



Solution

**SECONDARY 4
2025 PRELIMINARY EXAMINATION**

**MATHEMATICS
Paper 1**

4052/01

26 August 2025 (Tuesday)

2 hour 15 minutes

CANDIDATE
NAME

CLASS

S	4 -		
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INDEX NUMBER

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Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number in the spaces above.
Write in dark blue or black pen in the space provided for each question.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any question, it must be shown in the space below the question.

Omission of essential working will result in loss of marks.

The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142.**For Examiner's Use**

Q1	2	
Q2	4	
Q3	4	
Q4	6	
Q5	2	
Q6	3	
Q7	4	
Q8	3	
Q9	3	
Q10	3	
Q11	2	
Q12	4	
Q13	4	
Q14	2	
Q15	4	
Q16	6	
Q17	5	
Q18	5	
Q19	4	
Q20	6	
Q21	2	
Q22	8	
Q23	4	
Total	90	

This document consists of 28 printed pages including the cover page.

[Turn over

Mathematical Formulae*Compound Interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer all the questions.

1. $\xi = \{\text{integers } x : 3 < x < 18\}$
 $P = \{\text{multiples of } 5\}$
 $Q = \{\text{perfect squares}\}$

- (a) List the elements in $(P \cup Q)'$.

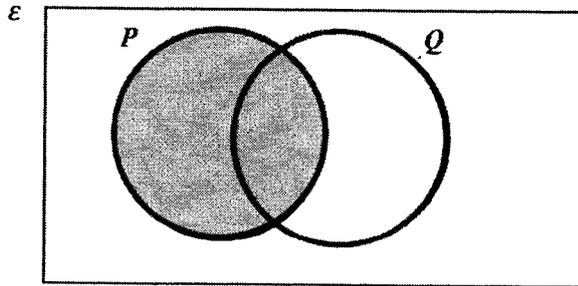
$$P \cup Q = \{4, 5, 9, 10, 15, 16\}$$

$$(P \cup Q)' = \{6, 7, 8, 11, 12, 13, 14, 17\}$$

Answer [1]

- (b) On the Venn diagram, shade the region which represents $(P \cap Q) \cup (P \cap Q)'$.

Answer



[1]

[Turn over

2. (a) Use prime factors to explain why 18×63 is not a perfect square.

Answer

$$18 = 2 \times 3^2$$

$$63 = 7 \times 3^2$$

$$18 \times 63 = 2 \times 3^2 \times 7 \times 3^2$$

$$18 \times 63 = 2 \times 3^4 \times 7$$

Since the indices of the prime factors of 18×63 are not all even numbers, 18×63 is not a perfect square.

.....

[2]

- (b) The number $18p$ is a perfect cube.
 Find the smallest positive integer value of p .

$$18 = 2 \times 3^2$$

$$18p = 2 \times 3^2 \times p$$

$$p = 2^2 \times 3$$

$$p = 12$$

Answer $p = \dots\dots\dots$ [2]

3. (a) Given that $2^{m-1} \times 3^m = 1$, find the value of 36^m .

$$2^{m-1} \times 3^m = 1$$

$$2^m \times 3^m = 2$$

$$6^m = 2$$

$$36^m = 2^2$$

$$36^m = 4$$

Answer $36^m = \dots\dots\dots$ [2]

- (b) Simplify $\frac{(2xy)^{-2}}{12} \div \frac{x^3}{4y^2}$.

$$\begin{aligned} & \frac{(2xy)^{-2}}{12} \div \frac{x^3}{4y^2} \\ &= \frac{1}{48x^2y^2} \times \frac{4y^2}{x^3} \\ &= \frac{1}{12x^5} \end{aligned}$$

Answer $\dots\dots\dots$ [2]

[Turn over

4. (a) Solve the equation $x^2 - 4x - 11 = 0$ by completing the square.
Give your answer correct to 2 decimal places.

$$\begin{aligned}x^2 - 4x - 11 &= 0 \\(x^2 - 4x + 2^2 - 2^2) - 11 &= 0 \\(x - 2)^2 - 4 - 11 &= 0 \\(x - 2)^2 &= 15 \\x - 2 &= \pm\sqrt{15} \\x &= 2 \pm \sqrt{15} \\x &= 5.87 \text{ (2 d.p.) or } x = -1.87 \text{ (2 d.p.)} \quad [\end{aligned}$$

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$ [3]

- (b) (i) Factorise completely $4x^2 + 5x - 6$.

$$4x^2 + 5x - 6 = (4x - 3)(x + 2)$$

Answer $\dots\dots\dots$ [1]

- (ii) Hence, factorise completely $4(2t + 2)^2 + 5(2t + 2) - 6$.
Write your answer as simply as possible.

$$\begin{aligned}4(2t + 2)^2 + 5(2t + 2) - 6 \\= [4(2t + 2) - 3][(2t + 2) + 2] \\= (8t + 5)(2t + 4) \\= 2(8t + 5)(t + 2)\end{aligned}$$

Answer $\dots\dots\dots$ [2]

5. It is given that y is directly proportional to x^2 .
Find the percentage increase in y if x is increased by 25% of its original value.

$$y = kx^2, \text{ where } k \text{ is a constant}$$

$$\text{Let new } x = x_1$$

$$\text{Let new } y = y_1$$

$$x_1 = 1.25x$$

$$y_1 = k(1.25x)^2$$

$$y_1 = 1.5625kx^2$$

Percentage increase in y

$$\begin{aligned} &= \frac{y_1 - y}{y} \times 100\% \\ &= \frac{1.5625kx^2 - kx^2}{kx^2} \times 100\% \\ &= \frac{kx^2(1.5625 - 1)}{kx^2} \times 100\% \\ &= 56.25\% \end{aligned}$$

Answer % [2]

6. Express as a single fraction in its simplest form

$$\frac{4(x-2)}{x^2-4} - \frac{2(3x-1)}{3x^2+5x-2}$$

$$\begin{aligned} &\frac{4(x-2)}{x^2-4} - \frac{2(3x-1)}{3x^2+5x-2} \\ &= \frac{4(x-2)}{4(x-2)} - \frac{2(3x-1)}{(3x-1)(x+2)} \\ &= \frac{4}{x+2} - \frac{2}{x+2} \\ &= \frac{2}{x+2} \end{aligned}$$

Answer [3]

[Turn over

7. In the sequence, the difference between any two consecutive terms is the same number.

$$17 \quad x \quad y \quad z \quad 9 \quad \dots$$

(a) Find the values of x , y and z .

$$\begin{aligned} T_1 &= 17 \\ T_2 &= x = 17 + 1(-2) = 15 \\ T_3 &= y = 17 + 2(-2) = 13 \\ T_4 &= z = 17 + 3(-2) = 11 \\ T_5 &= 9 = 17 + (5 - 1)(-2) = 9 \end{aligned}$$

Alternative method

$$\begin{aligned} \text{Common difference} &= 2 \\ x &= 17 - 2 = 15 \\ y &= 17 - 4 = 13 \\ z &= 17 - 6 = 11 \end{aligned}$$

Answer $x = \dots\dots\dots$

$y = \dots\dots\dots$

$z = \dots\dots\dots$ [2]

(b) Write down an expression for the n th term of the sequence.

$$\begin{aligned} T_n &= 17 + (-2)(n - 1) \\ T_n &= 19 - 2n \end{aligned}$$

Answer $\dots\dots\dots$ [1]

(c) Explain why -317 is a term of this sequence.

Answer

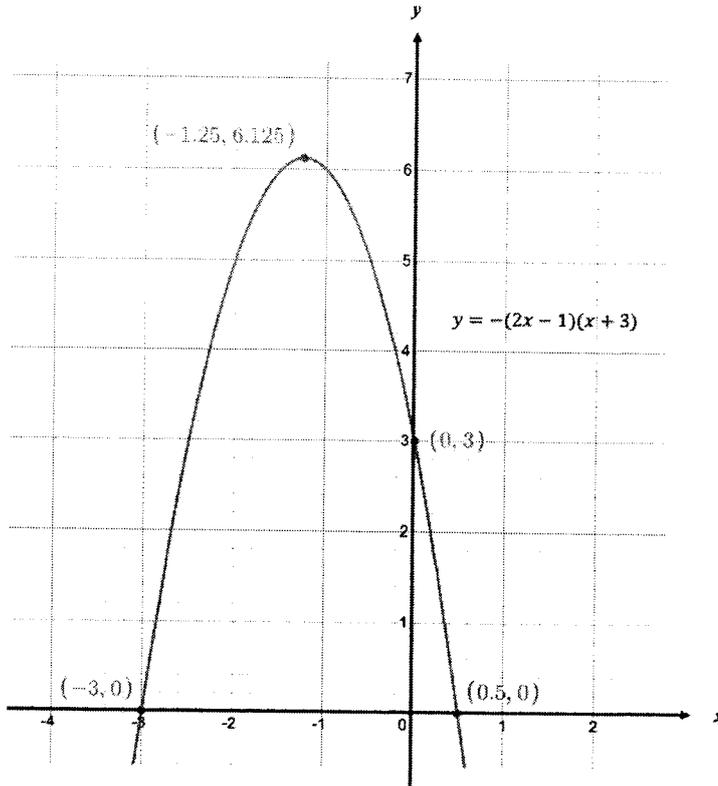
$$\begin{aligned} 19 - 2n &= -317 \\ n &= 168 \end{aligned}$$

Since n is a positive integer, -317 is the 168th term of the sequence.

OR Since n is a positive integer, -317 is a term of the sequence.

$\dots\dots\dots$ [1]

8. Sketch the graph of $y = -(2x - 1)(x + 3)$ on the axes below. Indicate clearly the coordinates of the turning point and where the graph crosses the axes.



[3]

9. Diana has two outlets that sell donuts of three flavours.
The number of donuts sold in the outlets over a two-hour period is given by the matrix D .

$$D = \begin{array}{ccc} \text{Blueberry} & \text{Chocolate} & \text{Apple} \\ \left(\begin{array}{ccc} 75 & 86 & 48 \\ 108 & 56 & 36 \end{array} \right) & \begin{array}{l} \text{Outlet 1} \\ \text{Outlet 2} \end{array} \end{array}$$

- (a) Each blueberry donut is sold at \$3.50.
Each chocolate donut is sold at \$4.50.
Each apple donut is sold at \$2.50.

Represent the selling price in a 3×1 column matrix P .

$$P = \begin{pmatrix} 3.5 \\ 4.5 \\ 2.5 \end{pmatrix}$$

Answer $P = \dots\dots\dots$ [1]

- (b) Evaluate the matrix $M = DP$.

$$\begin{aligned} M &= DP \\ &= \begin{pmatrix} 75 & 86 & 48 \\ 108 & 56 & 36 \end{pmatrix} \begin{pmatrix} 3.5 \\ 4.5 \\ 2.5 \end{pmatrix} \\ &= \begin{pmatrix} (75 \times 3.5) + (86 \times 4.5) + (48 \times 2.5) \\ (108 \times 3.5) + (56 \times 4.5) + (36 \times 2.5) \end{pmatrix} \\ &= \begin{pmatrix} 769.5 \\ 720 \end{pmatrix} \end{aligned}$$

Answer $M = \dots\dots\dots$ [1]

- (c) Explain what each element in matrix M represents.

769.5 represents the total amount in dollars collected by Diana from selling donuts of the 3 flavours in outlet 1 over a two-hour period.

720 represents the total amount collected in dollars by Diana from selling donuts of the 3 flavours in outlet 2 over a two-hour period.

Answer $\dots\dots\dots$ [1]

10. A bag contains some blue marbles, some green marbles and some red marbles.
 The probability of choosing a green marble is 0.2 .
 The probability of choosing a red marble is 0.35 .

- (a) Find the probability of choosing a blue marble at random.

$$\begin{aligned} P(\text{blue marble}) \\ &= 1 - 0.35 - 0.2 \\ &= 0.45 \end{aligned}$$

45% : not accepted

Answer [1]

- (b) In the bag, there are 9 more red marbles than green marbles.
 Find the total number of marbles in the bag.

$$\begin{aligned} \text{Total number of marbles in the bag} \\ &= \frac{9}{0.35 - 0.2} \\ &= 60 \end{aligned}$$

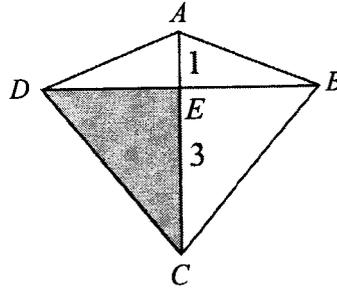
Answer marbles [2]

[Turn over

11. The diagram shows a kite $ABCD$.

AC and AB are the diagonals of the kite.

E is a point on AC such that $AE : EC = 1 : 3$.



Not drawn to scale

Find the fraction of the kite that is shaded.

$$\frac{\text{Area of smaller triangle}}{\text{Area of bigger triangle}} = \frac{\frac{1}{2} \times B \times H_1}{\frac{1}{2} \times B \times H_2}$$

$$\frac{\text{Area of smaller triangle}}{\text{Area of bigger triangle}} = \frac{1}{3}$$

$$\frac{\text{Area of Shaded triangle}}{\text{Diagram}} = \frac{1.5}{4} = \frac{3}{8}$$

Alternative Solution

Let the height of the smaller triangle be H_1 .

Let the height of the larger triangle be H_2 .

$$H_2 = 3H_1$$

Fraction of the kite that is shaded

$$= \frac{\frac{1}{2} \left(\frac{1}{2} \times H_2 \times B \right)}{\left(\frac{1}{2} \times H_1 \times B \right) + \left(\frac{1}{2} \times H_2 \times B \right)}$$

$$= \frac{\frac{1}{2} \left(\frac{1}{2} \times 3H_1 \times B \right)}{\left(\frac{1}{2} \times H_1 \times B \right) + \left(\frac{1}{2} \times 3H_1 \times B \right)}$$

$$= \frac{\frac{1}{2} \left(\frac{1}{2} \times 3H_1 \times B \right)}{\left(\frac{1}{2} \times H_1 \times B \right) (1+3)}$$

$$= \frac{3}{8}$$

Answer% [2]

12. (a) Ben sold a painting to Carl at a profit of 20% and Carl then sold it to Don at a profit of 40%. If Don paid \$12800 for the painting, calculate how much Ben paid for the painting.

Let the cost of the painting be \$ x .

$$\frac{140}{100} \left(\frac{120}{100} x \right) = 12800$$

$$1.4 \times 1.2x = 12800$$

$$1.68x = 12800$$

$$x = \frac{12800}{1.68}$$

$$x = 7619.05 \text{ (2 d.p.)}$$

Answer \$..... [2]

- (b) Some bacteria were introduced into a culture.

The number, N , of bacteria, t hours after being introduced is given by $N = 2 \times 5^t$.

Find the increase in the number of bacteria after 5 hours as a percentage of the number of bacteria originally introduced.

Percentage increase in the number of bacteria at the end of 5 hours

$$= \frac{2 \times 5^5 - 2 \times 5^0}{2 \times 5^0} \times 100\%$$

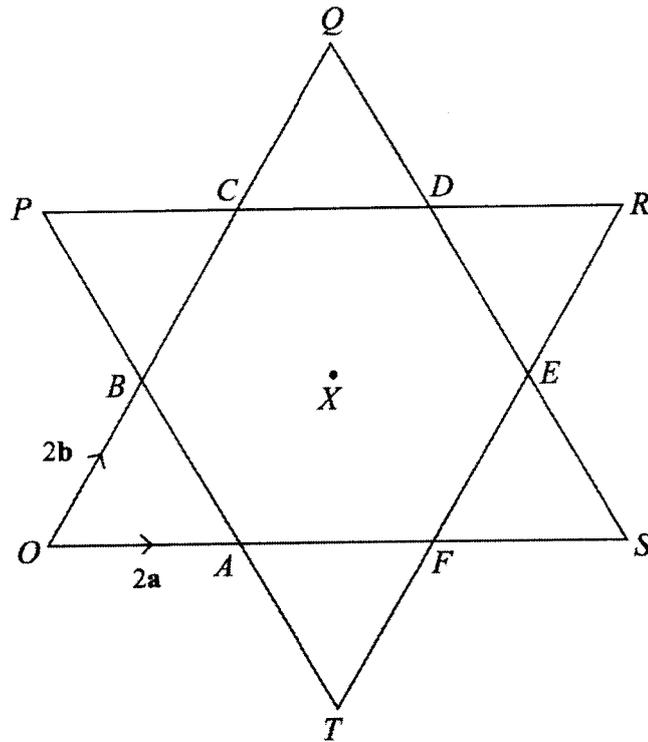
$$= \frac{2(5^5 - 1)}{2} \times 100\%$$

$$= 3124 \times 100\%$$

$$= 312400\%$$

Answer% [2]

13.



A star is made up of a regular hexagon $ABCDEF$, centre X , surrounded by equilateral triangles.

$\vec{OA} = 2\mathbf{a}$ and $\vec{OB} = 2\mathbf{b}$.

(a) Express the following vectors in terms of \mathbf{a} and / or \mathbf{b} , giving your answers in the simplest form.

(i) \vec{OR}
 $\vec{OR} = \vec{OA} + \vec{AF} + \vec{FE} + \vec{ER}$
 $\vec{OR} = 4\mathbf{a} + 4\mathbf{b}$

Answer [1]

(ii) \vec{FC}
 $\vec{FC} = \vec{FE} + \vec{ER} + \vec{RD} + \vec{DC}$
 $\vec{FC} = 4\mathbf{b} - 4\mathbf{a}$

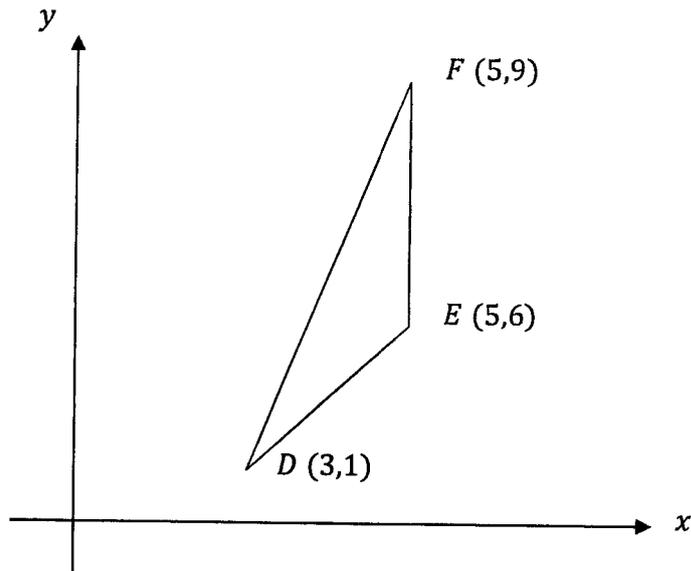
Answer [1]

(b) When $|\mathbf{a}| = 6$ units, find the perimeter of the star $OATFSERDQCPBO$.

Perimeter of the star
 $= 12 (2 \times 6)$
 $= 144$ units

Answerunits [2]

14. The diagram shows the points $D(3, 1)$, $E(5, 6)$ and $F(5, 9)$.



- (a) Find the coordinates of G such that $DEFG$ is a parallelogram.

$$G(3, 4)$$

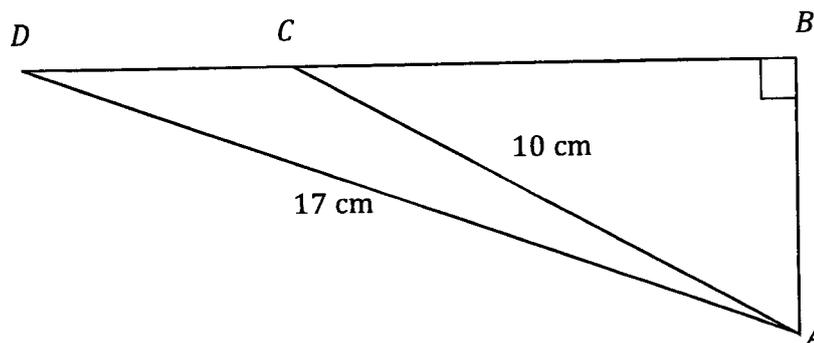
Answer $G(\dots\dots\dots, \dots\dots\dots)$ [1]

- (b) Find the length of the line DF .

$$\begin{aligned} DF &= \sqrt{(5-3)^2 + (9-1)^2} \\ &= 8.2462 \\ &= 8.25 \text{ units (3 s.f.)} \end{aligned}$$

Answer $DF = \dots\dots\dots$ units [1]

15. In the triangle ABD , $AD = 17$ cm, $AC = 10$ cm and $\angle ABC = 90^\circ$.



- (a) Given that $\sin \angle ACD = \frac{3}{5}$, find the exact value of
(i) AB

$$\begin{aligned} \sin \angle ACD &= \sin \angle ACB = \frac{3}{5} \\ \frac{AB}{AC} &= \frac{AB}{10} = \frac{3}{5} \\ AB &= 6 \text{ cm} \end{aligned}$$

Answer [1]

- (ii) $\tan \angle ACB$

$$\begin{aligned} BC &= \sqrt{10^2 - 6^2} = 8 \text{ cm} \\ \tan \angle ACB &= \frac{AB}{BC} = \frac{6}{8} = \frac{3}{4} \end{aligned}$$

Answer [2]

- (b) A circle Q_1 is drawn such that it passes through A , B and C .
A second circle Q_2 is drawn such that it passes through A , B and D .
Find the ratio of the circumference of Q_1 to circumference of Q_2 .

Note that AC is the diameter of Q_1 (right angle in a semicircle)
And AD is the diameter of Q_2 (right angle in a semicircle)

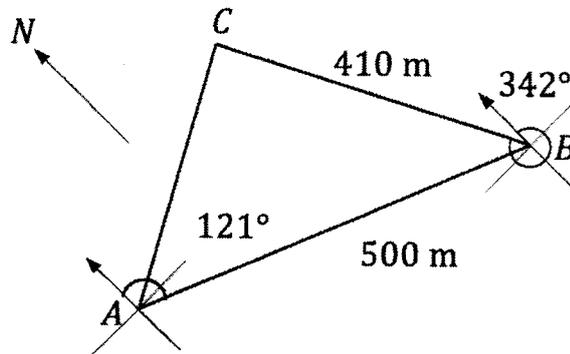
$$\begin{aligned} \text{Therefore} \\ \frac{\text{circumference of } Q_1}{\text{circumference of } Q_2} &= \frac{10}{17} \end{aligned}$$

The ratio of the circumference of Q_1 to circumference of Q_2 is $10 : 17$.

Answer : [1]

16. Aris cycled from his house at Alley Way, point A , to meet Meng at point B , which is 500 m away on a bearing of 121° .

After that, they both cycled to the School of Science and Technology (SST), point C , on a bearing of 342° , which is 410 m from B .



- (a) Show that angle ABC is 41° .

$$\angle ABN = 180^\circ - 121^\circ = 59^\circ \text{ (interior angles, parallel lines)}$$

$$\angle CBN = 360^\circ - 342^\circ = 18^\circ \text{ (angles at a point)}$$

$$\therefore \angle ABC = 59^\circ - 18^\circ = 41^\circ \text{ (adjacent angles) (shown)}$$

[2]

- (b) Calculate the distance AC .

$$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos 41^\circ$$

$$AC^2 = 500^2 + 410^2 - 2(500)(410)\cos 41^\circ$$

$$AC = 329.64 = 330 \text{ m (correct to 3 significant figures)}$$

Answer m [2]

- (c) Calculate the shortest distance from A to BC .

Let the shortest distance from A to BC be h .

$$\sin \angle ABC = \frac{h}{500}$$

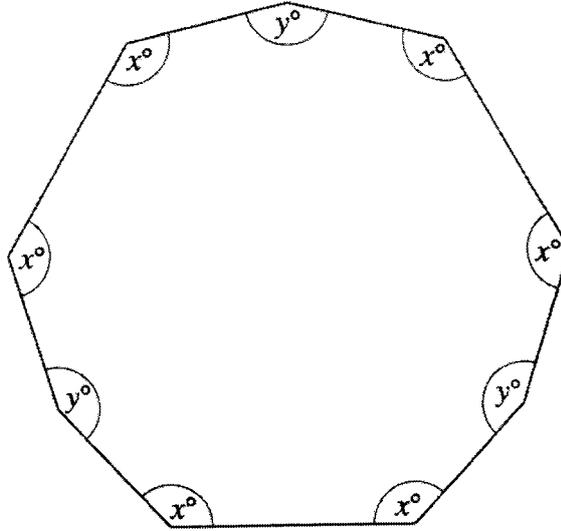
$$h = 500 \sin 41^\circ$$

$$h = 328.02 = 328 \text{ m (correct to 3 significant figures)}$$

Answer m [2]

[Turn over

17. The diagram shows a nonagon.
Six interior angles are each x° . The remaining three interior angles are each y° .



- (a) State the number of lines of symmetry if $x^\circ \neq y^\circ$.

Lines of symmetry = 3

Answer [1]

- (b) Find the sum of the interior angles of the nonagon.

$$(9 - 2) \times 180^\circ = 1260^\circ$$

Answer° [1]

- (c) Hence, show that $y = -2x + 420$.

$$\begin{aligned} \text{Use } 6x^\circ + 3y^\circ &= 1260^\circ \\ y &= -2x + 420 \quad (\text{shown}) \end{aligned}$$

[1]

- (d) Given that $x : y = 13 : 16$, find x .

$$\frac{x}{y} = \frac{13}{16}$$

$$y = \frac{16}{13}x \text{ ----- (1)}$$

Sub (1) into equation into $y = -2x + 420$

$$-2x + 420 = \frac{16}{13}x$$

$$-26x + 5460 = 16x$$

$$42x = 5460$$

$$x = 130$$

Answer $x = \dots\dots\dots$ [2]

18. (a) Ellie went for a school exchange in the United Kingdom. She changed 250 Singapore dollars (\$) to UK pounds (£). She spent £100 during her trip and converted the remaining pounds back to Singapore dollars.

Exchange Rate	Buying Rate for One Singapore Dollar (\$)	Selling Rate for One Singapore Dollar (\$)
UK Pounds (£)	0.5746	0.5830

Using the exchange rate given, calculate how much Singapore dollars she had left. Give your answer to the nearest dollar.

Selling rate of UK pounds: SGD 1 = £ 0.5830
 Conversion of SGD250 to pounds
 = \$250 × 0.580
 = £ 145.75 (2 d.p.)

Buying rate of UK pounds: SGD 1 = £ 0.5746
 Conversion of pounds to SGD
 = \$ [(145.75 - 100) ÷ 0.5746]
 = \$79.62
 = \$80 (to the nearest dollar).

Answer \$ [2]

- (b) Jane intends to borrow \$88000 for her business venture and is considering the loan schemes by Bank A and Bank B.

Bank A	Bank B
Interest rate of 2.38% per annum compound interest compounded half-yearly	Interest rate of 3.28% per annum simple interest

If Jane wishes to take a loan for a period of 5 years, determine which bank should she borrow from.

Explain your answer with mathematical calculations.

	Bank A	Bank B
Total amount to repay	Total $= \$88000 \left(1 + \frac{2.38}{100}\right)^{5 \times 2}$ $= 88000 \left(1 + \frac{1.19}{100}\right)^{10}$ $= \$99050.95$ (2 d p.) [M1]	Total $= 88000 + 88000 \left(\frac{3.28}{100} \times 5\right)$ $= 88000 + 14432$ $= \$102432$ [M1]

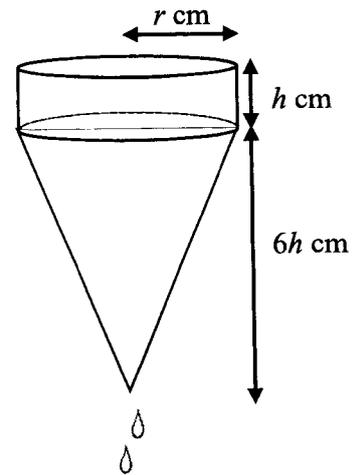
Jane should borrow from Bank A as the total amount she has to repay is \$99050.95 which is less than the total amount of \$102432 from Bank B.

[3]

[Turn over

19. A water tank is formed from a cylinder and a cone.
 The cylinder has radius r cm and height h cm.
 The cone has a base radius of r cm and height $6h$ cm.

The water tank is filled with water to the brim of the tank.
 Water is drained out at a constant rate and the tank is completely emptied in 60 minutes.



- (a) Find the time taken, in minutes, for the water in the cylinder to drain completely.

Answer

$$\text{Volume of cylinder} = \pi r^2 h$$

$$\text{Volume of cone} = \frac{1}{3} \pi r^2 6h = 2\pi r^2 h$$

Time for water in the cylinder to leak completely

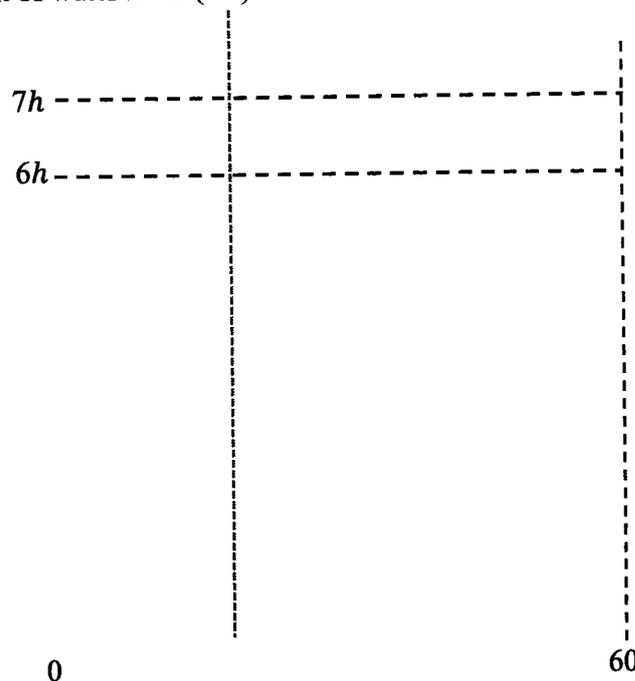
$$= \frac{\pi r^2 h}{3\pi r^2 h} \times 60 = 20 \text{ minutes}$$

Answermins [2]

- (b) On the grid provided, sketch the graph of the height of the water level in the tank against the time taken (in min).

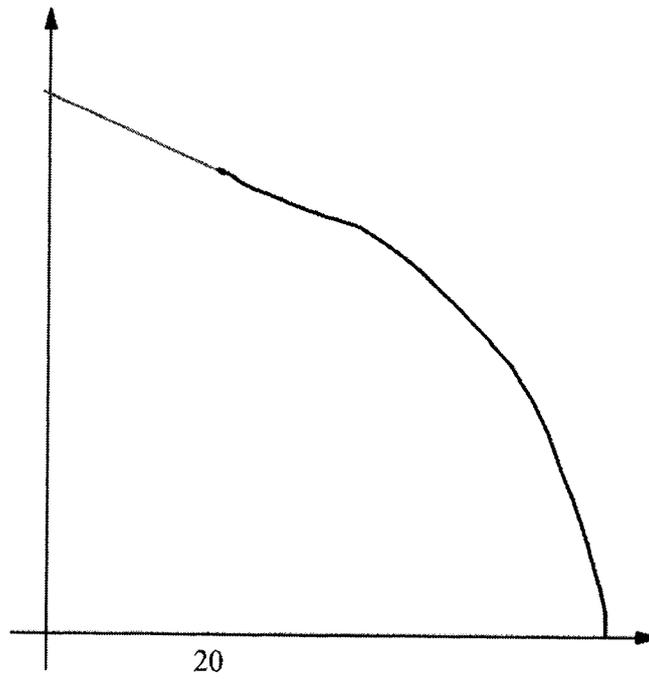
Answer

Height of water level (cm)



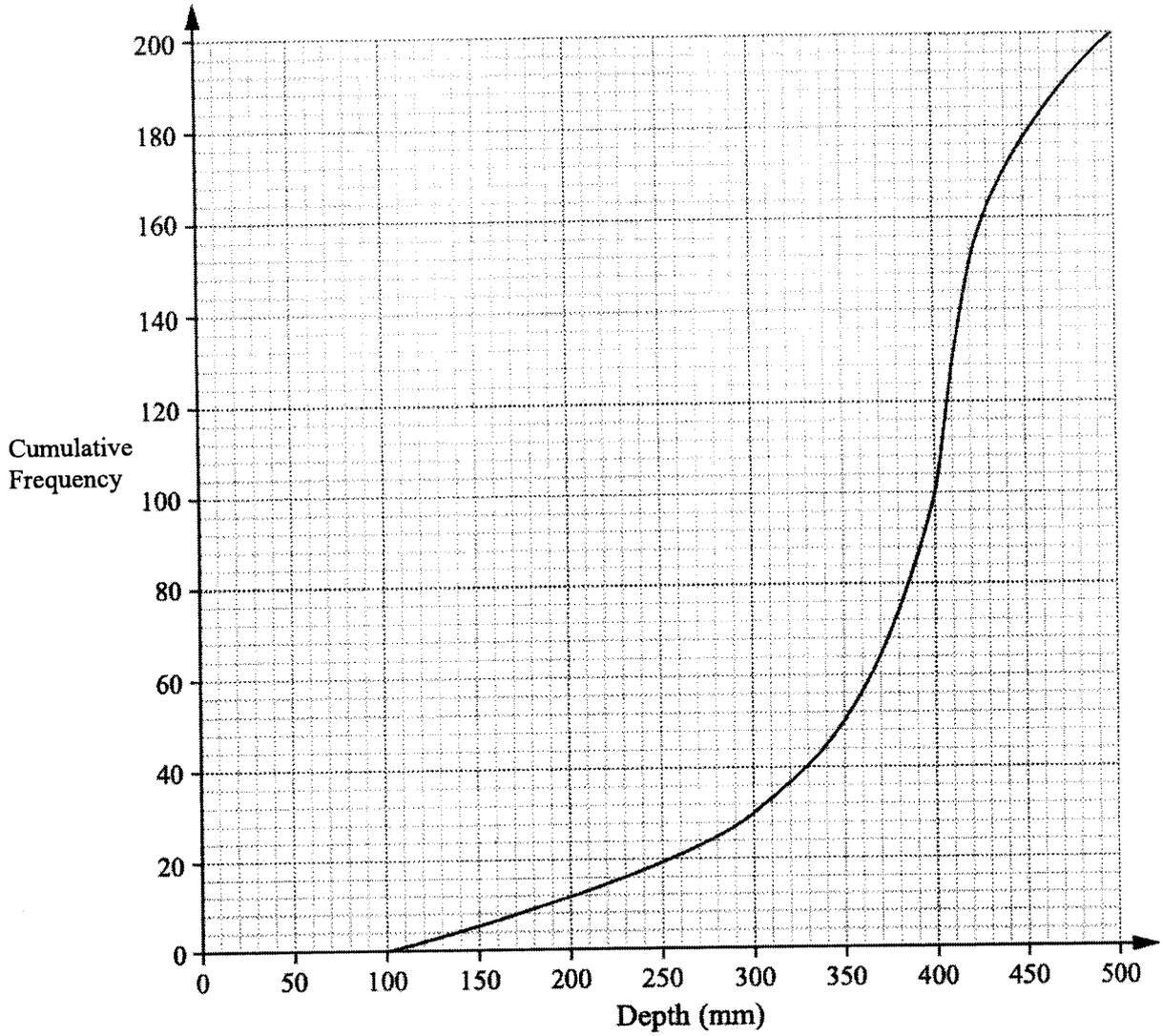
[2]

21



20. The cumulative frequency curve shows the distribution of the depth, d mm, of a canal measured daily for a period of 200 days.

[Turn over



(a) Use the curve to estimate

(i) the median depth of the canal,
Median = 400 mm.

Answer mm [1]

(ii) the interquartile range of the depth ,
IQR
= 420 – 350
= 70 mm

Answer mm [1]

(iii) the depth at the 30th percentile,
Reading at 60 days = 365 mm

Answer mm [1]

(iv) the number of days when the depth of the canal was at least 450 mm.

$$\begin{aligned}
&\text{Number of days when the depth of the canal was at least 450 mm} \\
&= 200 - 178 \\
&= 22
\end{aligned}$$

Answer days [1]

(b) The information on the cumulative frequency curve is shown in this table.

Depth (mm)	$100 < d \leq 200$	$200 < d \leq 300$	$300 < d \leq 400$	$400 < d \leq 500$
No. of days	12	m	n	100

(i) Find the values of m and n .

$$\begin{aligned}
m &= 30 - 12 = 18 \\
n &= 70
\end{aligned}$$

Answer $m = \dots\dots\dots n = \dots\dots\dots$ [1]

(ii) Calculate an estimate of the mean depth of the canal.

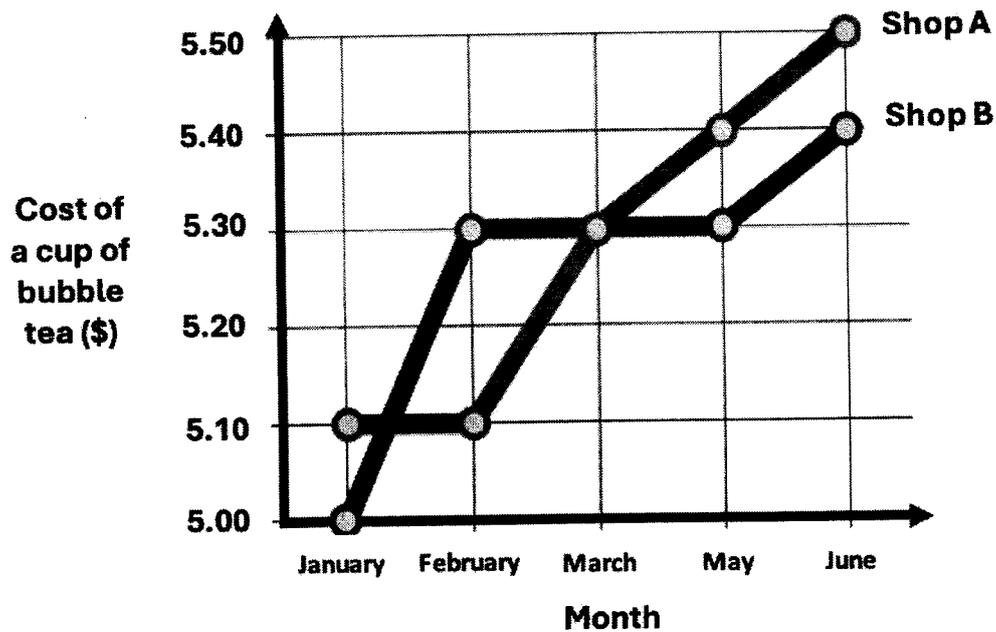
$$\begin{aligned}
&\text{Estimated mean} \\
&= \frac{(150 \times 12) + (250 \times 18) + (350 \times 70) + (450 \times 100)}{200} \\
&= \frac{1800 + 4500 + 24500 + 45000}{200} \\
&= \frac{77000}{200} \\
&= 379 \text{ mm}
\end{aligned}$$

Answermm [1]

21.

[Turn over

Cost of Bubble Tea has Risen Drastically



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

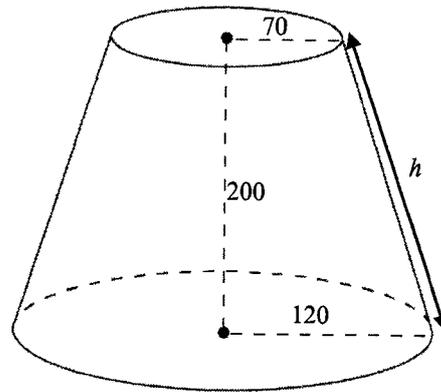
Answer

Possible answers:

- (i) The vertical axis did not start from zero. This exaggerates the differences between the price difference of \$0.10.
- (ii) The title is biased. This misguides the readers to think that the price of bubble tea has increased significantly and does not allow readers to make their own judgement.
- (iii) Information from the month of April is missing. The reader may assume that the price of bubble tea for Shop B has remained constant compared to the the price of bubble tea for Shop A which has increased.

[2]

25



The diagram shows a lampshade in the shape of a frustum.
 The bottom radius of the frustum is 120 mm.
 The top radius of the frustum is 70 mm.
 The vertical height of the frustum is 200 mm.

- (a) Find the slant height h of the frustum.

Answer

$$\begin{aligned}
 &\text{Slant height} \\
 &= \sqrt{(120 - 70)^2 + 200^2} \\
 &\text{use of Pythagoras' Theorem} \\
 &= \sqrt{42500} \\
 &= 206.15 \\
 &= 206 \text{ mm (3 significant figures)}
 \end{aligned}$$

Answer mm [2]

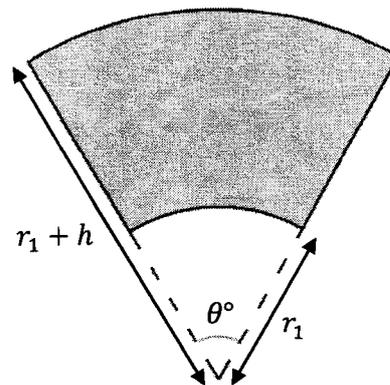
[Turn over

- (b) The shaded area represents the net of the lampshade.

The radius of the inner arc is r_1 mm.

The radius of the outer arc is $(r_1 + h)$ mm.

Show that the radius of the inner arc is 288.6 mm.



Answer

$$\frac{\theta}{360^\circ} \times 2\pi r_1 = 2\pi(70) \quad \text{--- (1)}$$

$$\frac{\theta}{360^\circ} \times 2\pi(r_1 + h) = 2\pi(120) \quad \text{--- (2)}$$

From (1),

$$\frac{\theta}{360^\circ} = \frac{2\pi(70)}{2\pi r_1}$$

$$\frac{\theta}{360^\circ} = \frac{70}{r_1}$$

From (2),

$$\frac{\theta}{360^\circ} = \frac{2\pi(120)}{2\pi(r_1 + h)}$$

$$\frac{\theta}{360^\circ} = \frac{120}{r_1 + h}$$

Hence,

$$\frac{70}{r_1} = \frac{120}{r_1 + \sqrt{42500}}$$

$$r_1 = 70\sqrt{17} = 288.6 \text{ mm (shown)}$$

[3]

- (c) Hence, find angle θ .

Answer

$$\begin{aligned} \theta &= \frac{2\pi(70) \times 360}{2\pi(70\sqrt{17})} = 87.312^\circ \\ &= 87.3^\circ \text{ (1 d.p.)} \end{aligned}$$

$$\text{Answer } \theta^\circ = \dots\dots\dots^\circ \quad [1]$$

- (d) Calculate the amount of material required to make the lampshade, leaving your answer in square centimeters.

Answer

Total surface area

$$= \frac{87.312^\circ}{360^\circ} \times \pi(70\sqrt{17} + 50\sqrt{17})^2$$

$$- \frac{87.312^\circ}{360^\circ} \times \pi(70\sqrt{17})^2$$

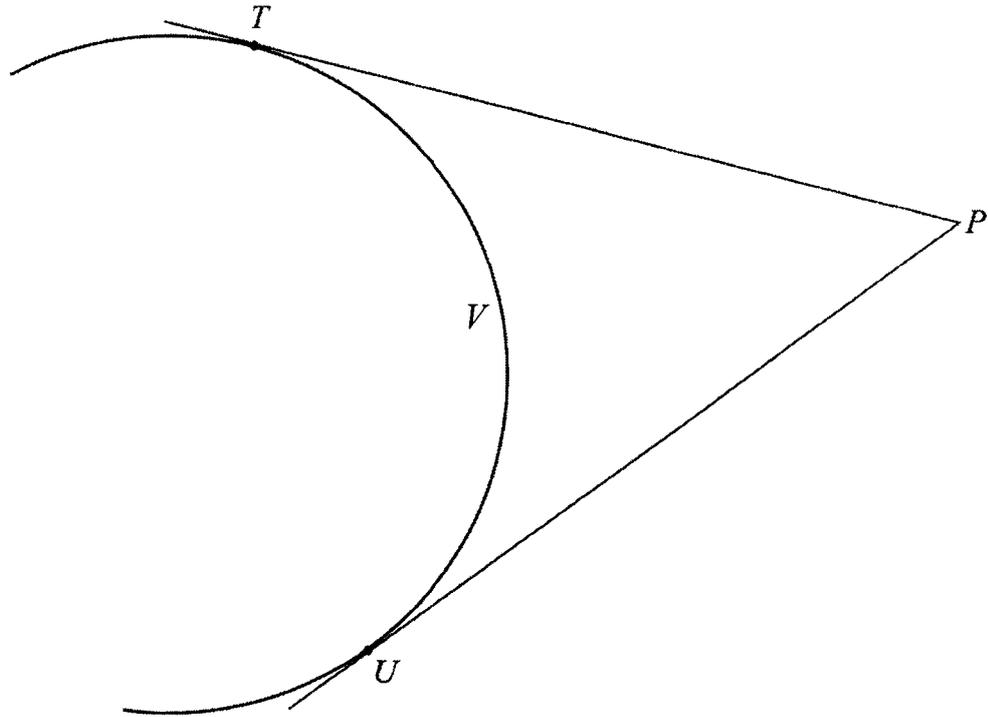
$$= 123053.4615 \text{ mm}^2$$

$$= 1230 \text{ cm}^2 \text{ (3 significant figures)}$$

$$\text{Answer } \dots\dots\dots \text{ cm}^2 \quad [2]$$

23. The diagram shows a plot of land, $PTVU$, on a campsite.

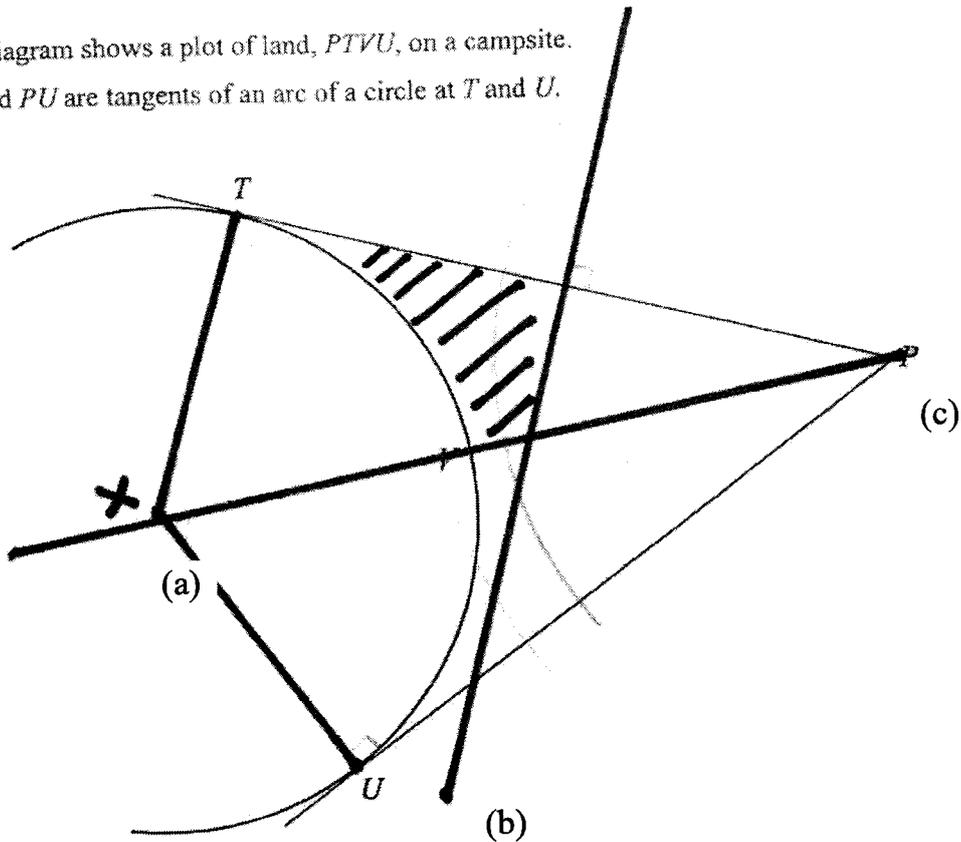
PT and PU are tangents of an arc of a circle at T and U .



- (a) Find the center of the circle and label it X . [1]
- (b) Construct the perpendicular bisector of TP . [1]
- (c) Construct the angle bisector of angle TPU . [1]
- (d) A lookout tower is to be built on the campsite $PTVU$, nearer to T than P and nearer to PT than PU .

Shade the region where the lookout tower is to be built. [1]

23. The diagram shows a plot of land, $PTVU$, on a campsite. PT and PU are tangents of an arc of a circle at T and U .



- (a) Find the center of the circle and label it X .
- Construction of perpendicular lines at T and U
 - Labelling of intersection of perpendicular lines at T and U as point X
- (b) Construction of the perpendicular bisector of TP with working
- (c) Construction of the angle bisector of angle TPU with working
- (d) A lookout tower is to be built on the campsite $PTVU$, nearer to T than P and nearer to PT than PU .
Shading of the region where the lookout tower is to be built.

----- End of Paper -----



SECONDARY 4
2025 PRELIMINARY EXAMINATION

MATHEMATICS
Paper 2

4052/02

27 August 2025 (Wednesday)

2 hour 15 minutes

CANDIDATE
NAME

Student's version

CLASS

4	-		
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INDEX NUMBER

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Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number in the spaces above.

Write in dark blue or black pen in the space provided for each question.

You may use a HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any question, it must be shown in the space below the question.

Omission of essential working will result in loss of marks.

The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142.

For Examiner's Use		
Q1	10	
Q2	12	
Q3	10	
Q4	11	
Q5	10	
Q6	9	
Q7	10	
Q8	8	
Q9	10	
Total	90	

This document consists of **24** printed pages including the cover page.

[Turn over

*Mathematical Formulae**Compound Interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer all the questions.

N1 (N1.7 & N1.8) : Numbers and Operations [10]

N2 (N2.2 & N2.4) : Ratio and Proportion

N3 (N3.4) : Percentage

1. (a) (i) The number of electric cars produced in country *Y* by company *X* in 2015 was 3 857 000. Write this number in standard form.

Answer

$$3.857 \times 10^6$$

[1]

- (ii) In 2005, 2.96 million electric cars were produced in country *Y*. Calculate the percentage increase in the car production from 2005 to 2015.

Answer

$$\begin{aligned} &\text{Percentage increase} \\ &= \frac{3.857 \times 10^6 - 2.96 \times 10^6}{2.96 \times 10^6} \times 100\% \end{aligned}$$

Show computation for percentage increase

$$= 30.3\% \quad (3 \text{ significant figures})$$

- (iii) In 2015, 84.7% of all cars produced in country *Y* were non-electric cars. Calculate the total number of cars produced in 2015, giving your answer to the nearest million.

Answer

$$\begin{aligned} &\text{Total cars in 2015} \\ &= \frac{3.857 \times 10^6}{100 - 84.7} \times 100 \\ &= 2.5209 \times 10^7 \\ &= 25 \text{ million cars (nearest million)} \\ &\text{Or } 25\,000\,000 \end{aligned}$$

- (iv) In 2015, there were 420 cars per 1000 people in country *Y*. Estimate the population of the country in 2015. Give your answer in standard form correct to **three significant figures**.

Answer

$$\begin{aligned} &\text{Population} \\ &= \frac{2.5209 \times 10^7}{420} \times 1000 \\ &= 6.00 \times 10^7 \quad (3 \text{ significant figures}) \end{aligned}$$

[Turn over

- (b) A design team creates a scale model for a museum exhibit.
The diameter of an asteroid is 253 km.
In a scale model, the diameter of the asteroid is 11 m.

- (i) Find the scale used for the model.
Give your answer in the form $1 : n$.

Answer

$$11 \text{ m} : 253 \text{ km}$$

$$1 \text{ m} : 23 \text{ km}$$

$$1 : 23\,000$$

Be careful with conversion
to km.

- (ii) Drawn to the same scale as the asteroid, the diameter of the asteroid's moon is 1.2 m.
Find the **actual surface area of the moon**, leaving your answer in terms of π .
Assume that the moon is spherical.

Answer

$$1 \text{ m} : 23 \text{ km}$$

$$1.2 \text{ m} : 27.6 \text{ km}$$

Actual surface area

$$= 4\pi \left(\frac{27.6}{2}\right)^2$$

$$= 761.76\pi \text{ km}^2$$

Surface area of a sphere =
 $4\pi r^2$
From page 2

N7 (7.6, 7.7, 7.8) : Equations and Inequalities [12]

2. (a) Solve $7(x - 2) - 3(2x - 1) = 8(3x + 5) - 6$.

Answer

$$7x - 14 - 6x + 3 = 24x + 40 - 6$$

$$x = -\frac{45}{23} = -1\frac{22}{23}$$

Based on context,
Answer must be
exact and shown as
mixed number

- (b) Solve the inequality $-3x + 4 < -2x - 6$.

Answer

$$\begin{aligned} -x &< -10 \\ x &> 10 \end{aligned}$$

- (c) It is given that

$$p = \frac{r^2q - 2}{q - r^2}.$$

- (i) Find p when $r = \frac{1}{2}$ and $q = -1$.

$$p = \frac{\left(\frac{1}{2}\right)^2 (-1) - 2}{(-1) - \left(\frac{1}{2}\right)^2} = \frac{-9/4}{-5/4} = \frac{9}{5} = 1\frac{4}{5}$$

Or 1.8

Show substitution of
values

- (ii) Express r in terms of p and q .

$$\begin{aligned} p(q - r^2) &= r^2q - 2 \\ pq - pr^2 &= r^2q - 2 \\ pr^2 + r^2q &= pq + 2 \\ r^2(p + q) &= pq + 2 \\ r^2 &= \frac{pq + 2}{p + q} \end{aligned}$$

This is a quadratic
relationship.

$$r = \pm \sqrt{\frac{pq + 2}{p + q}}$$

6

(d) Solve the equation $\frac{x+3}{2x^2-8x+6} + \frac{5x}{3-x} = \frac{2}{x-1}$.

Give your solutions correct to two decimal places.

Answer

$$\frac{x+3}{2(x-3)(x-1)} + \frac{5x}{3-x} = \frac{2}{x-1}$$

$$\frac{x+3}{2(x-3)(x-1)} - \frac{5x}{x-3} = \frac{2}{x-1}$$

$$\frac{x+3}{2(x-3)(x-1)} - \frac{5x(2)(x-1)}{5x(2)(x-1)} = \frac{2(2)(x-3)}{2(x-3)(x-1)}$$

$$x+3-10x(x-1) = 4(x-3)$$

$$10x^2 - 7x - 15 = 0$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(10)(-15)}}{2(10)}$$

$$x = 1.62 \quad \text{or} \quad -0.92$$

$$(2 \text{ d.p.}) \quad (2 \text{ d.p.})$$

Note:

$$2x^2 - 8x + 6$$

$$= 2(x-3)(x-1)$$

Always check back when using the calculator to factorise.

$$3-x = -(x-3)$$

N5: finding the value of an unknown quantity in a given formula

N7: solving fractional equations that can be reduced to quadratic equations, formulating equations to solve problems [10]

3. Two water taps, X and Y , together can fill a tank in 6 hours.

Tap X takes x hours to fill the tank alone.

Tap Y takes 25 hours less than the Tap X to fill the tank alone.

- (a) Write down an expression, in terms of x , for the fraction of the tank that Tap X will fill up in one hour.

$$\frac{1}{x}$$

- (b) Write an expression, in terms of x , for the fraction of the tank that Tap Y will fill up in one hour.

$$\frac{1}{x - 25}$$

- (c) Write down an equation to represent the rate of filling the tank by the 2 taps and show that it reduces to $x^2 - 37x + 150 = 0$.

$$\frac{1}{x} + \frac{1}{x - 25} = \frac{1}{6}$$

Equation that represents given context.

$$\frac{x + x - 25}{x(x - 25)} = \frac{1}{6}$$

$$6(2x - 25) = x(x - 25)$$

$$12x - 150 = x^2 - 25x$$

$$x^2 - 25x - 12x + 150 = 0$$

$$x^2 - 37x + 150 = 0 \text{ (shown)}$$

8

- (d) Solve the equation $x^2 - 37x + 150 = 0$.

$$x^2 - 37x + 150 = 0$$

$$x = \frac{-(-37) \pm \sqrt{(-37)^2 - 4(1)(150)}}{2(1)}$$

Must show
substitution of values.

$$x = 32.365 \text{ or } x = 4.634$$

$$x = 32.4 \text{ or } x = 4.63 \text{ (3 s.f.)}$$

- (e) Calculate how long it would take to fill the tank from empty using Tap Y.
Give your answer in hours and minutes, correct to the nearest minute.

Reject $x = 4.63$ as the time taken,
 $x - 25$, cannot be negative.

$$\begin{aligned} \text{Time Tap Y takes to fill up the tank completely} \\ &= 32.365 - 25 \\ &= 7.365 \end{aligned}$$

Important to test both
values and justify any
rejection of solution.

$$= 7 \text{ hours } 22 \text{ minutes (nearest minute)}$$

N6 (6.10) : Functions and Graphs [11]
 N7 (N7.2) : Equations and Inequalities

4. (a) Complete the table of values for $y = \frac{x^2}{7} + \frac{2}{x} - 5$.
 Values are given to one decimal place where appropriate.

x	0.2	0.5	1	2	3	4	5	6
y		-1.0	-2.9	-3.4	-3.0	-2.2	-1.0	0.5

[1]

Answer

5.0 (1 d.p.)

- (b) On the grid opposite, draw the graph of $y = \frac{x^2}{7} + \frac{2}{x} - 5$ for $0 < x \leq 6$. [2]

Answer

[points plotted correctly]

[smoothness of curve **and** label]

- (c) Use your graph to write down an inequality in x to describe the range of values where $y < -2$.

Answer $0.7 < x < 4.2$ (± 0.1)

- (d) (i) On the same grid, draw the graph of $2y + 4x = 9$ for $0 < x \leq 6$. [2]

Answer

[at least 3 plotted points]

[graph **and** label]

- (ii) Write down the x -coordinates of the points where the line intersects the curve.

Answer0.2, 3.55 (± 0.1)

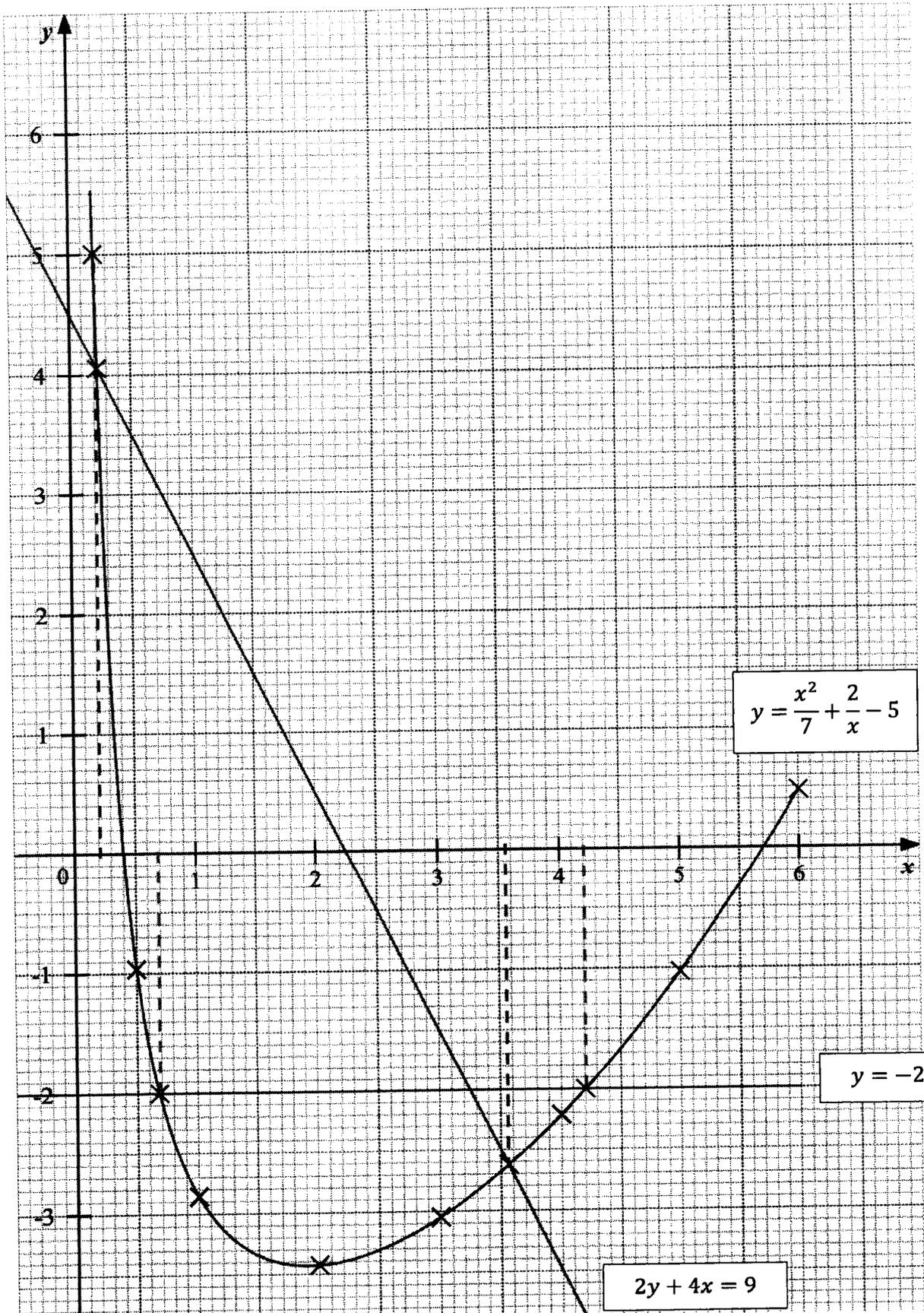
- (iii) These values of x are the solutions of the equation $2x^3 + Ax^2 - Bx + 28 = 0$.
 Find the value of A and the value of B .

$$\frac{x^2}{7} + \frac{2}{x} - 5 = -2x + \frac{9}{2} \quad \text{[equate both functions]}$$

$$2x^3 + 28 - 70x = -28x^2 + 63x$$

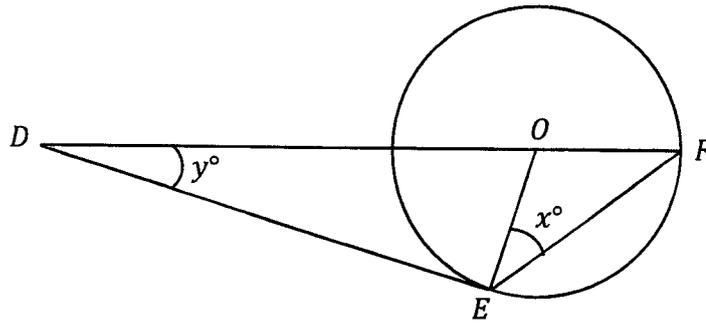
$$2x^3 + 28x^2 - 133x + 28 = 0$$

$$A = 28, \quad B = 133$$



G3 (3.6, 3.8, 3.9) : Properties of Circle [10]

5. (a)



E and F are points on the circle, centre O .
 DOF is a straight line and DE is a tangent at E .
 Angle $FEO = x^\circ$ and angle $EDO = y^\circ$.

- (i) Show that $y = 90 - 2x$.
 State all the reasons clearly.

Answer

$\angle OFE = x^\circ$ (Base angles of isosceles triangle since OE and OF are radii of same circle.)

$\angle OED = 90^\circ$ (Tangent to a circle)

$y + 90 + x + x = 180$ (Angles sum of triangle)

$\therefore y = 90 - 2x$ (Shown)

- (ii) Given that $y + x = 60$, find angle EOD .

$$y = 90 - 2x \dots(1)$$

$$y + x = 60. \dots(2)$$

Substitute (1) into (2) :

$$90 - 2x + x = 60$$

$$x = 30, y = 30$$

$$\therefore \angle EOD = 90^\circ - y^\circ$$

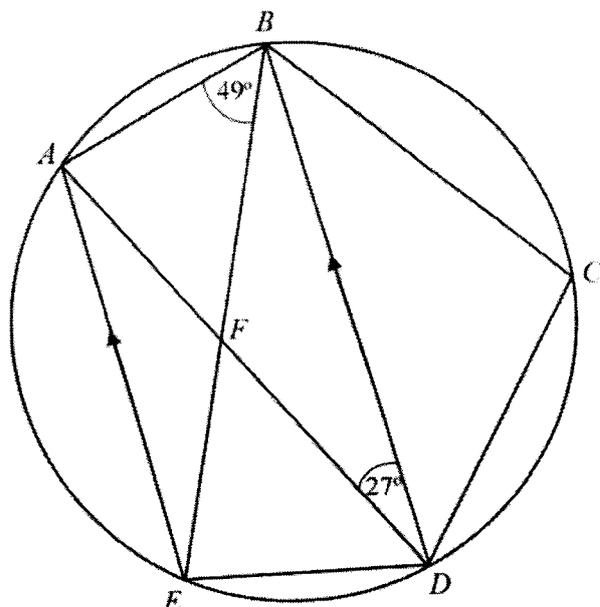
$$\therefore \angle EOD = 90^\circ - 30^\circ = 60^\circ$$

- (b) The diagram shows a circle that passes through A, B, C, D and E .

[Turn over

12

AFD and BFE are straight lines.
 EA and DB are parallel.
 Angle $ABE = 49^\circ$ and angle $ADB = 27^\circ$.



Find, give a reason for each step of your working,

(i) angle AFE ,

$$\begin{aligned} \angle AEB &= \angle ADB = 27^\circ \\ &\text{(Angles in the same segment)} \\ \angle EAD &= \angle ADB = 27^\circ \\ &\text{(Alternate angles, } BD \text{ parallel } AE) \\ \angle AFE + \angle AEB + \angle EAD &= 180^\circ \\ &\text{(Angles sum of triangle)} \end{aligned}$$

$$\angle AFE = 126^\circ$$

(ii) angle BCD .

$$\begin{aligned} \angle AFE &= \angle DAB + \angle ABE \text{ (Exterior angle)} \\ \angle DAB &= 126^\circ - 49^\circ = 77^\circ \end{aligned}$$

$$\begin{aligned} &\text{(Opposite angles of a cyclic quadrilateral)} \\ \angle BCD &= 180^\circ - \angle DAB \\ &= 180^\circ - 77^\circ \\ &= 103^\circ \end{aligned}$$

G5 (5.1) : Mensuration [4]

G4 (4.3) : Trigonometry and Pythagoras Theorem

G6 (6.1) : finding the gradient of a straight line given the coordinates of two points on it [5]

6. (a)

Figure 1

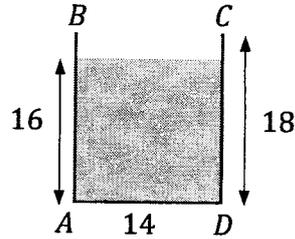
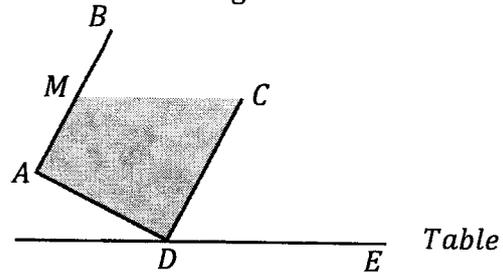


Figure 2



The diagram (Figure 1) shows one side $ABCD$ of a square-based prism. Each side of the base is 14 cm. The height of the container DC is 18 cm. The prism is placed on a table and filled with water until the height of the water reaches 16 cm.

Figure 2 shows the square-based prism tilted until the water level is at MC . During this process of tilting, some water is spilled out.

Given that M is the mid-point of AB , find

- (i) the amount of water that was spilled out from the square-based prism during the tilt.

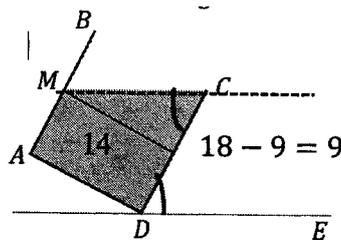
$$\begin{aligned} \text{Amount of water spilled} &= 14^2 \times 16 - \frac{1}{2}(9 + 18) \times 14 \times 14 \\ &= 3136 - 2646 \\ &= 490 \text{ cm}^3 \end{aligned}$$

Context: it is about volume NOT area.

- (ii) angle CDE , the angle between the square-based prism and the table.

$$\angle MCD = \tan^{-1} \left(\frac{14}{18-9} \right) = 57.264^\circ.$$

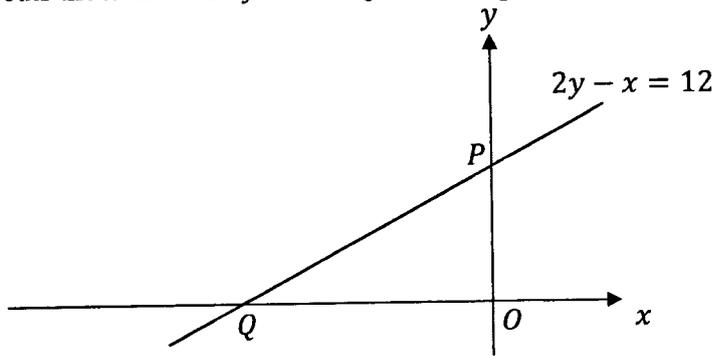
$$\begin{aligned} \angle CDE &= \angle MCD = 57.3^\circ \text{ (1 d.p.)} \\ &\text{(Alternate angles, } MC \text{ parallel } DE.) \end{aligned}$$



Context: water level is parallel to the surface of the table.

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- (b) The equation of the line, L , is $2y - x = 12$.
It cuts the x -axis and y -axis at Q and P respectively.



- (i) Write down the gradient of PQ ,
 $2y - x = 12$
 $y = \frac{1}{2}x + 6$

Is gradient equivalent to tangent in this context?

$$\text{Gradient of } PQ = \frac{1}{2}$$

- (ii) A point K lies on the line L , and is equidistant from x -axis and y -axis.
Find the coordinates of K .

2 possibilities

<p>Since point is equidistant from x-axis and y-axis, therefore $x = -y$. From</p> $2y - x = 12$ $2y + y = 12$ $y = 4, x = -4$ <p>$K = (-4, 4)$</p>	<p>Since point is equidistant from x-axis and y-axis, therefore $x = y$. From</p> $2y - x = 12$ $2y - y = 12$ $y = 12, x = 12$ <p>$K = (12, 12)$</p>
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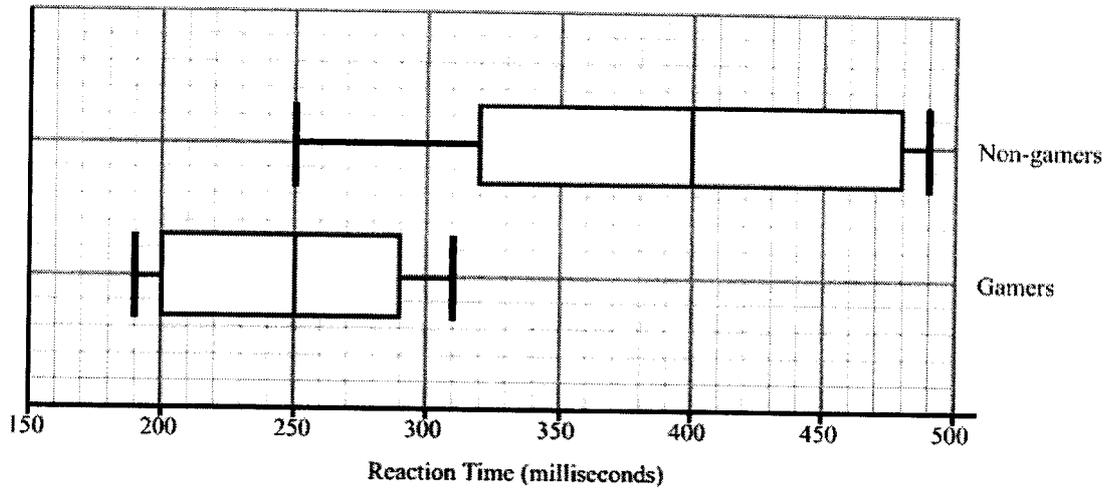
- (iii) Find acute angle QPO .

$$\tan \angle QPO = \frac{OQ}{OP} = \frac{12}{6} = 2$$

$$\text{acute } \angle QPO = 63.4^\circ (\text{to 1 d.p.})$$

S1(S1.11) : Data Handling and Analysis [4]
S2 (S2.3) : Probability [6]

7. (a) A study records the reaction times of gamers and non-gamers.
The box-and-whisker plots show the distributions of the results.



- (i) There are 15 non-gamers who had a reaction time slower than 480 milliseconds.

Work out the number of non-gamers in the study.

Answer

$$15 \times \frac{4}{3} = 20$$

- (ii) Make a comment comparing the averages and a comment comparing the distributions of the times taken by the gamers and non-gamers.
Use figures to support your answers.

Answer

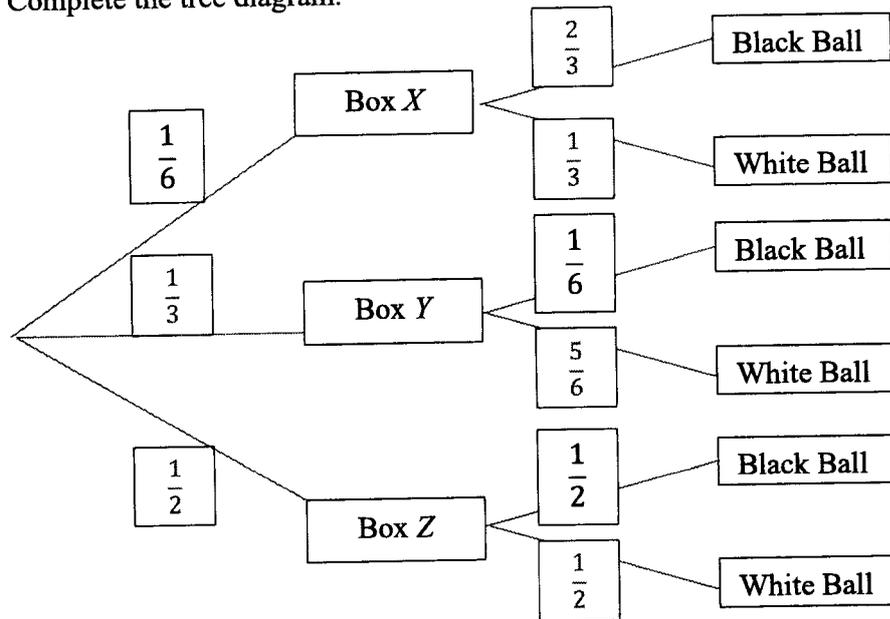
Gamers have a lower *median* of 250 milliseconds compared to 400 milliseconds for non-gamers. Hence, gamers generally had faster reaction times.

The *interquartile range* for gamers of $290 - 200 = 90$ milliseconds is smaller than that for non-gamers of $480 - 320 = 160$ milliseconds. Hence, non-gamers had a wider spread in reaction times, showing less consistency.

- (b) Three boxes, X , Y and Z contain the following balls.
 Box X : 2 black balls and 1 white ball.
 Box Y : 1 black ball and 5 white balls.
 Box Z : 2 black balls and 2 white balls.
 A ball is picked at random from a box.
 The box is chosen by throwing a die and applying the following rules.
1. If the outcome on the die is 6, box X will be chosen.
 2. If the outcome on the die is a 4 or a 5, box Y will be chosen.
 3. If the outcome on the die is a 1, a 2 or a 3, box Z will be chosen.

If a black ball is obtained, a prize of \$10 is awarded.
 There will be a \$2 fee to play the game.

- (i) Complete the tree diagram.



- (ii) Find the probability of winning the \$10 prize.

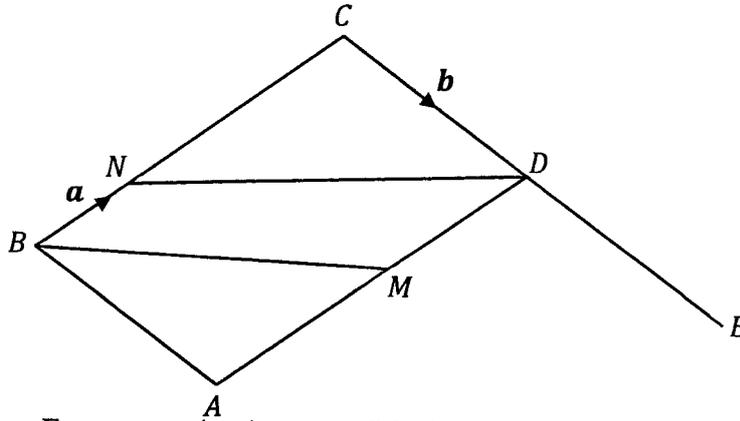
$$\frac{1}{6} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{2} = \frac{5}{12}$$

- (iii) Find the amount gained or lost after playing 12 games.

$$\begin{aligned} &5 \text{ wins in 12 games gains } \$50 \\ &\text{Fee incurred} = \$2 (12) = \$24 \\ &\text{Amount earned} = \$50 - \$24 \\ &= \$26 \text{ gained} \end{aligned}$$

G7 (7.2, 7.4, 7.6) : Vectors in 2-dimensions [9]

8. $ABCD$ is a parallelogram, and E lies on CD produced such that $CD:CE = 1:2$.
 M is the midpoint of AD .
 N is a point on BC such that $BN:BC = 1:4$.
 $\vec{BN} = \mathbf{a}$ and $\vec{CD} = \mathbf{b}$.



- (a) Express, as simply as possible, in terms of \mathbf{a} and/or \mathbf{b} ,

(i) \vec{AM} ,

$$\vec{AM} = \frac{1}{2}\vec{AD} = \frac{1}{2}\vec{BC} = \frac{1}{2}(4\mathbf{a})$$

(ABCD is a parallelogram)

$$\vec{AM} = 2\mathbf{a}$$

(ii) \vec{BM} ,

$$\vec{BM} = \vec{BA} + \vec{AM}$$

$$\vec{BM} = 2\mathbf{a} + \mathbf{b}$$

(iii) \vec{BE} .

$$\vec{BE} = \vec{BC} + \vec{CE}$$

$$\vec{BE} = 4\mathbf{a} + 2\mathbf{b}$$

Correct vector notation is crucial because it ensures clarity, precision, and the accurate representation of quantities with both magnitude and direction, preventing confusion between vectors and scalars and enabling correct calculations and understanding in physics, mathematics, and computer graphics.

- (b) Explain why B , M and E lie on a straight line.

Answer

$$\vec{BM} = 2\mathbf{a} + \mathbf{b}$$

$$\vec{BE} = 4\mathbf{a} + 2\mathbf{b} = 2(2\mathbf{a} + \mathbf{b}) = 2\vec{BM}$$

Since $\vec{BE} = 2\vec{BM}$ and B is a **common point**, B , M and E lie on a straight line.
 (Shown)

[2]

A proper statement and logical justification is important for such questions.

(c) Find the numerical value of

(i) $\frac{\text{Area of triangle } AMB}{\text{Area of triangle } DCN}$,

ABCD is a parallelogram, let h be the perpendicular height between AD and BC.

$$\begin{aligned} \frac{\text{Area of triangle } AMB}{\text{Area of triangle } DCN} &= \frac{\frac{1}{2} \times h \times AM}{\frac{1}{2} \times h \times NC} \\ &= \frac{AM}{NC} = \frac{2}{3} \end{aligned}$$

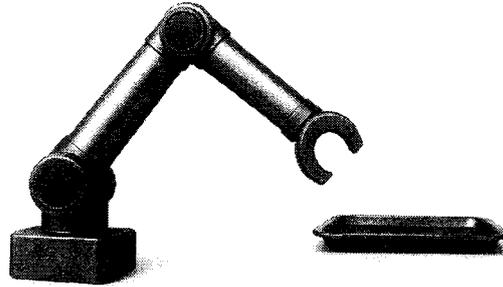
(ii) $\frac{\text{Area of triangle } EDM}{\text{Area of } DCBM}$.

$$\frac{\text{Area of triangle } EDM}{\text{Area of triangle } ECB} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\frac{\text{Area of triangle } EDM}{\text{Area of } DCBM} = \frac{1}{3}$$

PRWC [10]

9. A packing company uses robotic arms to pick up trays from a moving conveyor belt.



A robotic arm consists of two segments connected by joints.

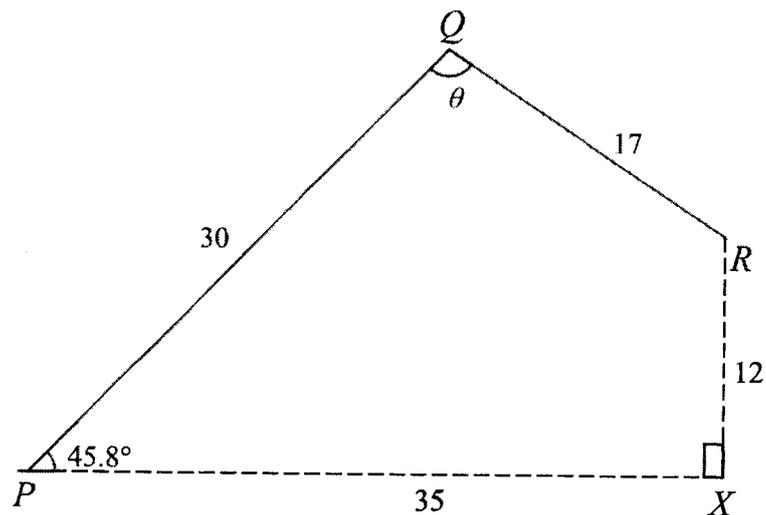
The length of the upper arm PQ is 30 cm.

At P , the shoulder joint is fixed at an angle of 45.8° .

The length of the forearm QR is 17 cm.

At Q , the elbow joint rotates at an angle θ .

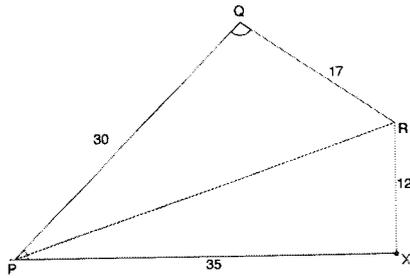
The robotic arm picked a tray at point R , which lies 35 cm horizontally and 12 cm vertically above X .



[Turn over

- (a) Calculate angle θ , that allows for the arm to pick the tray at R .

Answer



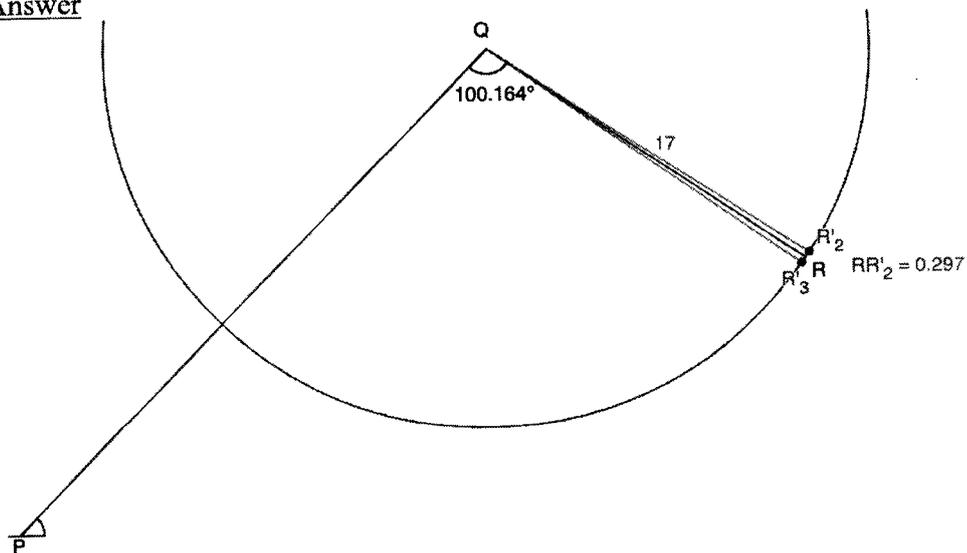
$$PR = \sqrt{35^2 + 12^2} = 37 \text{ cm}$$

$$\angle PQR = \cos^{-1} \left[\frac{30^2 + 17^2 - 37^2}{2(30)(17)} \right]$$

$$\theta = 100.2^\circ \text{ (1 d.p.)}$$

- (b) Due to calibration errors, there may be a 1° change in θ . Show that a 1° change in θ will lead to 0.297 cm difference from the intended position of R .

Answer



[1]

Arc length

$$= \frac{1^\circ}{360^\circ} \times 2\pi(17)$$

$$= 0.297 \text{ cm (3 significant figures)}$$

- (c) The company is deciding whether to adopt a new system that uses camera feedback to calculate the joint angle θ in real time.

They are considering two new systems B and C.

System	Description
System A (Original)	Uses pre-programmed joint angle θ . There is a 40% chance that a 1° error in θ will occur, which causes the arm to not be able to pick up the tray.
System B (Real-time)	Uses real-time camera feedback to recalculate θ for every tray. The angle is always accurate.
System C (Hybrid)	Uses pre-programmed joint angle θ . When a 1° error in θ is detected, it uses real-time camera feedback to recalculate the angle θ .

Additional notes:

- Using the original system, the robot handles 300 trays per hour.
- Each successful tray earns the company \$0.50 revenue.
- A 1° error in θ leads to a miss, as the arm cannot pick up the tray.
- Each missed tray incurs a \$0.30 cost.
- Using the camera feedback introduces a 0.4 second delay per tray.
- There is a fixed monthly cost of \$12 000 for using the camera feedback.
- The company operates 160 hours per month.

Determine which system the company should adopt.
Justify the decision you make and show your calculations clearly.

Answer

	A (Original)	B (Real-time)	C (Hybrid)
Trays/hour	Number of missed trays $= 40\% \times 300$ $= 120$ Number of successful trays $= \frac{60}{100} \times 300$ $= 180$ [M1]	Original time taken for one tray $= \frac{3600}{300} = 12 \text{ s}$ New time taken for each tray (includes delay) $= 12.4 \text{ s}$ Number of successful trays $= \frac{3600}{12.4}$ $= 290$ (nearest whole number) [M1]	Similar to A Successful trays = 180 Total time taken for successful trays $= \frac{3600}{300} \times 180$ $= 2160 \text{ s}$ Includes B Number of trays picked up after recalculation. $= \frac{3600 - 2160}{12.4}$ $= 116$ (nearest whole number) Total number of trays $= 180 + 116$ $= 296$ [M1]
Revenue/hour	$180 \times \$0.50 = \90	$290 \times \$0.50 = \145	$296 \times \$0.50 = \148
Missed cost/hour	$120 \times \$0.30 = \36	\$0	\$0
Camera cost/hour	\$0	$\frac{\$12000}{160} = \75	$\frac{\$12000}{160} = \75
Net profit/hour	$\$90 - \$36 = \$54$	$\$145 - \$75 = \$70$	$\$148 - \$75 = \$73$
Monthly profit (160 hour / month)	$\$54 \times 160 = \8640 [M1]	$\$70 \times 160 = \11200 [M1]	$\$73 \times 160 = \11680 [M1]

Since System C generates the most profit, the company should adopt System C.

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..... [7]

----- End of Paper -----