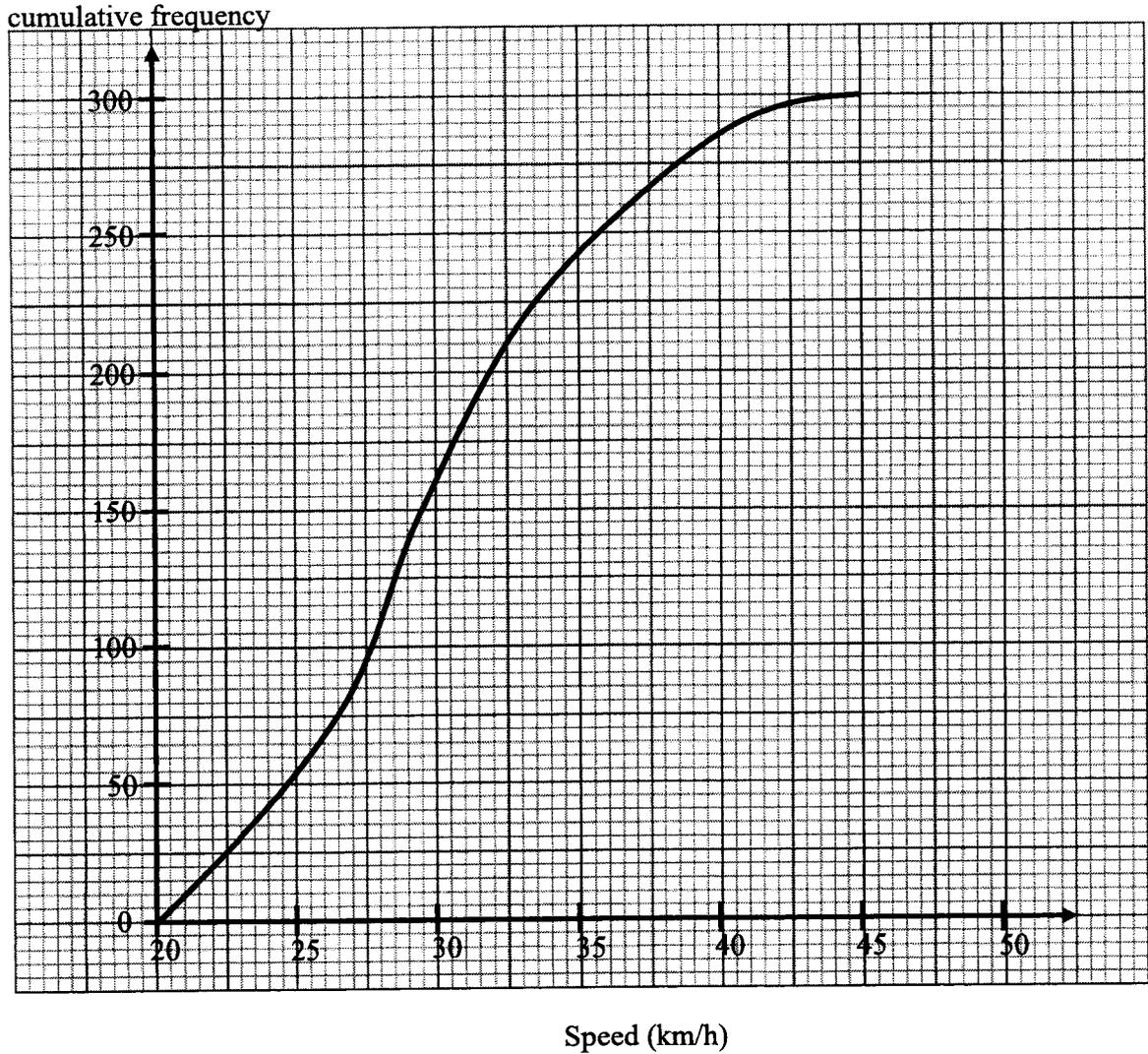
(i) Use the curve to estimate

(a) the median speed,

29.5		B1

Answer .... km/h [1]

(b) the interquartile range of the speeds,

33.5 − 26.5	M1
= 7	A1

Answer .... km/h [2]

(c) 70th percentile.

32.5	B1

Answer .... km/h [1]

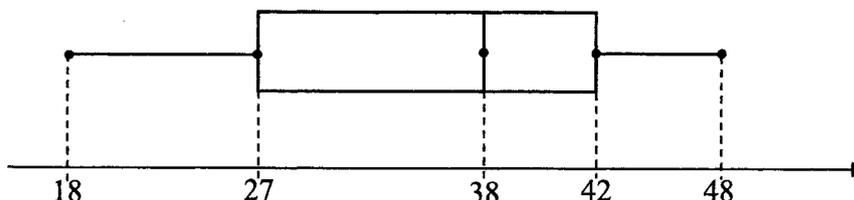**(ii)** Given that 10% of the cars have a speed of more than p km/h, find the value of p.

38	B1

Answer p = ... [1]

(iii) The speeds of 300 cars passing the same road in the afternoon were also recorded.
The box-and-whiskers plot shows the distribution of the speeds, in km/h.

Make two comments comparing the speeds of the cars in the morning and in the afternoon.

Median speed in the morning = 29.5 km/h Median speed in the afternoon = 38 km/h The cars in the morning travel at a lower speed as the median speed in the lower in the morning	B1
Interquartile range in the morning = 7 km/h Interquartile range in the afternoon = 15 km/h The speeds of the cars in the morning are more consistent as the interquartile range in the morning is smaller.	B1

Answer ....

.... [2]

(b) Ten identical cards are labelled 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 respectively.
Two cards are drawn at random, one after the other, without replacement.
Find the probability that

(i) both cards picked have a number that is odd,

$\dfrac{5}{10} \times \dfrac{4}{9} = \dfrac{2}{9}$	B1

Answer .... [1]

(ii) one card picked has a number that is odd and the other card has a number that is even,

$\dfrac{5}{10} \times \dfrac{5}{9} \times 2$	M1
$= \dfrac{5}{9}$	A1

Answer .... [2]

(iii) the product of the numbers on both cards is even.

$1 - \dfrac{2}{9}$	
$= \dfrac{7}{9}$	B1

Answer .... [1]

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7   The diagram shows a cylindrical solid with a hemispherical top with the same radius r cm.
The height of the cylinder is $2r$ cm and the volume of the solid is 576π cm^3.
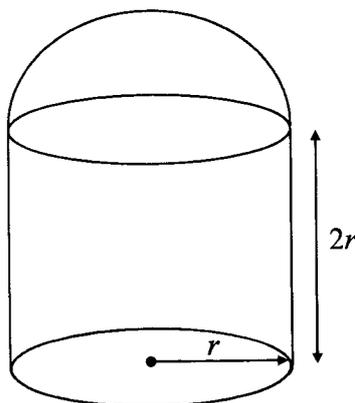
(a)   Show that $r = 6$.

$\dfrac{2}{3}\pi r^3 + \pi r^2 (2r) = 576\pi$	M1
$\dfrac{8}{3} r^3 = 576$	
$r^3 = 216$	M1
$r = 6$	A1

[3]

(b) If the solid is melted to form a solid cone such that its radius and height are the same, find the total surface area of the cone.

$\frac{1}{3}\pi r_c^{\,2}(r_c) = 576\pi$	M1
$\frac{1}{3}r^3 = 576$	
$r_c^3 = 1728$	
$r_c = 12$	M1
$\pi r_c l + \pi r_c^{\,2} = \pi(12)\sqrt{12^2 + 12^2} + \pi(12)^2$	M1
$= 1090 \text{ cm}^2$	A1

Answer .... cm² [4]

8 A water heater is programmed to heat up water in a tank to a temperature of 38°C.

(a) In the morning, the temperature of the water before heating is 25°C.
The water heater heats up water at an average rate of x°C per minute.

Write down an expression, in terms of x, for the time taken to heat up the water.

$\dfrac{38-25}{x} = \dfrac{13}{x}$	B1

Answer .... min [1]

(b) In the afternoon, the temperature of the water before heating is 28°C.
The water heater heats up water at an average rate of $(x+2)$°C per minute.

Write down an expression, in terms of x, for the time taken to heat up the water.

$\dfrac{38-28}{x+2} = \dfrac{10}{x+2}$	B1

Answer .... min [1]

(c) The difference between the time taken to heat up the water in the morning and in the afternoon is 2 minutes.

Write down an equation in x to represent this information, and show that it reduces to
$$2x^2 + x - 26 = 0.$$

$\dfrac{13}{x} - \dfrac{10}{x+2} = 2$	M1
$13(x+2) - 10x = 2x^2 + 4x$	M1
$13x + 26 - 10x = 2x^2 + 4x$	
$2x^2 + x - 26 = 0$	A1

[3]

(d) Solve the equation $2x^2 + x - 26 = 0$, giving your answers to two decimal places.

$2x^2 + x - 26 = 0$	
$x = \dfrac{-1 \pm \sqrt{1^2 - 4(2)(-26)}}{2(2)}$	M2
$= 3.36$ or -3.86	A1

Answer .... [3]

(e) Find the time taken to heat up the water in the morning.
Give your answer in minutes and seconds, correct to the nearest second.

$\dfrac{13}{3.36} = 3.869 \text{ min} \times 60 \text{ s}$	
$= 3 \text{ min } 0.869 \times 60 \text{ s}$	B1
$= 3 \text{ min } 52 \text{ s (nearest second)}$	

Answer .... min .... s [1]

9   A is the point $(-3,-1)$ and B is the point $(5,-7)$.

Find

(a)   the equation of the line AB,

$m = \dfrac{-1-(-7)}{-3-5}$ $= -\dfrac{3}{4}$	M1
$y-(-1) = -\dfrac{3}{4}[x-(-3)]$ $y = -\dfrac{3}{4}x - \dfrac{9}{4} - 1$	
$y = -\dfrac{3}{4}x - \dfrac{13}{4}$	A1

Answer .... [2]

(b)   the value of k if the point $(k, 3k)$ lies on the line AB,

$3k = -\dfrac{3}{4}k - \dfrac{13}{4}$ $\dfrac{15}{4}k = -\dfrac{13}{4}$	M1
$k = -\dfrac{13}{15}$	A1

Answer $k = $ .... [2]

(c)   the value of a if the line $4y = 3ax + 4$ is parallel to the line AB.

$4y = 3ax + 4$	
$y = \dfrac{3a}{4} + 1$	M1
$\dfrac{3a}{4} = -\dfrac{3}{4}$	
$a = -1$	A1

Answer $a =$ ....…........ [2]

10 Caroline is interested to buy a new car.
   She has shortlisted 2 cars, which she intends to drive for 10 years.
   The following tables show the details of the specifications, car loans, applicable road taxes and expenditure of owning the cars.

Car	Model A	Model B
Engine Capacity (EC)/ cc	1598	1798
Power Rating (PR)/ kW	96	72
Fuel Type	Petrol	Petrol-Electric
Fuel consumption (km/L)	15.6	27
Cost	$180 888	$189 300
Annual maintenance fee	$463	$389
Car loan: Simple interest rate per annum	1.88%	2.18%

Road Tax (For Petrol Cars)

Engine Capacity (EC) in cc	Annual Road Tax Formula
$EC \leq 600$	S\$400 x 0.782
$600 < EC \leq 1,000$	[S\$400 + S\$0.25(EC - 600)] x 0.782
$1,000 < EC \leq 1,600$	[S\$500 + S\$0.75(EC - 1,000)] x 0.782
$1,600 < EC \leq 3,000$	[S\$950 + S\$1.5(EC - 1,600)] x 0.782

Road Tax (For Electric Cars)

Power Rating (PR) in kW	Annual Road Tax Formula
$PR \leq 7.5$	S\$400 x 0.782
$7.5 < PR \leq 30$	[S\$400 + S\$4(PR – 7.5)] x 0.782
$30 < PR \leq 230$	[S\$500 + S\$7.5(PR - 30)] x 0.782

Road Tax (For Petrol-Electric Cars)

To find out your road tax, calculate road tax twice: first based on your car's engine capacity, then based on its Power Rating (PR). The higher amount will be the road tax you have to pay.

a) Calculate the annual road tax payable for Model A.

$[500 + 0.75 \times (1598 - 1000)] \times 0.782$	
$= 741.727$	
$= 741.73$ (2d.p.)	B1

Answer $ .... [1]

b) Show that annual road tax payable for Model B is $975.15 correct to two decimal places.

$[950+1.5\times(1798-1600)]\times0.782$ $=975.154$ $[500+7.5\times(72-30)]\times0.782$ $=637.33$ $975.154 > 637.33$	M1
Hence, road tax for Model B is $975.15 (2d.p.).	A1

[2]

c) Caroline estimates that she will travel approximately 20 000 km per year and petrol prices are at $2.57 per litre.

Caroline owns a credit card which gives her a discount of 21% off petrol prices.
She will pay $60 000 cash for the downpayment for the car and take a car loan for 7 years.

Assuming both car models offer comparable performance over 10 years, which car model offers better value? Justify your answer with clear working.

Model A	M1
$180\ 888 - 60000 = 120888$	
$\text{interest} = \dfrac{120888\times1.88\times7}{100}$	
$=15908.8608$	
$180888 + 15908.8608 = 196796.8608$	
petrol	
$\dfrac{20000}{15.6}\times2.57\times79\% = 2602.948718$	M1
road tax and maintenance	
$741.727 + 463 = 1204.727$	
Total cost $= 196796.8608 + 2602.948718\times10 + 1204.727\times10$	M1
$= 234873.618$	
$= 234873.62$	

For Model B $189300 - 60000 = 129300$ $\text{interest} = \dfrac{129300 \times 2.18 \times 7}{100}$ $= 19731.18$	M1
$189300 + 19731.18 = 209031.18$ petrol $\dfrac{20000}{27} \times 2.57 \times 79\% = 1503.925926$	M1
$975.154 + 389 = 1364.154$ Total cost $= 209031.18 + 1503.925926 \times 10 + 1364.154 \times 10$ $= 237711.9793$	M1
$234873.62 < 237711.9793$ Model A offers better value	A1

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[7]

End of Paper