BP~557

Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/01**
Paper 1

25 August 2025 Monday 2 hours 15 min

2025 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)
PRELIMINARY EXAMINATIONS

SUGGESTED ANSWERS

	For Examiner's Use													
Qn	1	2	3	4	5	6	7	8	9	10	11	12	13	*Marks Deducted*
Marks														

Category	Accuracy	Units	Notations	Others
Question No.				

TOTAL MARKS
90

This question paper consists of **23** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1	(a)	Differentiate $\ln\left(\frac{3x}{x^2+1}\right)$ with respect to x.	[3]
	(b)	Hence find $\int \frac{2x}{x^2+1}\,dx$.	[2]

1	(a)	$Let\ y=\ln\left(\frac{3x}{x^2+1}\right)=\ln(3x)-\ln(x^2+1)$ $\frac{dy}{dx}=\frac{1}{x}-\frac{2x}{x^2+1}$ OR $\frac{dy}{dx}=\frac{\frac{3(x^2+1)-3x(2x)}{(x^2+1)^2}}{\frac{3x}{x^2+1}}$ $=\frac{3x^2+3-6x^2}{(x^2+1)^2}\times\frac{x^2+1}{3x}$ $=\frac{1-x^2}{x(x^2+1)}$	
	(b)	$\int\left(\frac{1}{x}-\frac{2x}{x^2+1}\right)dx=\ln\left(\frac{3x}{x^2+1}\right)+c$ $\int\frac{2x}{x^2+1}\,dx=\int\frac{1}{x}\,dx-\ln\left(\frac{3x}{x^2+1}\right)+c$ $=\ln x-\ln\left(\frac{3x}{x^2+1}\right)+c$ OR $\frac{dy}{dx}=\frac{1-x^2}{x(x^2+1)}=\frac{A}{x}+\frac{Bx+C}{x^2+1}$	

		Solve for A, B & C. $A=1,\ B=-2,\ C=0$ $\frac{1-x^2}{x\left(x^2+1\right)}=\frac{1}{x}-\frac{2x}{x^2+1}$ $\int\left(\frac{1}{x}-\frac{2x}{x^2+1}\right)\mathrm{dx}=\ln\left(\frac{3x}{x^2+1}\right)+c$ $\int\frac{2x}{x^2+1}\,dx=\int\frac{1}{x}\,\mathrm{dx}-\ln\left(\frac{3x}{x^2+1}\right)+c$ $=\ln x-\ln\left(\frac{3x}{x^2+1}\right)+c$	

2	It is given that $\cos A = \frac{1}{2}$ and $\sin B = -\frac{1}{\sqrt{2}}$ where $0° < A < 90°$ and $180° < B < 270°$. Find, without using a calculator, the exact value of $\cos(A - B)$, leaving your answer in the form $p\sqrt{2} + q\sqrt{6}$, where p and q are real numbers.	[4]

2	$\cos A = \frac{1}{2},\ \sin A = \frac{\sqrt{3}}{2}$ $\sin B = -\frac{1}{\sqrt{2}},\ \cos B = -\frac{1}{\sqrt{2}}$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$ $= \left(\frac{1}{2}\right)\left(-\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{\sqrt{2}}\right)$ $= -\frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$ $= \frac{-1-\sqrt{3}}{2\sqrt{2}}$ $= -\frac{1}{4}\sqrt{2} - \frac{1}{4}\sqrt{6}$	

3	Baking powder is poured onto a flat surface at a constant rate of 2π cm^{3}s^{-1}, forming a right circular cone. The radius of the cone is always $\frac{1}{18}$ of its height. Find the rate of change of the radius of the cone after 3 seconds of pouring. $\left[\text{Volume of cone} = \frac{1}{3}\pi r^2 h\right]$	[5]

Volume of cone, $V = \frac{1}{3}\pi r^2 (18r) = 6\pi r^3$

$\frac{dV}{dr} = 18\pi r^2$

After 3s, $V = 2\pi \times 3 = 6\pi$

$$6\pi r^3 = 6\pi$$
$$r = 1$$

When $t = 3s$, $r = 1cm$, $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$$2\pi = 18\pi(1)^2 \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{9} \text{ cm/s}$$

Rate of change of radius of cone after 3 s is $\frac{1}{9}$ cm/s

4	A and B are the points of intersection of the line $4y = 2x + 1$ and the curve $3y - x = 4xy$.		
	(a)	Find the coordinates of A and of B.	[4]
	(b)	Henry says that the line $2y - 4x = 5$ is perpendicular to the line AB. Is he correct? Justify your answer with workings.	[3]

4 (a)

$4y = 2x + 1$ ....(1)

$y = \frac{2x+1}{4}$ ....(2)

$3y - x = 4xy$ ....(3)

Substitute (2) into (3):

$3\left(\frac{2x+1}{4}\right) - x = 4x\left(\frac{2x+1}{4}\right)$

$\frac{3}{2}x + \frac{3}{4} - x = 2x^2 + x$

$6x + 3 - 4x = 8x^2 + 4x$

$8x^2 + 2x - 3 = 0$

$(2x-1)(4x+3) = 0$

$x = \frac{1}{2}$ *or* $x = -\frac{3}{4}$

$y = \frac{1}{2}$ *or* $y = -\frac{1}{8}$

$A = \left(\frac{1}{2}, \frac{1}{2}\right)$ $B = \left(-\frac{3}{4}, -\frac{1}{8}\right)$

(b)

Gradient of $2y - 4x = 5$ is 2

Gradient of AB $= \dfrac{-\frac{1}{8} - \frac{1}{2}}{-\frac{3}{4} - \frac{1}{2}} = \frac{1}{2}$

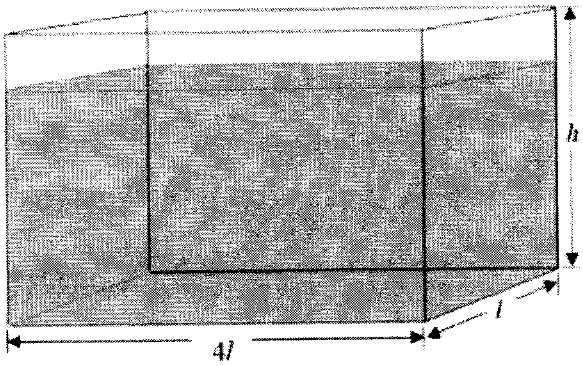
Since $2 \times \frac{1}{2} \neq -1$, the 2 lines are not perpendicular to each other. Henry is not correct.

5	(a)	Write down and simplify the first three terms in the expansion, in descending powers of x, of $\left(2-\frac{3}{x}\right)^8$.	[2]
	(b)	Given that there is no x term in the expansion of $(1-2x-kx^2)\left(2-\frac{3}{x}\right)^8$, find the constant term in the expansion.	[4]

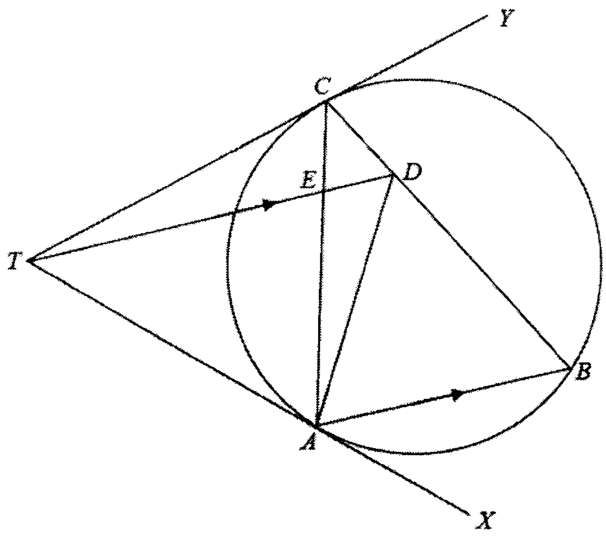
5	(a)	$\left(2-\frac{3}{x}\right)^8 = 2^8 + \binom{8}{1}(2^7)\left(-\frac{3}{x}\right) + \binom{8}{2}(2^6)\left(-\frac{3}{x}\right)^2 + \dots\dots$ $= 256 - \frac{3072}{x} + \frac{16128}{x^2} + \dots\dots$	
	(b)	$(1-2x-kx^2)\left(1-\frac{2}{x}\right)^8$ $= (1-2x-kx^2)\left(256 - \frac{3072}{x} + \frac{16128}{x^2} + \dots\dots\right)$ $= 256 - 2(256)x + 2(3072) + 3072kx - 16128k + \dots\dots$ $= 256 - 512x + 6144 + 3072kx - 16128k + \dots\dots$ Since there is no x term in the expansion, $3072k - 512 = 0$ $k = \frac{1}{6}$ The constant term $= 256 + 6144 - 16128k$ $= 6400 - 2688$ $= 3712$	

6	**(a)**	Express $y = 4x - 4x^2 - 3$ in the form $p(x+q)^2 + r$ where p, q and r are constants.	[2]
	(b)	Hence, explain whether $4x - 4x^2 - 3 = 0$ has any real solutions.	[2]

6	**(a)**	$-4x^2 + 4x - 3$ $= -4(x^2 - x) - 3$ $= -4\left[x^2 - x + \left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)^2\right] - 3$ $= -4\left(x - \frac{1}{2}\right)^2 + 1 - 3$ $= -4\left(x - \frac{1}{2}\right)^2 - 2$	
	(b)	$4x - 4x^2 - 3 = 0$ $y = 0\ (x - axis)$ Since the maximum value of y is -2, the graph of $y = 4x - 4x^2 - 3$ will not intersect the x-axis, $4x - 4x^2 - 3 = 0$ has no real solutions. OR $-4\left(x - \frac{1}{2}\right)^2 - 2 = 0$ $\left(x - \frac{1}{2}\right)^2 = -\frac{1}{2}$ $\left(x - \frac{1}{2}\right) = \pm\sqrt{-\frac{1}{2}}$ (*no real solutions*)	

7	Peter constructed an open fish tank with a rectangular base of length $4l$ m, breadth l m, and height h m. He wanted the total outer surface area of the fish tank to be 5 m^2. 		
	(a)	Show that the volume of the tank, V m^3, is given by $V = \frac{2}{5}\left(5l - 4l^3\right)$.	[3]
	(b)	Determine the area of the rectangular base for the tank to contain the maximum amount of water when filled to the brim.	[4]

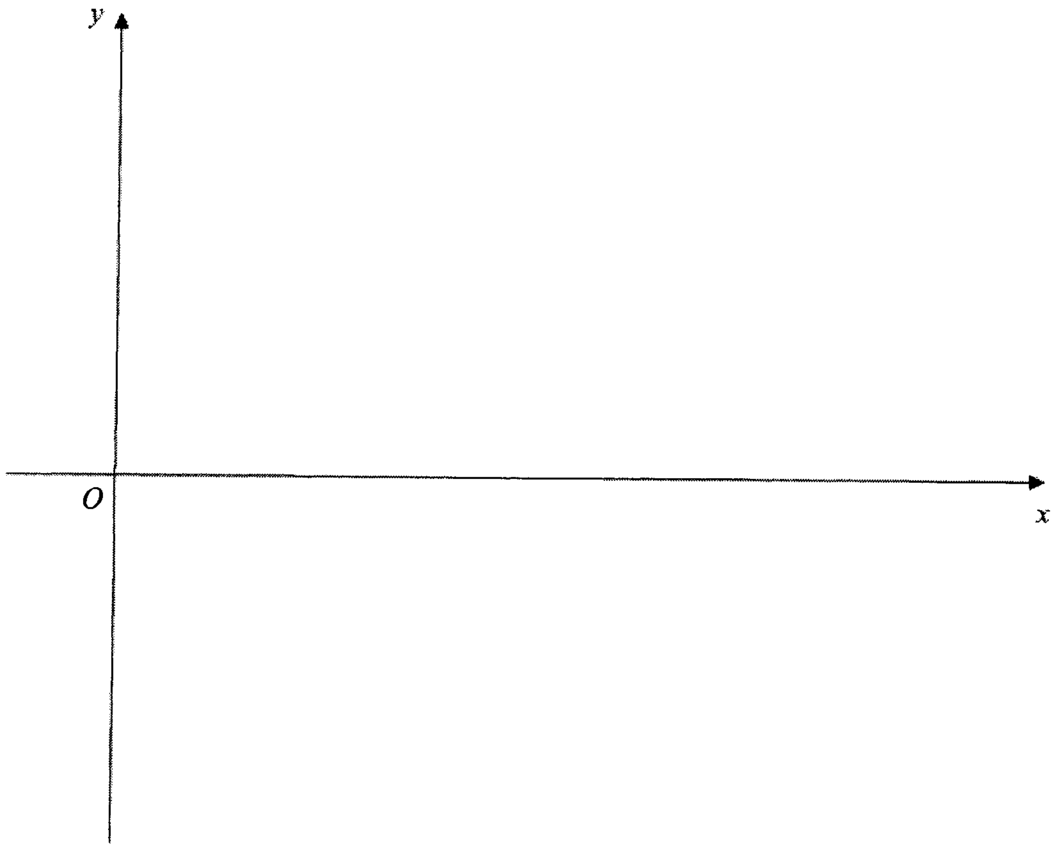
7	(a)	Total outer surface area $= 5\,m^2$ $2(4lh) + 2(lh) + 4l^2 = 5$ $10lh + 4l^2 = 5$ $h = \dfrac{5-4l^2}{10l}$ $V = 4l \times l \times h$ $= 4l^2 \times \dfrac{5-4l^2}{10l}$ $= 2l \times \dfrac{5-4l^2}{5}$ $= \dfrac{2}{5}(5l - 4l^3)$ *(Shown)*	
	(b)	$V = \dfrac{2}{5}(5l - 4l^3)$ $\dfrac{dV}{dl} = \dfrac{2}{5}(5 - 12l^2)$ *or* $2 - \dfrac{24}{5}l^2$ $\dfrac{d^2V}{dl^2} = -\dfrac{48}{5}l < 0$ (maximum V) $\dfrac{dV}{dl} = 0$ $\dfrac{2}{5}(5 - 12l^2) = 0$ $l^2 = \dfrac{5}{12}$ $l = \sqrt{\dfrac{5}{12}}$ Area of rectangular base $= 4l^2 = 4\left(\dfrac{5}{12}\right) = \dfrac{5}{3}$ *or* $1.67\ m^2$	

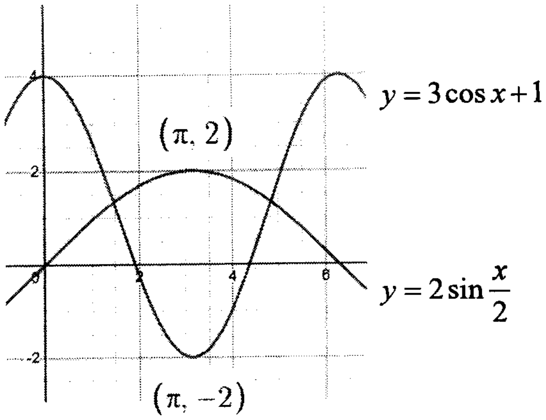
8	In the diagram below, TAX and TCY are tangents to the circle at A and C respectively. AC meets TD at E and D is on BC such that TD is parallel to AB. 		
	(a)	Prove that angle ACB is equal to angle ATD.	[2]
	(b)	Using the result from part **(a)**, explain whether a circle can be drawn passing through the points T, A, D and C.	[2]
	(c)	Prove that $CE \times EA = DE \times TE$.	[3]

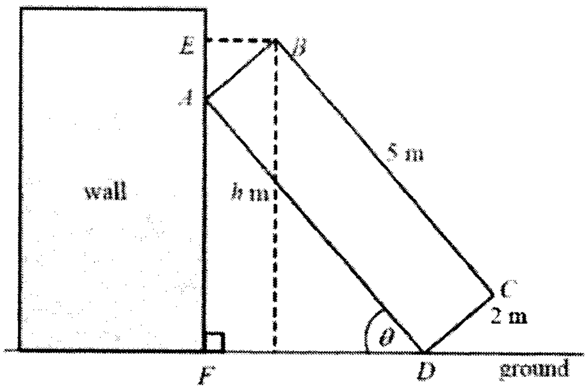
8	(a)	$\angle ATD = \angle XAB$ (corresponding angles, $AB // TD$) $\angle XAB = \angle ACB$ (Alternate Segment Theorem) $\angle ACB = \angle ATD$ (Proved)	
	(b)	From Part (a), $\angle ACD = \angle ATD$, the properties of circles tell us that angles in the same segment of a circle are equal. Therefore, we can draw a circle passing through points T, A, D and C.	
		$\angle ATE = \angle DCE$ (from Part (a)) $\angle TEA = \angle CED$ (vertically opposite angles) Therefore ΔATE is similar to ΔDCE (AA Similarity test) $\frac{TE}{CE} = \frac{EA}{DE}$ $CE \times EA = DE \times TE$	

9	**(a)**	A square has an area $\left(17-8\sqrt{2}\right)$ cm^2. The length of each side of the square can be expressed in the form $\left(a+b\sqrt{2}\right)$ cm, where a and b are integers. Show that $2b^4-17b^2+16=0$.	[4]
	(b)	[The area of a sector is $\frac{1}{2}r^2\theta$ and the arc length of a sector is $r\theta$.] The sector of a circle with radius, r, has an arc length of $\left(\sqrt{15}-\sqrt{3}\right)$ cm and an area of $\left(3\sqrt{3}-\sqrt{15}\right)$ cm^2. Show that $r=\dfrac{6\sqrt{3}-2\sqrt{15}}{\sqrt{15}-\sqrt{3}}$ and hence express r in the form $\left(p+q\sqrt{5}\right)$ cm, where p and q are integers.	[5]

9	(a)	$\left(a+b\sqrt{2}\right)^2=17-8\sqrt{2}$ $a^2+2ab\sqrt{2}+2b^2=17-8\sqrt{2}$ $\Rightarrow a^2+2b^2=17$ ....(1) $2ab=-8$ $a=-\frac{4}{b}$ ....(2) Substitute (2) into (1): $\left(-\frac{4}{b}\right)^2+2b^2=17$ $\frac{16}{b^2}+2b^2=17$ $16+2b^4=17b^2$ $2b^4-17b^2+16=0$ (Shown)	
	(b)	$A=\frac{1}{2}r^2\theta\ ,\ S=r\theta$ $\frac{A}{S}=\frac{r}{2}$ $r=\frac{2A}{S}$ $r=\frac{2\left(3\sqrt{3}-\sqrt{15}\right)}{\sqrt{15}-\sqrt{3}}=\frac{6\sqrt{3}-2\sqrt{15}}{\sqrt{15}-\sqrt{3}}$ *(showed)* $r=\frac{6\sqrt{3}-2\sqrt{15}}{\sqrt{15}-\sqrt{3}}\times\frac{\sqrt{15}+\sqrt{3}}{\sqrt{15}+\sqrt{3}}$ $=\frac{6\sqrt{45}+18-30-2\sqrt{45}}{15-3}$ $=\frac{4\sqrt{45}-12}{12}$ $=\frac{\sqrt{9}\sqrt{5}}{3}-1$ $=\sqrt{5}-1$	

10	It is given that $f(x) = 2\sin\frac{x}{2}$ and $g(x) = 3\cos x + 1$, where $0 \le x \le 2\pi$.		
	(a)	State the period of $f(x)$.	[1]
	(b)	State the smallest value of $f(x)$.	[1]
	(c)	State the largest value of $g(x)$.	[1]
	(d)	Sketch on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0 \le x \le 2\pi$. Label your graphs clearly.	[4]
			
	(e)	The solutions to the equation $f(x) = g(x)$ for $0 \le x \le 2\pi$ are a and b, where $a < b$. State, in terms of a and b, the range of values of x for which $f(x) > g(x)$.	[1]

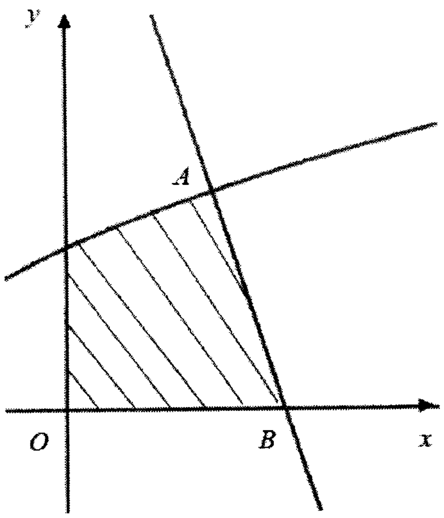
10	(a)	Period of $f(x) = 720°$ *or* 4π	
	(b)	Smallest value of $f(x) = 0$	
	(c)	Smallest value of $g(x) = 4$	
	(d)		
	(e)	$a < x < b$	

11	The diagram shows a rectangular wooden plank *ABCD*, 5 m by 2 m, leaning against a vertical wall. *AD* makes an acute angle θ with the horizontal ground. *E* is a point on the vertical wall such that *EB* is parallel to the ground and *F* is a point on the ground such that *AF* is perpendicular to *FD*. The point *B* is *h* m vertically above the ground. 	
(a)	Show that $h = 5\sin\theta + 2\cos\theta$.	[2]
(b)	Express *h* in the form $R\sin(\theta + \alpha)$, where $R > 0$, and $0° < \alpha < 90°$.	[3]
(c)	Find the greatest possible value of *h* and the value of θ at which it occurs.	[2]
(d)	Find the value of θ for which $h = \sqrt{15}$ m.	[2]

11	(a)	In ΔABE, $\angle EAB = \theta$, $AB = 2\,\text{cm}$, $EA = 2\cos\theta$ In ΔAFD, $\angle ADF = \theta, AD = 5\,\text{cm}$, $AF = 5\sin\theta$ $h = AF + EA = 5\sin\theta + 2\cos\theta$ (showed)	
	(b)	$h = 5\sin\theta + 2\cos\theta$ $R = \sqrt{5^2 + 2^2} = \sqrt{29}$ $\alpha = \tan^{-1}\dfrac{2}{5} = 21.801°$ $h = \sqrt{29}\sin(\theta + 21.8°)$	
	(c)	$h = \sqrt{29}\sin(\theta + 21.8°)$ Greatest possible value of $h = \sqrt{29}$ $\sin(\theta + 21.8°) = 1$ $\theta + 21.8° = 90°$ $\theta = 68.2°$	
	(d)	$\sqrt{29}\sin(\theta + 21.8014°) = \sqrt{15}$ $\sin(\theta + 21.8014°) = \sqrt{\dfrac{15}{29}}$ $\theta + 21.8014° = 45.98805°$ $\theta = 24.2°$	

12	(a)	By using long division, divide $4x^3+5x^2+x-1$ by $x^2(x+1)$.	[1]
	(b)	Hence, express $\dfrac{4x^3+5x^2+x-1}{x^2(x+1)}$ in partial fractions.	[5]
	(c)	Hence, find $\int \dfrac{4x^3+5x^2+x-1}{x^2(x+1)}\,dx$.	[3]

12	(a)	$$\begin{array}{r l} & \quad 4 \\ x^3+x^2 \big) & 4x^3+5x^2+x-1 \\ & 4x^3+4x^2 \\ \hline & \quad\quad x^2+x-1 \end{array}$$	
	(b)	$\dfrac{4x^3+5x^2+x-1}{x^2(x+1)}=4+\dfrac{x^2+x-1}{x^2(x+1)}$ $\dfrac{x^2+x-1}{x^2(x+1)}=\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x+1}$ $x^2+x-1=Ax(x+1)+B(x+1)+Cx^2$ When $x=0$, $B=-1$ When $x=-1$, $C=-1$ When $x=1$, $2A-2-1=1$ $A=2$ $\dfrac{4x^3+5x^2+x-1}{x^2(x+1)}=4+\dfrac{2}{x}-\dfrac{1}{x^2}-\dfrac{1}{x+1}$	
	(c)	$\int \dfrac{4x^3+5x^2+x-1}{x^2(x+1)}\,dx$ $=\int 4+\dfrac{2}{x}-\dfrac{1}{x^2}-\dfrac{1}{x+1}\,dx$ $=4x+2\ln x+\dfrac{1}{x}-\ln(x+1)+c$	

13	The diagram shows part of the curve $y=\sqrt{2x+5}$. A is a point on the curve and the x-coordinate of A is 2. The normal to the curve at A meets the x-axis at B. 		
	(a)	Find the equation of the normal to the curve at A.	[5]
	(b)	Find the area of the shaded region bounded by the normal AB, the curve, the x-axis and the y-axis.	[5]

13	(a)	$y=\sqrt{2x+5}$ Coordinates of $A=(2,\,3)$ $\frac{dy}{dx}=\frac{1}{2}(2x+5)^{-\frac{1}{2}}(2)=\frac{1}{\sqrt{2x+5}}$ At A, $\frac{dy}{dx}=\frac{1}{3}$ Equation of normal AB is $y-3=-3(x-2)$ $y=-3x+9$	
	(b)	At $B, y=0$ $-3x+9=0$ $x=3$ $B=(3,\,0)$ Area of shaded region $=\frac{1}{2}(3-2)(3)+\int_0^2\left(\sqrt{2x+5}\right)dx$ $=1.5+\left[\frac{(2x+5)^{\frac{3}{2}}}{\frac{3}{2}\times 2}\right]_0^2$ $=1.5+\left[\left(\frac{9^{\frac{3}{2}}}{3}\right)-\left(\frac{5^{\frac{3}{2}}}{3}\right)\right]$ $=1.5+(5.27322)$ $=6.77\ units^2$	

BP~579

Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/02**
Paper 2

26 August 2025 Tuesday 2 hours 15 min

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2025 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)
PRELIMINARY EXAMINATIONS

WORKED SOLUTIONS

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The equation of a curve is $y = \dfrac{5x}{6-2x}$. Find the x-coordinates of the points at which the gradient of the curve is 7.5. [4]

$$y = \frac{5x}{6-2x}$$

$$\frac{dy}{dx} = \frac{(6-2x)(5) - 5x(-2)}{(6-2x)^2}$$

$$= \frac{30}{(6-2x)^2}$$

$$\frac{30}{(6-2x)^2} = 7.5$$

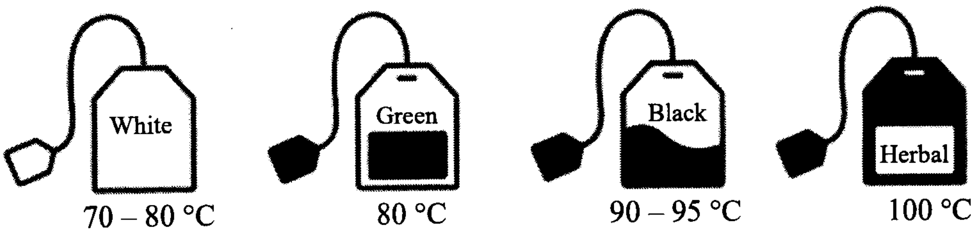
$$(6-2x)^2 = 4$$

$$6 - 2x = \pm 2$$

$$-2x = -4 \quad or \quad -8$$

$$x = 2 \quad or \quad 4$$

2 The temperature of water in a tea cup, T °C, t minutes after boiling water is poured into the empty cup can be modelled by the formula $T = 25 + 75\mathrm{e}^{-kt}$.



The diagram above shows the ideal brewing temperature for various teas at a particular teahouse. At this teahouse, green tea leaves are placed into the tea cup for brewing at the ideal temperature when $t = 6$. Determine whether the teahouse's standards for brewing tea would be adhered to if black tea leaves are placed into the teacup when $t = 2$. [4]

When $t = 6$, $T = 80$,

$$80 = 25 + 75e^{-6k}$$

$$\frac{55}{75} = e^{-6k}$$

$$\ln\frac{55}{75} = \ln e^{-6k}$$

$$k = -\frac{1}{6}\ln\frac{55}{75} = 0.0516925$$

Alternative:

$$\ln 55 = \ln\left(75e^{-6k}\right)$$

$$\ln 55 = \ln 75 - 6k$$

$$k = \frac{1}{6}(\ln 75 - \ln 55)$$

When $t = 2$, $T = 92.6$ °C

Since the temperature of the water would be within the ideal range of 90 – 95 °C, the standards for brewing black tea **would** be adhered to.

3 It is given that $f'(x) = 6\cos 3x - 12\sin 3x$ and $f\left(\frac{\pi}{3}\right) = 0$.

(a) Find f(x). [4]

$$f'(x) = 6\cos 3x - 12\sin 3x$$

$$f(x) = \frac{6\sin 3x}{3} + \frac{12\cos 3x}{3} + c$$

$$= 2\sin 3x + 4\cos 3x + c$$

$$f\left(\frac{\pi}{3}\right) = 0$$

$$2\sin \pi + 4\cos \pi + c = 0$$

$$c = 4$$

$$\therefore\ f(x) = 2\sin 3x + 4\cos 3x + 4$$

(b) Find the exact value of $f''\left(\frac{\pi}{12}\right)$. [2]

$$f''(x) = -18\sin 3x - 36\cos 3x$$

$$f''\left(\frac{\pi}{12}\right) = -18\sin\left(\frac{\pi}{4}\right) - 36\cos\left(\frac{\pi}{4}\right)$$

$$= -18\left(\frac{\sqrt{2}}{2}\right) - 36\left(\frac{\sqrt{2}}{2}\right)$$

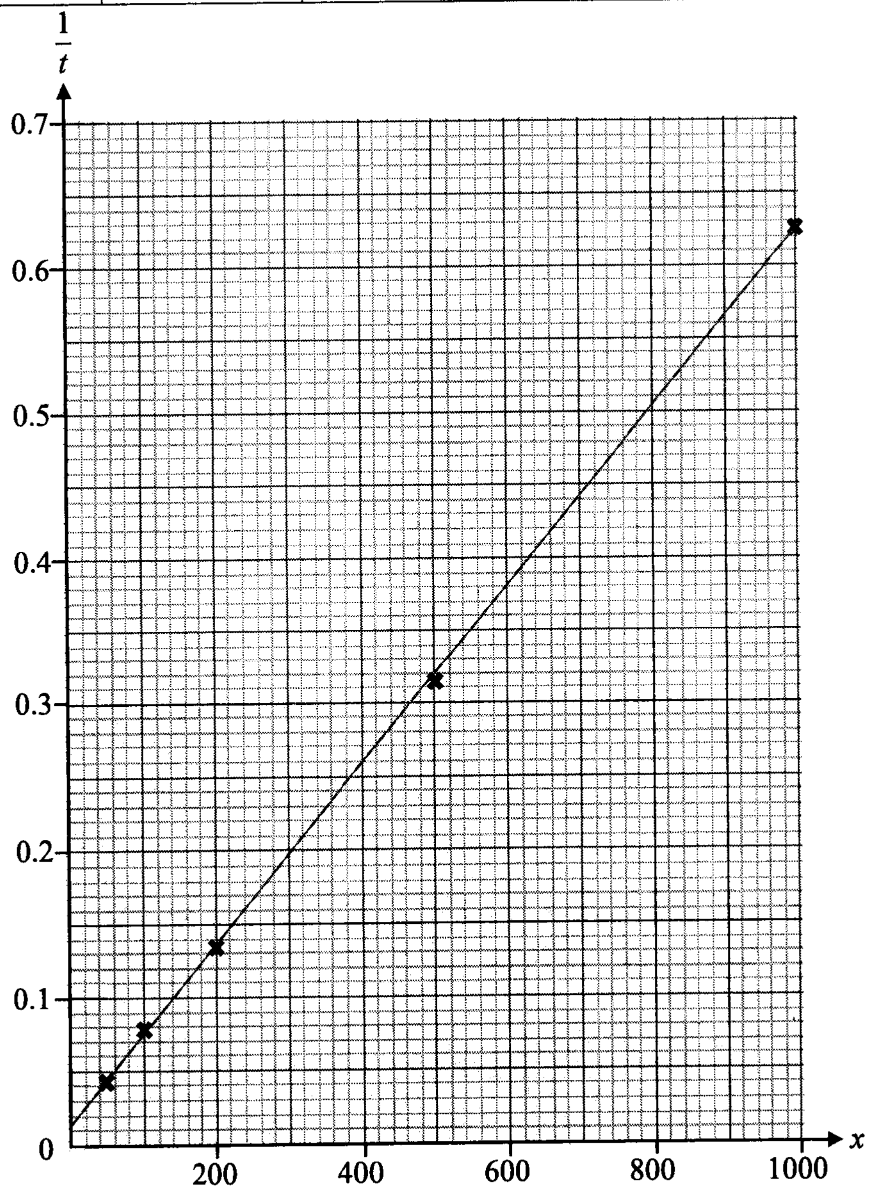
$$= -27\sqrt{2}$$

4 **(a)** The time taken, t in minutes, to download a particular gaming software is related to internet speed, x Mbps. The variables x and t are related by the formula $t = \frac{a}{b+x}$, where a and b are positive constants. The data below shows some measured values of x and t.

x (Mbps)	50	100	200	500	1000
t (minutes)	23	13	7.3	3.2	1.6

(i) Plot $\frac{1}{t}$ against x and draw a straight line graph. [2]

$\frac{1}{t}$	0.043	0.077	0.137	0.313	0.625



(ii) Use the graph to estimate the value of a and of b, correct to 2 significant figures. [4]

$$\frac{1}{t}=\frac{b}{a}+\frac{1}{a}x$$

$$\text{Gradient}=\frac{0.62-0.02}{1000}=0.0006$$

$$\frac{1}{a}=0.0006$$

$$a=\frac{1}{0.0006}=1700\ (2\ s.f.)$$

$$\frac{b}{a}=0.02$$

$$b=\frac{0.02}{0.0006}=33\ (2\ s.f.)$$

(iii) Use the graph to estimate the time taken for John to download the gaming software if the speed of his internet is 240 Mbps. [2]

From the graph,

$$\frac{1}{t}=0.16$$

$$t=6.25\ \text{min}$$

(iv) Suggest an alternative set of variables to be plotted on each axis to draw a straight line to represent the formula. [1]

$$tb+xt=a$$

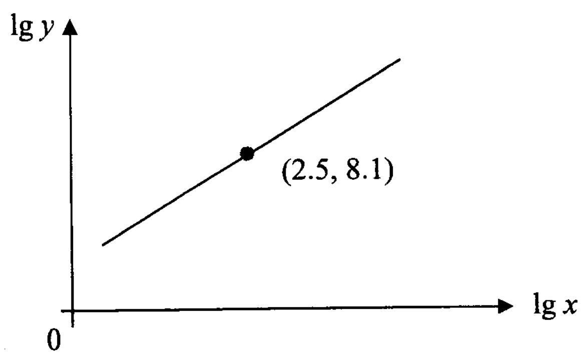
$$xt=-bt+a$$

An alternative would be to plot

xt against t **OR** t against xt

(b) The diagram shows part of a straight line graph drawn to represent the equation $y = px^q$.

Given that the line passes through the point (2.5, 8.1) and has a gradient of 3, find the value of p correct to the nearest integer. [3]



$y = px^q$

$\lg y = \lg p + q \lg x$

When $\lg y = 8.1$, $\lg x = 2.5$,

$8.1 = \lg p + 3(2.5)$

$\lg p = 0.6$

$p = 10^{0.6} = 4$ (to nearest integer)

5 **(a)** Find the range of values of the constant k for which $k\left(x^2+x\right)+2\left(x^2+x+1\right)=0$ has no real roots. [4]

$(k+2)x^2+(k+2)x+2=0$

Since there are no x-intercepts,

$(k+2)^2-4(k+2)(2)<0$

$k^2+4k+4-8k-16<0$

$k^2-4k-12<0$

$(k-6)(k+2)<0$

$\therefore\ -2<k<6$

(b) Hence, explain why $-\left(x^2+x\right)+2\left(x^2+x+1\right)$ is always positive. [2]

Since the **discriminant < 0 when k = –1** and the **coefficient of x^2** , i.e. 1, is **positive,**

$-\left(x^2+x\right)+2\left(x^2+x+1\right)$ is always positive

6 **(a)** By using a suitable substitution, or otherwise, solve the equation

$$3\log_6 x = \log_x 3 + \log_x 2 + \log_5 25.$$ [4]

$3\log_6 x = \log_x 3 + \log_x 2 + \log_5 25$

$3\log_6 x = \log_x 6 + 2$

$3\log_6 x = \dfrac{1}{\log_6 x} + 2$

Let $y = \log_6 x$

$3y = \dfrac{1}{y} + 2$

$3y^2 - 2y - 1 = 0$

$(3y+1)(y-1) = 0$

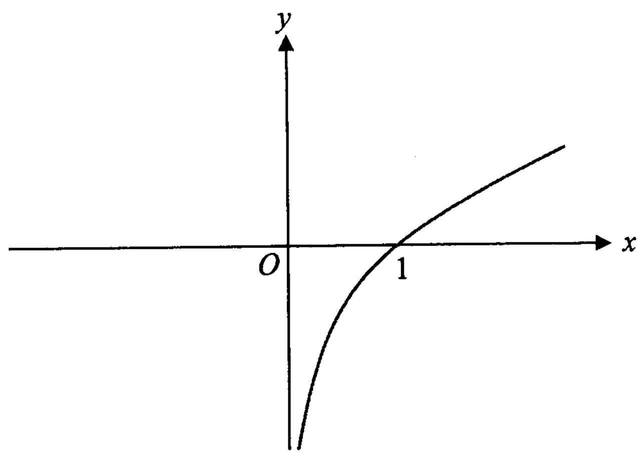
$y = -\dfrac{1}{3}$ *or* $y = 1$

$\log_6 x = -\dfrac{1}{3}$ *or* $\log_6 x = 1$

$x = 6^{-\frac{1}{3}}$ *or* $x = 6$

$= 0.550$ ($3s.f.$)

(b) Sketch the graph of $y = \log_6 x$ on the given axes. Label any axial intercepts. [1]



(c) **(i)** It is given that the equation of a curve is $y = e^{2x+1} + 2e^{x+1} - 3e$.

Explain why the curve has no stationary point. [3]

$\frac{dy}{dx} = 2e^{2x+1} + 2e^{x+1}$

Since $e^{2x+1} > 0$ and $e^{x+1} > 0$ for all real values of x,

$2e^{2x+1} + 2e^{x+1} > 0,\ \frac{dy}{dx} \neq 0$

Therefore the curve has no stationary points

(ii) Find the x-intercept of the curve $y = e^{2x+1} + 2e^{x+1} - 3e$. [3]

$e^{2x} \cdot e + 2e^{x} \cdot e - 3e = 0$

$e^{2x} + 2e^{x} - 3 = 0$

Let $y = e^{x}$

$y^2 + 2y - 3 = 0$

$(y+3)(y-1) = 0$

$e^{x} = -3$ *(rej)* *or* $e^{x} = 1$

$x = 0$

7 **(a)** Factorise $x^3 - 125$. [1]

$x^3 - 125 = (x-5)\left(x^2 + 5x + 25\right)$

(b) The expression $-2x^3 + 7x^2 + ax + b$, where a and b are integers, has a factor of $x - 5$ and a remainder of -11 when divided by $2x + 1$. Find the value of a and of b. [4]

Let $f(x) = -2x^3 + 7x^2 + ax + b$.

$f(5) = 0$

$-2(5)^3 + 7(5)^2 + a(5) + b = 0$

$5a + b = 75$ ------ (1)

$f\left(-\frac{1}{2}\right) = -11$

$-2\left(-\frac{1}{2}\right)^3 + 7\left(-\frac{1}{2}\right)^2 + a\left(-\frac{1}{2}\right) + b = -11$

$2 - \frac{1}{2}a + b = -11$

$a = 2b + 26$ ------ (2)

Subst (2) into (1):

$5(2b + 26) + b = 75$

$11b = -55$

$b = -5,\ a = 16$

(c) Hence, or otherwise, show that the equation $x^3 - 125 = -2x^3 + 7x^2 + ax + b$ has only one real root. [4]

$x^3 - 125 = -2x^3 + 7x^2 + 16x - 5$

$(x-5)(x^2 + 5x + 25) = (x-5)(-2x^2 - 3x + 1)$

$(x-5)(3x^2 + 8x + 24) = 0$

$x - 5 = 0$ *or* $3x^2 + 8x + 24 = 0$

$x = 5$ $\quad b^2 - 4ac = 8^2 - 4(3)(24)$

$= -224$

< 0

There is no real root for $3x^2 + 8x + 24 = 0$.

Therefore there is only one real root $x = 5$.

<u>Alternative</u>

$x^3 - 125 = -2x^3 + 7x^2 + 16x - 5$

$3x^3 - 7x^2 - 16x - 120 = 0$

$(x-5)(3x^2 + 8x + 24) = 0$

8 The equation of a curve is $y = e^{-2x} \tan x$.

(a) Show that $\frac{dy}{dx} = e^{-2x}(1 - \tan x)^2$. [3]

$$\frac{dy}{dx} = e^{-2x}\sec^2 x - 2e^{-2x}\tan x$$
$$= e^{-2x}\left(\tan^2 x + 1 - 2\tan x\right)$$
$$= e^{-2x}(\tan x - 1)^2$$

(b) Find, in terms of π, the x-coordinate of the stationary point for $0 < x < \frac{\pi}{2}$. [2]

$e^{-2x}(\tan x - 1)^2 = 0$

Since $e^{-2x} > 0$ for all real values of x,

$(\tan x - 1)^2 = 0$

$\tan x = 1$

$x = \frac{\pi}{4}$

(c) Explain why y is never decreasing. [2]

Since $e^{-2x} > 0$ and $(\tan x - 1)^2 \geq 0$ for all real values of x,

$\frac{dy}{dx} \geq 0$ therefore y is never decreasing.

(d) What does your answer to **part (c)** imply about the stationary point found in **part (b)**? Explain your answer. [2]

The stationary point at $x = \frac{\pi}{4}$ is a **point of inflexion.**

Since y is never decreasing, the **gradient of the curve must be positive (or dy/dx > 0) slightly to the left and right** of the stationary point.

9 **(a)** State

(i) the principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, [1]

$-60°$ or $-\frac{\pi}{3}$

(ii) the values between which the principal value of $\tan^{-1}x$ must lie, [1]

$-90° < \tan^{-1}x < 90°$

(iii) the range of values of k for which the equation $4\sin^2 x = k$ has a solution. [1]

$0 \leq k \leq 4$

(b) **(i)** Prove the identity $\cot x - \cot x \tan^2 x + \tan x = \dfrac{1+\cos 2x}{\sin 2x}$. [4]

$$\cot x - \cot x \tan^2 x + \tan x$$
$$= \cot x - \frac{1}{\tan x}\tan^2 x + \tan x$$
$$= \frac{\cos x}{\sin x} - \tan x + \tan x$$
$$= \frac{2\cos^2 x}{2\sin x \cos x}$$
$$= \frac{1+\cos 2x}{\sin 2x}$$

(ii) Hence solve the equation $\cot x - \cot x \tan^2 x + \tan x = \operatorname{cosec} 2x - 3$ for $0° \le x \le 180°$.

[5]

$$\frac{1+\cos 2x}{\sin 2x} = \operatorname{cosec} 2x - 3$$
$$\frac{1+\cos 2x}{\sin 2x} = \frac{1}{\sin 2x} - 3$$
$$1+\cos 2x = 1 - 3\sin 2x$$
$$\tan 2x = -\frac{1}{3}$$
$$\tan\alpha = \frac{1}{3}$$
$$\alpha = \tan^{-1}\left(\frac{1}{3}\right) = 18.434°$$
$$2x = 180° - 18.434°,\ 360° - 18.434°$$
$$2x = 161.566°,\ 341.566°$$
$$x = 80.8°,\ 170.8°$$

10 **(a)** Points $P(-2, 0)$, $Q(5, 1)$, $R(6, -6)$ and $S(-1, -7)$ lie on a circle.

PQ and SR are parallel chords of the same length.

Show that the centre of the circle is at $(2, -3)$. [2]

Centre of circle

$$= \left(\frac{-2+6}{2}, \frac{0-6}{2}\right)$$

$$= (2, -3)$$

OR

$$\text{Midpoint of } PQ = \left(\frac{-2+5}{2}, \frac{0+1}{2}\right)$$

$$= \left(\frac{3}{2}, \frac{1}{2}\right)$$

$$\text{Midpoint of } SR = \left(\frac{6-1}{2}, \frac{-6-7}{2}\right)$$

$$= \left(\frac{5}{2}, -\frac{13}{2}\right)$$

$$\text{Centre of circle} = \left(\frac{\frac{3}{2}+\frac{5}{2}}{2}, \frac{\frac{1}{2}-\frac{13}{2}}{2}\right)$$

$$= (2, -3)$$

(b) Find the equation of the circle. [3]

Radius

$$= \sqrt{(5-2)^2 + (1+3)^2}$$

$$= 5 \text{ units}$$

$$(x-2)^2 + (y+3)^2 = 25$$

OR

$$x^2 - 4x + y^2 + 6y - 12 = 0$$

(c) A second circle has centre *C*. It is given that the *y*-axis is the perpendicular bisector of *CR* and is also a tangent to the second circle.

(i) State the coordinates of *C*. [1]

Centre of second circle = (–6, –6)

(ii) Hence explain why the *x*-axis is a tangent to the second circle. [2]

Since the *y*-axis is a tangent to the second circle, the radius of the circle is 6 units. The highest point, *H*, of the second circle is (–6, 0). Since the circle touches the *x*-axis at exactly one point at (–6, 0), the *x*-axis is a tangent to the second circle. Or Since *CH* is a vertical radius of the circle and *H* lies on the *x*-axis, the *x*-axis, which is horizontal, must be a tangent to the second circle (rad⊥tan)

11 A particle moves from point P towards point O. The speed of the particle, t seconds after passing point P, can be modelled by the formula $v = (k - 2t)^n$, where k and n are constants.

(a) Find an expression for the acceleration of the particle in terms of k, t and n. [1]

$a = -2n(k - 2t)^{n-1}$

The particle has a speed of 27 m/s when $t = 3.5$ and comes to instantaneous rest 4.8 metres to the right of point O when $t = 8$.

(b) Find the value of k and of n. [3]

When $t = 8$, $v = 0$,

$0 = (k - 16)^n$

$k = 16$

When $t = 3.5$, $v = 27$,

$27 = (16 - 7)^n$

$27 = 9^n$

$3^3 = 3^{2n}$

$n = 1.5$

(c) Using the values of k and n found in **part (b)**, find the total distance travelled by the particle in the first 8 seconds. [4]

$v = (16 - 2t)^{1.5}$

$s = \dfrac{(16 - 2t)^{2.5}}{(2.5)(-2)} + c$

When $t = 8$, $s = 4.8$

$4.8 = c$

$s = -\dfrac{1}{5}(16 - 2t)^{2.5} + 4.8$

When $t = 0$, $s = -200$

Total distance = 200 + 4.8 = 204.8 m

(d) Explain whether the given model can continue to describe the motion of the particle after 8 seconds. [1]

No since when $t > 8$, v and s are undefined.

END OF PAPER