ST JOSEPH'S INSTITUTION
PRELIMINARY EXAMINATION 2025
(YEAR 4) MA4052 Paper 1 Solutions

Q/N	Solutions
1	$\underline{26450 \le x \le 26549}$
2	$I = \dfrac{50000 \times r \times \frac{q}{12}}{100}$ $= \underline{\dfrac{125rq}{3}}$ or $\underline{41\frac{2}{3}rq}$
3	$\cos \angle BDC = \dfrac{9^2 + 10^2 - 8^2}{2(9)(10)}$ $= \dfrac{13}{20}$ $\underline{\cos \angle BDC = -\dfrac{13}{20}}$
4(a)	$\text{P(walks to school)} = 1 - \dfrac{1}{5}$ $= \underline{\dfrac{4}{5}}$
4(b)	$\text{No. of days} = \dfrac{4}{5}(190)$ $= \underline{\mathbf{152}}$
5	$\dfrac{(2n-2)\times 180}{2n} - \dfrac{(n-2)\times 180}{n} = 20$ $(2n-2)(180) - (2n-4)(180) = 40n$ $360 = 40n$ $n = \underline{\mathbf{9}}$
6(a)	Median mark $= \underline{\mathbf{34}}$
6(b)	Max mark $-17 = 32$

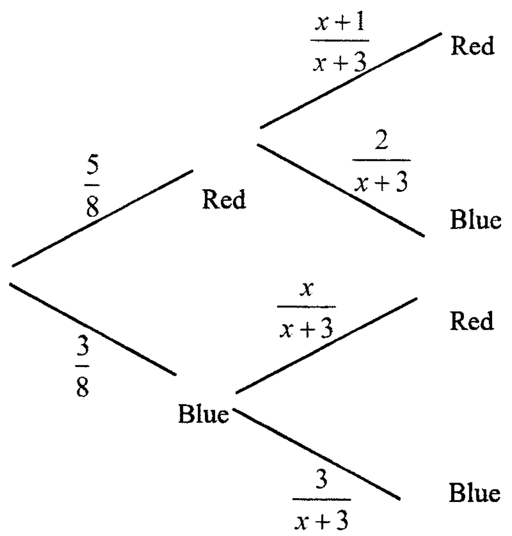
	Max mark $=49$ $\therefore x=\underline{9}$
7(a)	% reduction $=\dfrac{142-130}{142}\times 100\%$ $=\underline{\mathbf{8.45\%}}$ (to 3 sf)
7(b)	The **<u>graph gives the impression</u>** that water consumption in **<u>2021 doubled</u>** compared to 2018/19, **<u>when the actual increase is only about one-third</u>**. This misinterpretation arises because the **<u>vertical axis does not start from zero</u>**, exaggerating the fluctuations and making **the changes appear more significant than they really are**. **<u>OR</u>** It misleads us into **<u>thinking that there is a gradual downward trend in water consumption</u>** from 2024 to 2030, **<u>which might not be the case.</u>**
8	$\left(\dfrac{16m^6}{9q^8}\right)^{-\frac{3}{2}}=\left(\dfrac{9q^8}{16m^6}\right)^{\frac{3}{2}}$ $=\underline{\dfrac{27q^{12}}{64m^9}}$
9	$18=2\times 3^2$ $N=(2\times 3)^6$ $=\underline{\mathbf{46\ 656}}$
10	$\dfrac{1}{2-x}-\dfrac{3x+2}{x^2-7x+10}=\dfrac{-1}{x-2}-\dfrac{3x+2}{(x-2)(x-5)}$ $=\dfrac{-x+5-3x-2}{(x-2)(x-5)}$ $=\underline{\dfrac{-4x+3}{(x-2)(x-5)}}$
11(a)	$T_5=17^2+5^2=\underline{\mathbf{314}}$
11(b)	$T_n=(4n-3)^2+n^2$

	$=16n^2-24n+9+n^2$ $\underline{=17n^2-24n+9}$
12(a)	$x=\underline{9}$
12(b)	$3+x=8+6+2+1$ $\underline{x=14}$
13	Let x be the total distance travelled. $\dfrac{x}{\dfrac{0.35x}{20}+\dfrac{0.65x}{30}}=x\div\left[\dfrac{1.05x+1.3x}{60}\right]$ $=x\div\dfrac{2.35x}{60}$ $=$ **<u>25.5 km/h</u>** (to 3 sf)
14(a)	**Set B** has the larger standard deviation. All values in **<u>Set A</u>** are the same, so there is no spread and the **<u>standard deviation is 0</u>**. **<u>Set B's values are spread out around the mean, so its standard deviation is greater than 0</u>**.
14(b)	**<u>14</u>** *Adding 10 to each score does **not** change the standard deviation, so it remains 7. Multiplying every score by 2 **doubles** the standard deviation.*
15(a)	**<u>6, 9, 12, 15, 18, 21, 24</u>**
15(b)	Probability $=\dfrac{9}{24}$ $=\underline{\dfrac{3}{8}}$
16(a)	$\angle BAD=90^\circ$ (tan $\perp$ rad) $\angle ACB=90^\circ$ ($\angle$ in semicircle) $\angle ACD=90^\circ$ (adj $\angle$ s on a str line) $\angle BAD=\angle ACD=90^\circ$ $\angle BDA=\angle ADC$ (common angle) ΔCDA and ΔADB are similar (AA Similarity Test)
16(b)	$\dfrac{x}{9}=\dfrac{25}{x}$ $x^2=225$ $\underline{x=15}$ $(x>0)$

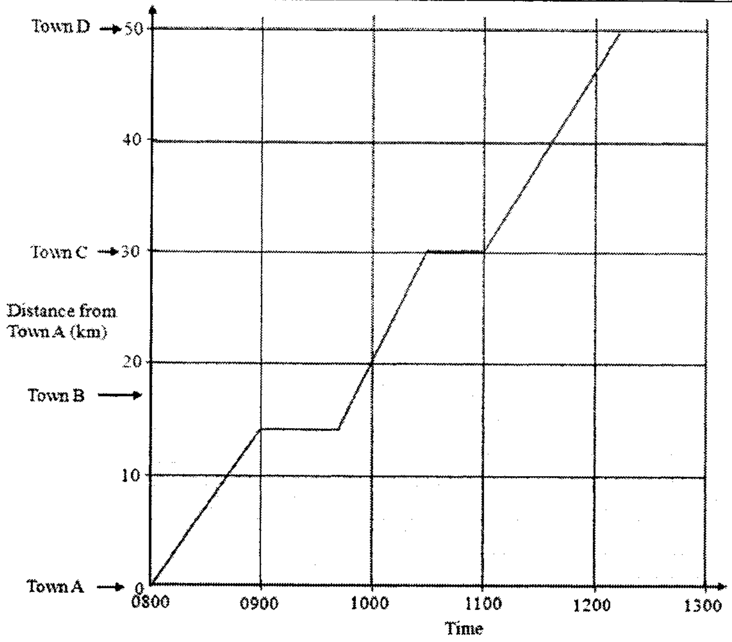
17	$F = kv^2$ $F_{new} = k(v_{new})^2$ $F_{new} = k(0.8v)^2$ Percentage change in $F = \dfrac{k(0.8v)^2 - kv^2}{kv^2} \times 100\%$ $= \underline{-36\%}$
18(a)	$x^2 - 169 = \underline{(x+13)(x-13)}$
18(b)	$1431 = 40^2 - 169$ $= (40+13)(40-13)$ $= (53)\,(27)$ 2 factors are **<u>27</u>** and **<u>53</u>**.
19(a)	6.25 cm^2 rep 1.44 km^2 2.5 cm rep $\sqrt{1.44}$ km $1 : \dfrac{1.2 \times 100000}{2.5}$ **<u>1: 48 000</u>**
19(b)	Length (on the map) $= \dfrac{100x}{48000}$ $= \underline{\dfrac{x}{480}}$ cm
20(a)	$120 = 2^3 \times 3 \times 5$ $84 = 2^2 \times 3 \times 7$ $96 = 2^5 \times 3$ Largest possible length $= 2^2 \times 3$ $= \underline{\mathbf{12}}$
20(b)	No. of cubes $= \dfrac{120}{12} \times \dfrac{84}{12} \times \dfrac{96}{12}$ $= \underline{\mathbf{560}}$

21(a)	$2x^2-4xy-x+2y = 2x(x-2y)-(x-2y)$ $= \underline{(x-2y)(2x-1)}$
21(b)	$2x^2-8x-7=0$ $x^2-4x-\frac{7}{2}=0$ $(x-2)^2-2^2-\frac{7}{2}=0$ $(x-2)^2=\frac{15}{2}$ $x-2=\pm\sqrt{\frac{15}{2}}$ $\underline{x=4.74 \quad \text{or} \quad x=-0.74}$
22(a)	$\angle ADO=\frac{\pi}{8}$ (tangents from ext point where line OD bisects $\angle ADB$) $\tan\frac{\pi}{8}=\frac{8}{AD}$ $AD=\frac{8}{\tan\frac{\pi}{8}}$ ≈ 19.3137 $=$ **<u>19.3 cm</u>** (to 3 sf)
22 (b)	Shaded area $= 2\times\frac{1}{2}\times 19.3137\times 8-\frac{1}{2}(8)^2\left(2\pi-\frac{\pi}{2}-\frac{\pi}{2}-\frac{\pi}{4}\right)$ ≈ 79.111 $=$ **<u>79.1 cm²</u>** (to 3 sf)
23(a)	$\underline{\mathbf{N}=\begin{pmatrix}4\\2\\x\end{pmatrix}}$

23(b)	$\mathbf{L}=\begin{pmatrix}2&1&3\\3&2&1\\1&4&2\end{pmatrix}\begin{pmatrix}4\\2\\x\end{pmatrix}$ $=\underline{\begin{pmatrix}10+3x\\16+x\\12+2x\end{pmatrix}}$
23(c)	Matrix L represents **<u>the total litres of red, blue, and yellow base colors, respectively</u>**, **<u>needed by the shop to create the three shades</u>**, A, B, and C. OR $10+3x$ represents the **<u>total litres of red base colour needed by the shop to create the 3 shades</u>**, A, B and C. $16+x$ represents the **<u>total litres of blue base colour needed by the shop to create the 3 shades</u>**, A, B and C. $12+2x$ represents the **<u>total litres of yellow base colour needed by the shop to create the 3 shades</u>**, A, B and C.
23(d)	Each colour must use at most 25 litres. For Red: $10+3x\le 25$ $x\le 5$ For Blue: $16+x\le 25$ $x\le 9$ For Yellow: $12+2x\le 25$ $x\le 6.5$ Largest integer of $x=\underline{5}$

24 (a)	
24 (b)	$\frac{5}{8}\times\frac{x+1}{x+3}=\frac{5}{12}$ $\frac{x+1}{x+3}=\frac{2}{3}$ $3x+3=2x+6$ $x=3$
24 (c)	As x increases, the **number of red balls in Bag B increases (or decreases)**, while the number of blue balls remains the same. This makes it less likely to draw a blue ball from B. The **probability of drawing 2 blue balls decreases (or increases)** as x increases (decreases).

25(a)	$\underline{x=1}$
25(b)	$m=\frac{6-0}{0-6}$ $=\underline{-1}$
25(c)	The **number of** real **solutions** to the question is **equal to the number of points** where the horizontal line $\underline{y=m}$ **meets the graph**.
25(d)	Draw $y=-0.5x+2.5$ From the graph, $\underline{x=-1 \text{ or } 5}$
26(a)	Lily **stopped** (or rest / not moving / at Town B).
26(b)	Average speed = $\frac{30-14}{\frac{48}{60}}$ = **20 km/h**
26(c)	Time taken = $\frac{20}{16\frac{2}{3}}$ = 1h 12 min **Correct straight line drawn.**

<table>
<tr><td></td><td>
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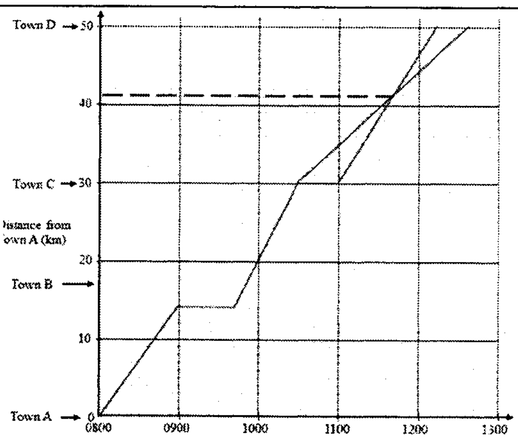
<tr><td>26(d)</td><td>
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<tr><td>26(e)</td><td>Distance from Town D = <u>9 km</u></td></tr>
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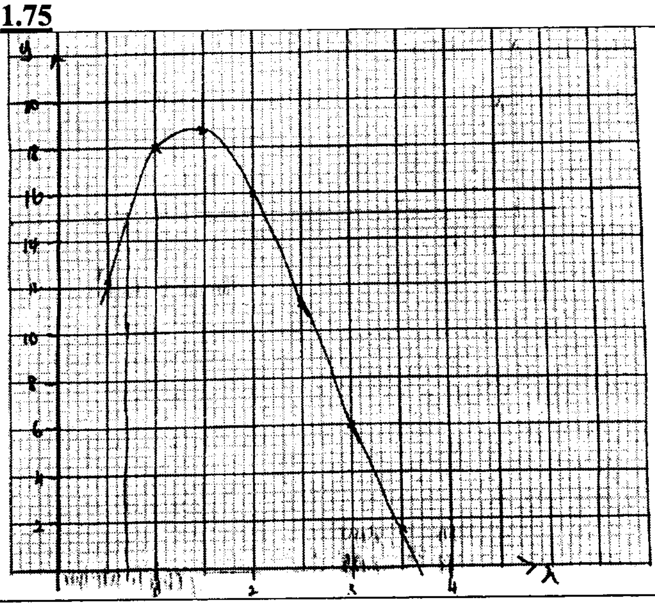
(YEAR 4) MA4052 Paper 2 Solutions

Q/N	Solutions
1(a)(i)	$a=\sqrt{\frac{3n}{2n+5k}}$ $=\sqrt{\frac{3\left(\frac{1}{3}\right)}{2\left(\frac{1}{3}\right)+5\left(\frac{2}{3}\right)}}$ $=\underline{\frac{1}{2}}$
1(a)(ii)	$a=\sqrt{\frac{3n}{2n+5k}}$ $a^2=\frac{3n}{2n+5k}$ $2a^2n+5a^2k=3n$ $2a^2n-3n=-5a^2k$ $n(2a^2-3)=-5a^2k$ $\underline{n=\frac{-5a^2k}{2a^2-3}}$ or $\underline{n=\frac{5a^2k}{3-2a^2}}$

1(b)	$\frac{4-5x}{6} > 1 - \frac{x+5}{4}$ $\frac{4-5x}{6} > \frac{4-(x+5)}{4}$ $\frac{4-5x}{6} > \frac{4-x-5}{4}$ $\frac{4-5x}{6} > \frac{-x-1}{4}$ $4-5x > \frac{6(-x-1)}{4}$ $4(4-5x) > 6(-x-1)$ $16-20x > -6x-6$ $-14x > -22$ $x < \frac{22}{14}$ $\underline{x < \frac{11}{7}}$ or $\underline{x < 1\frac{4}{7}}$
1(c)	$\frac{x-2}{x+1} - \frac{x+3}{x-1} = 5$ $\frac{(x-2)(x-1)-(x+3)(x+1)}{(x+1)(x-1)} = 5$ $\frac{(x^2-x-2x+2)-(x^2+x+3x+3)}{x^2-1} = 5$ $\frac{-3x+2-4x-3}{x^2-1} = 5$ $\frac{-7x-1}{x^2-1} = 5$ $-7x-1 = 5(x^2-1)$ $5x^2-5+7x+1=0$ $5x^2+7x-4=0$ $x = \frac{-7 \pm \sqrt{(7)^2-4(5)(-4)}}{2(5)}$ $x = \frac{-7 \pm \sqrt{129}}{10}$ $x \approx 0.4357816692$ or -1.835781669 $x \approx$ **$\underline{0.44}$** or **$\underline{-1.84}$** (2dp)

2(a)(i)	114.9 million $= 114.9 \times 10^6$ $= 1.149 \times 10^2 \times 10^6$ $\approx \underline{1.15 \times 10^8}$
2(a)(ii)	Population density in 2025 $= \dfrac{116.79 \times 10^6}{300000}$ $= 389.3$ Since **<u>389.3 > 350</u>** The Philippines **<u>had exceeded</u>** the government's population density.
2(b)	% change $= \dfrac{(2.8x + 1.5x + 5x) - 10x}{10x} \times 100\%$ $= \dfrac{9.3x - 10x}{10x} \times 100\%$ $= \underline{\mathbf{-7\%}}$
2(c)	Total cost $= x \times 12 \times 1.1 \times 1.09$ $= 14.388x$ Total bill $= 30 \times 11 + 29.7 = 14.388x$ $\underline{x = 25}$
3(a)	Volume of the solid $= \frac{1}{2}(4-x)(2x)(4-x) + (4-x)^2 x$

	$= x\left(16-8x+x^2\right)+x(16-8x+x^2)$ $= x^3-8x^2+16x+x^3-8x^2+16x$ $= 2x^3-16x^2+32x$
3(b)	The **dimensions** of the figure **must be greater than 0**. Therefore, $4-x>0$ $x<4$ and $x>0$ $\therefore 0<x<4$
3(c)	**1.75**
3(d)	
3(e)	**0.65**
3(f)	From the graph the **maximum value is 18.9**, the volume cannot be greater than 18.9 cm^3. Hence the volume of the solid **cannot be 20 cm^3**.

4(a)	Using similarity, $\dfrac{h}{h-10}=\dfrac{24}{18}$ $18h=24h-240$ $h=40$ Volume of frustum = $\dfrac{1}{3}\pi(12)^2\times40-\dfrac{1}{3}\pi(9)^2\times30$ $=1110\pi\text{ cm}^3$ $=\underline{1.11\pi\text{ litres}}$
4(b)	$\dfrac{3}{4}\pi r^2(20)=1110\pi$ $r^2=74$ $r=\sqrt{74}\approx8.6023$ Therefore, area not in contact of water = $2\pi\left(\sqrt{74}\right)\left[\dfrac{1}{4}(20)\right]$ ≈270.250 $=\underline{\mathbf{270\text{ cm}^2}}$ (to 3 sf)
5(a)	$3x-2y=6$ --- (1) $x+y=5$ --- (2) From (2) $x=5-y$ --- (2*) Sub (2*) into (1) $3(5-y)-2y=6$ $y=\dfrac{9}{5}$ Sub $y=\dfrac{9}{5}$ into (2*) $x=5-\left(\dfrac{9}{5}\right)=\dfrac{16}{5}$ T is $\underline{\left(3\dfrac{1}{5},1\dfrac{4}{5}\right)}$

5(b)	$x+y=5$ A is $(5,0)$ B is $(0,5)$ Length of $AB=\sqrt{(5)^2+(5)^2}$ $=5\sqrt{2}$ $(=7.07106)$ [Using area of triangle OAB] $\frac{1}{2}(7.07106)h=\frac{1}{2}(5)(5)$ $\underline{h=3.54}$ units (to 3 sf)
5(c)	Let m be the distance from T $\frac{1}{2}(5+m)\times 3.2=18$ $m=6.25$ $\underline{y=-4.45}$ or $\underline{y=8.05}$
6(a)	$DF=AF$ (given F midpoint of AD) $\angle BFD=\angle BFA=90°$ ($\perp$ bisector of chord) BF is common $\therefore$ triangle $ABF\equiv$ triangle DBF **(SAS)**
6(b)(i)	$\angle DAC \doteq \angle DBC=\underline{\mathbf{45}}°$ **(<u>$\angle$ s in the same segment)</u>**
6(b)(ii)	$\angle ADC=90°$($\angle$ in a semicircle) $\angle DCA=180°-90°-45°$ **($\angle$ s sum of a triangle)** $=\underline{\mathbf{45}°}$
6(b)(iii)	$\angle DBA \doteq \angle DCA=45°$ ($\angle$ s in the same segment) $\angle OBA=\frac{45°}{2}$(triangle $ABF\equiv$ triangle DBF) $=22.5°$ $\angle OAB=\angle OBA=22.5°$ (base $\angle$ s of an isosceles triangle) $\angle AOB=180°-22.5°-22.5°$ **($\angle$ s sum of a triangle)** $=\underline{\mathbf{135}°}$

6(b)(iv)	$\angle ACB = \frac{135°}{2}$ ($\angle$ at centre = 2 $\angle$s at circumference) $= 67.5°$ $\angle CEB = 180° - 67.5° - 45°$ ($\angle$s sum of a triangle) $= 67.5°$ $\angle AED = \angle CEB =$ **67.5° (vertically opposite $\angle$s)**
7(a)	$\frac{BD}{\sin 44.2°} = \frac{200}{\sin 95°}$ $BD = \frac{200}{\sin 95°} \times \sin 44.2°$ ≈ 139.965 ≈ 140 **m**
7(b)	$3300 = \frac{1}{2}(48)(139.965)\sin \angle CBD$ $\sin \angle CBD = \frac{3300 \times 2}{48(139.965)}$ $\angle CBD \approx 79.230$ bearing of C from $B = 360° - (180° - 45.8°) - 95° - 79.230°$ ≈ 051.57 $=$ **051.6°** (to 1 dp)
7(c)	Let h be the height of the vertical tower. $\sin 79.230° = \frac{opp}{48}$ $opp = 48 \times \sin 79.230°$ ≈ 47.1544 $\tan \theta_e = \frac{80}{47.1544}$ $\theta_e = 59.5°$ (to 1 dp)

8(a)	$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \underline{3\mathbf{b} - 2\mathbf{a}}$
8(b)	$\overrightarrow{XZ} = \frac{1}{2}\overrightarrow{AZ} = \frac{1}{2}\left(\overrightarrow{AC} + \overrightarrow{CZ}\right)$ $= \frac{1}{2}(3\mathbf{b} - 2\mathbf{a} + \mathbf{a})$ $= \underline{\frac{3}{2}\mathbf{b} - \frac{1}{2}\mathbf{a}}$
8(c)	$\overrightarrow{CX} = \overrightarrow{CZ} + \overrightarrow{ZX}$ $= \mathbf{a} + \frac{1}{2}\mathbf{a} - \frac{3}{2}\mathbf{b}$ $= \underline{\frac{3}{2}\mathbf{a} - \frac{3}{2}\mathbf{b}}$
8(d)	$\overrightarrow{CY} = \overrightarrow{CA} + \overrightarrow{AY}$ $= -3\mathbf{b} + 2\mathbf{a} + \mathbf{b}$ $= \underline{2\mathbf{a} - 2\mathbf{b}}$
8(e)	Since $\underline{\overrightarrow{CY} = \frac{4}{3}\overrightarrow{CX}}$ and **C is a common point,** C, X and Y are **collinear.**
8(f)	$\frac{\text{area of } \Delta ACY}{\text{area of } \Delta ACZ} = \frac{\text{area of } \Delta ACY}{\text{area of } \Delta ABC} \times \frac{\text{area of } \Delta ABC}{\text{area of } \Delta ACZ}$ $= \frac{1}{3} \times \frac{2}{1}$ $= \underline{\frac{2}{3}}$

9(a)(i)	**<u>56 minutes</u>**
9(a)(ii)	IQR = $65 - 51.5 =$ **<u>13.5 minutes</u>**
9(a)(iii)	Number of students who took more than 70 minutes $= 600 - 490$ $= 110$ Probability that a pupil chosen at random from the school took more than 70 minutes $= \frac{110}{600} = \frac{11}{60}$
9(b)(i)	The **<u>median time taken by students from School X</u>** to complete the task is **<u>56 minutes</u>**, while for **<u>School Y it is 52 minutes</u>**. This means that students from **<u>School Y, on average, complete the task faster</u>** than those from School X.
9(b)(ii)	For School X: Upper boundary = 85.25 For School Y: Upper boundary =89.25 **<u>88 minutes is considered an outlier for School X</u>**, but not for School Y. Therefore, 88 minutes would be considered **more unexpected for School X.**
9(c)	Since both tasks have the <u>same IQR</u>, **<u>the steepness of the curve will remain the same</u>**, but the curve **<u>will be shifted to the left</u>** because of the <u>smaller median</u>.

10(a)	Total Stamp Duties = $1\%\times180000+2\%\times180000+3\%\times640000+4\%\times500000+5\%\times500000$ $+\quad 17\%\times2000000$ = 69 600 + 340 000 = **$409 600**
10(b)	If sold in April 2025 (> 2 years) Cost incurred = 2 000 000 + 409 600 = 2 409 600 Proceeds after SSD = 2 300 000 – 0.04*2 300 000 = 2 208 000 Net Profit = 2 208 000 – 2 409 600 = – 201 600 ----------------------------- if Sold in **March 2026** (> 3 years) Future price $= 2000000\times(1.07)^3$ = 2 450 086 SSD (more than 3 years) dropped to 0% Proceeds after SSD = 2 450 086 Net Profit = 2 450 086– 2 409 600 = **$40 486** If Rachel in April 2025: Loss = 201 600 **Wait for another year:** Net profit = **$40 486** The drop in seller's SSD to 0 and potential capital appreciation (property rise of 7%) mean Rachel stands to gain much more by waiting until April 2026.