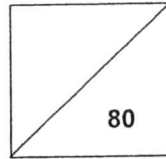




**NORTH VISTA SECONDARY SCHOOL**  
**MID-YEAR EXAMINATION 2017**



NAME: \_\_\_\_\_ ( ) CLASS : \_\_\_\_\_  
 SUBJECT: MATHEMATICS (PAPER 1) DATE : 2 MAY 2017  
 LEVEL/STREAM : SECONDARY 4 EXPRESS/  
 SECONDARY 5 NORMAL ACADEMIC TIME : 2 HOURS  
 CODE : 4016/01

**READ THESE INSTRUCTIONS FIRST**

Write your name, register number and class on all the work you hand in.  
 Write in dark blue or black pen.  
 You may use a pencil for any diagrams or graphs.  
 Do not use staples, paper clips, highlighters, glue or correction fluid.

**Answer all questions.**

If working is needed for any question it must be shown with the question.  
 Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.  
 If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.  
 For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

The number of marks is given in brackets [ ] at the end of each question or part question.  
 The total of the marks for this paper is 80.

For Examiner's Use	
Category	Question No.
Accuracy	
Brackets	
Fractions	
Units	
Others	
<b>Marks Deducted</b>	

**Mathematical Formulae**

*Compound Interest*

$$\text{Total amount} = P \left( 1 + \frac{r}{100} \right)^n$$

*Mensuration*

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

*Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bccosA$$

*Statistics*

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2}$$

Answer all the questions.

- 1 (a) Simplify  $10 - 2(x - 3) + x$ .

Answer ..... [1]

- (b) 150 g of beans contains  $x$  g of proteins. Find an expression for the amount of proteins, in grams, in terms of  $x$  and  $z$ , for  $z$  kg of beans.

Answer .....g [1]

- 2 Factorise completely  $3(2a - 1) - 4a^2 + 2a$ .

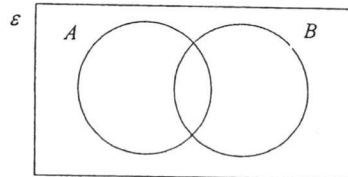
Answer ..... [2]

- 3 (a)  $\varepsilon = \{x : x \text{ is a positive integer and } x < 15\}$   
 $A = \{x : x \text{ is a factor of } 12\}$   
 $B = \{x : 15 - 2x > 1\}$

Find  $n(A' \cap B)$ .

Answer ..... [1]

- (b) On the Venn diagram, shade the region which represents  $(A \cap B) \cup A'$ . [1]



- 4 A worker needs to cover a wall completely with square tiles of the same size. The dimension of the wall is 245 cm by 280 cm.

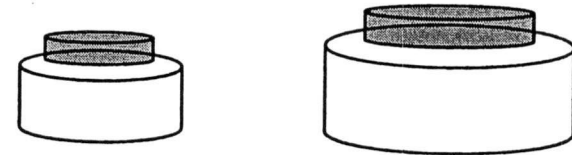
- (a) Find the largest possible length of the side of a square tile.

Answer ..... cm [1]

- (b) Find the total number of tiles that are needed to cover the wall completely.

Answer ..... [1]

- 5 A shop sells bottles of chilli sauce in two sizes as shown below. The two bottles are geometrically similar. The height of the larger bottle is 15 cm and the height of the smaller bottle is 12 cm.



The smaller bottle costs \$9.60. The cost of the bottles are proportional to their capacities. Calculate the cost of the larger bottle, given that a discount of 20% is given for buying the larger bottle.

Answer \$..... [2]

6 Write as a single fraction in its simplest form  $\frac{2}{9x^2-1} + \frac{1}{1-3x}$ .

Answer ..... [2]

7 The mean mass of a golf ball and a tennis ball is  $2.5 \times 10^{-2}$  kg.  
 The mass of the tennis ball is  $2 \times 10^{-3}$  kg.  
 Find the ratio of the mass of the golf ball to the mass of the tennis ball.

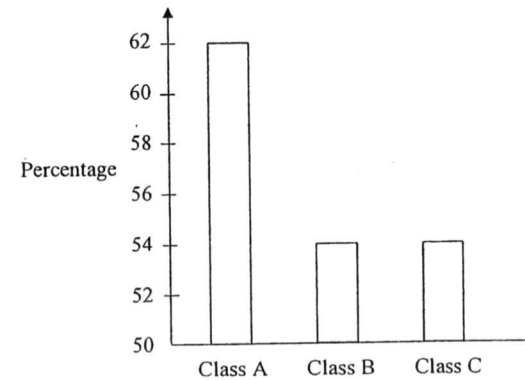
Answer ..... [2]

8 A company invested \$625000 into an investment account which pays 2% per annum compound interest compounded monthly. Find the total amount in the account after 2 years, giving your answer correct to the nearest 10 cents.

Answer \$..... [2]

9 Jordan draws this graph to show the percentage of students from class A, B and C who went overseas for holiday last December.

Students who went overseas for holiday last December



Show one aspect of the graph that is misleading and explain how this may lead to a misinterpretation of the graph.

Answer ..... [2]

10 (a) Simplify  $\left(\frac{x^6}{64}\right)^{-\frac{1}{3}}$ .

Answer ..... [1]

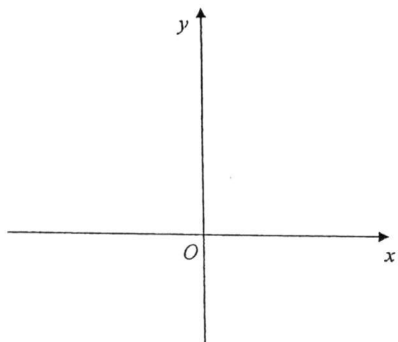
(b) Given that  $a^5 \times a^{-2} \div \sqrt[3]{a^2} = a^k$ , find the value of  $k$ .

Answer  $k =$  ..... [1]

11 (a) Express  $14 - 6x + x^2$  in the form  $(x - h)^2 + k$ .

Answer ..... [1]

(b) On the axes provided, sketch the graph of  $y = 14 - 6x + x^2$ .  
Answer [2]



(c) Write down the equation of the line of symmetry for the graph  $y = 14 - 6x + x^2$ .

Answer ..... [1]

12 A model of a space shuttle is made using a scale of 1 : 150.

(a) The actual length of the space shuttle is 35.5 m long. Find the length, in centimetres, of the model.

Answer .....cm [1]

(b) On the model, the area of the tail section painted red is 60 cm<sup>2</sup>. Find the actual area of the tail section that is painted red, giving your answer in square metres.

Answer .....m<sup>2</sup> [2]

13 The initial temperature of two substances X and Y at 08 00 were 28°C and -60°C respectively. Substance X was cooled at a steady rate. After 5 minutes, its temperature was -12°C. Substance Y was heated at a steady rate of 3°C per minute.

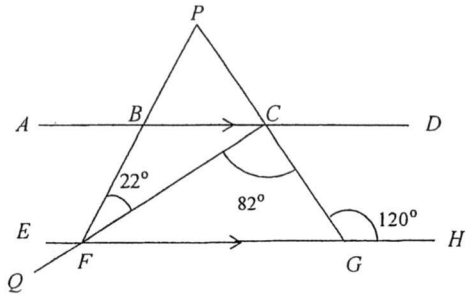
(a) Find the time, when the temperature of both substances will be the same.

Answer ..... [2]

(b) Hence, find the temperature at this time.

Answer .....°C [1]

- 14 In the diagram,  $ABCD$  and  $EFGH$  are parallel and  $QFC$ ,  $PBF$  and  $PCG$  are straight lines. Angle  $PFC = 22^\circ$ , angle  $FCG = 82^\circ$  and angle  $CGH = 120^\circ$ .



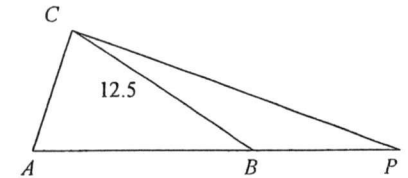
- (a) Calculate angle  $EFQ$ .

Answer ..... [1]

- (b) Explain why  $PB = PC$ .

Answer .....  
 .....  
 ..... [3]

- 15 In the diagram,  $ABC$  is an isosceles triangle,  $BA = BC = 12.5$  cm,  $\sin \angle PBC = \frac{7}{25}$  and  $AB$  is produced to  $P$ .



- (a) Calculate the area of triangle  $ABC$ .

Answer .....cm<sup>2</sup> [2]

- (b) Calculate the length from  $B$  where  $C$  is the nearest to the line  $AB$ .

Answer .....cm [2]

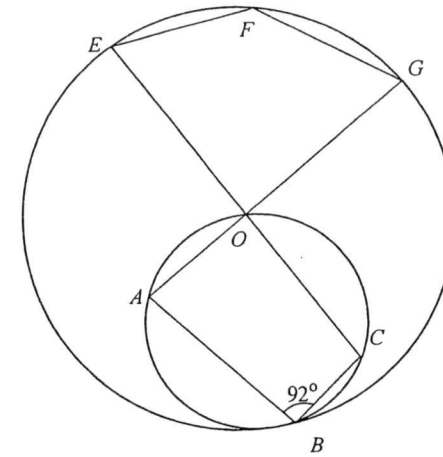
- 16 (a) The line  $ax + by + 1 = 0$  is parallel to the  $x$ -axis and passes through the point  $(-5, 4)$ . Find the values of  $a$  and  $b$ .

Answer  $a =$  .....  
 $b =$  ..... [2]

- (b)  $A$  is a point  $(2, -1)$  and  $B$  is the point  $(0, 5)$ . The line  $AB$  meets the straight line  $x = 3$  at  $C$ . Find the coordinates of  $C$ .

Answer (.....) [2]

- 17 The figure shows two circles. In  $ABCO$ ,  $\angle ABC = 92^\circ$ . Circle  $BEFG$  has centre  $O$ .  $EOC$  and  $AOG$  are straight lines.



- (a) Calculate the angle  $OEG$ , giving reasons for your answer.

Answer .....  
 .....  
 ..... [2]

- (b) Explain whether it is possible for a circle to be drawn passing through the points  $E, F, G$  and  $O$ .

Answer .....  
 .....  
 ..... [3]

18 When written as the product of their prime factors,

$$p = 3^2 \times 5, \quad q = 3^2 \times 7, \quad r = 3^m \times 5^n \times 7$$

(a) Find the LCM of  $p$  and  $q$ , giving your answer as the product of its prime factors.

Answer ..... [1]

(b) Given that  $\frac{pq}{k}$  is a perfect cube, find the smallest possible integer value of  $k$ .

Answer  $k =$  ..... [1]

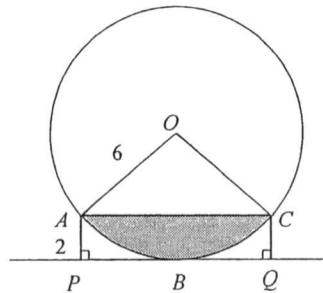
(c) Find the greatest number that will divide  $p$  and  $q$  exactly.

Answer ..... [1]

(d) State, with reason(s) if the product of  $p$ ,  $q$  and  $r$  is an odd or even number.

Answer ..... [1]

19 The diagram shows a circle with centre  $O$  and radius 6 cm passing through the points  $A$ ,  $B$  and  $C$ .  $AP = CQ = 2$  cm,  $\angle APB = \angle BQC = 90^\circ$  and the circle touches the line  $PQ$  at the point  $B$ . Calculate the area of the shaded region.

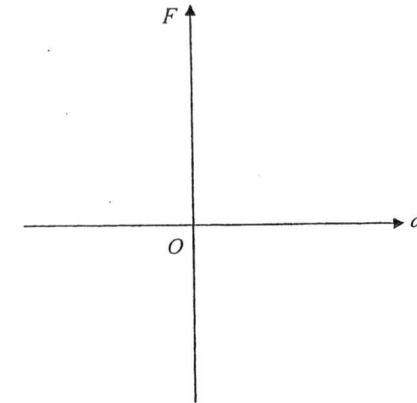


Answer .....cm<sup>2</sup> [5]

20 The force,  $F$  newtons, between 2 particles is inversely proportional to the square of the distance,  $d$  metres, between them.

(a) Sketch a graph to represent the relation between the force and the distance between the particles. [1]

Answer



The force is 3 newtons when the particles are 6 m apart.

(b) Find the equation for  $F$  in terms of  $d$ .

Answer ..... [2]

(c) Calculate the distance between the particles when the force is 12 newtons.

Answer .....m [1]

21 It is given that  $A$  is the point  $(2, -7)$ ,  $\overline{BA} = \begin{pmatrix} 12 \\ -16 \end{pmatrix}$  and  $\overline{CD} = \begin{pmatrix} -3 \\ m \end{pmatrix}$ .

Find

(a)  $|\overline{BA}|$ ,

Answer ..... [1]

(b) the coordinates of  $B$ ,

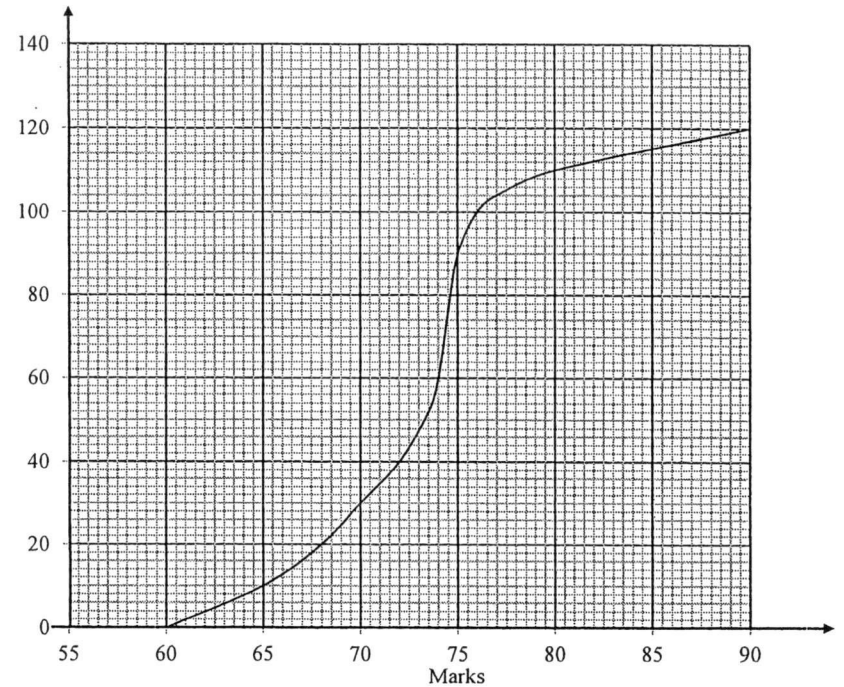
Answer ..... [1]

(c) the value of  $m$  if  $CD$  is parallel to  $BA$ .

Answer  $m =$  ..... [2]

22 The cumulative frequency curve below illustrates the marks obtained by 120 students in an English test.

Cumulative Frequency



(a) Use your graph to find the interquartile range.

Answer ..... [2]

(b) If only 15% of the students scored a distinction, find the least marks for a student to obtain a distinction.

Answer ..... [2]

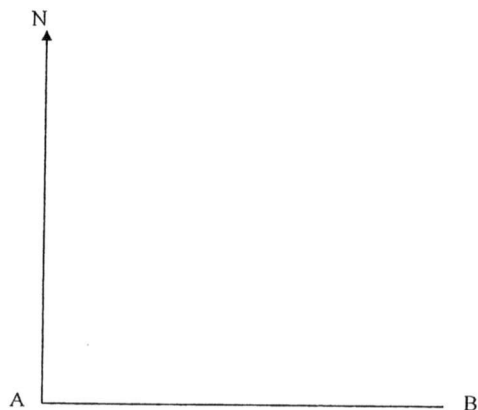
(c) Two students are chosen at random. Find the probability that one student scored above 80 marks and the other student scored below 65 marks.

Answer ..... [2]



23 The scale drawing shows the positions of two buildings A and B. Building B is due east of building A.

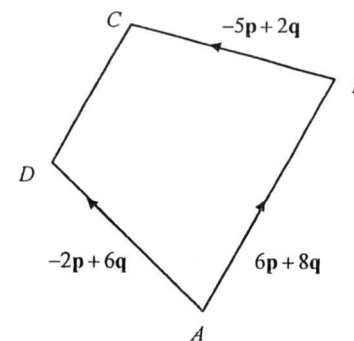
- (a) Building C is due north of building B and on a bearing of  $055^\circ$  from building A. Mark and label the position of building C. [1]
- (b) Building D is equidistant from the lines AB and AC and equidistant from the points B and C. By constructing suitable line(s), find and label the position of building D. [2]



(c) Given that the scale of the diagram is 1 cm to 100 m, calculate the actual distance of AD in metres.

Answer .....m [1]

24 In the diagram,  $\overrightarrow{AB} = 6p + 8q$ ,  $\overrightarrow{BC} = -5p + 2q$  and  $\overrightarrow{AD} = -2p + 6q$ .



(a) Express  $\overrightarrow{DC}$  in terms of p and q.

Answer ..... [1]

(b) Explain why ABCD is a trapezium.

Answer ..... [2]

(c) Find the ratio  $|\overrightarrow{DC}| : |\overrightarrow{AB}|$ .

Answer ..... [1]

(d) Find the numerical value of

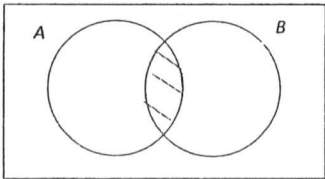
(i)  $\frac{\text{area of triangle } ADC}{\text{area of triangle } ABC}$

Answer ..... [1]

(ii)  $\frac{\text{area of triangle } ADC}{\text{area of trapezium } ABCD}$

Answer ..... [1]

Mid-year Exam 2017- E Math Paper 1 (Marking Scheme)

Qn.	Solutions	Marks
1(a)	$x - 2(3 - x) + 10$ $= x - 6 + 2x + 10$ $= 3x + 4$	B1
1(b)	$y$ hours – 5 km $60y$ mins – 5 km $1$ min – $\frac{5}{60y}$ km Distance = $\frac{5x}{60y} = \frac{x}{12y}$ km	B1
2(a)	$a^3 + a^2 - a - 1$ $= a^2(a+1) - (a+1)$ $= (a^2 - 1)(a+1) = (a-1)(a+1)^2$	M1 A1
3(a)	$\varepsilon = \{1, 2, 3, 4, 5, \dots, 14\}$ $A = \{1, 2, 3, 4, 6, 12\}$ $A' = \{5, 7, 8, 9, 10, 11, 13, 14\}$ $B = \{1, 2, 3, 4, 5, 6\}$ $A' \cap B = \{5\}$	B1
3(b)		B1
4(a)	HCF 245 and 280 = $5 \times 7$ Largest possible length = 35 cm	B1
4(b)	Total number of tiles = $8 \times 7 = 56$	B1
5	Cost of the bigger bottle = $\left(\frac{15}{12}\right)^3 \times 9.60$ $= \$18.75$ (exact) After discount = $\frac{18.75}{100} \times 80$ $= \$15$ (exact)	M1  A1

6	$\frac{2}{9x^2 - 1} + \frac{1}{1 - 3x}$ $= \frac{2}{(3x+1)(3x-1)} - \frac{1}{3x-1}$ $= \frac{2 - (3x+1)}{(3x+1)(3x-1)}$ $= \frac{2 - 3x - 1}{(3x+1)(3x-1)} = \frac{1 - 3x}{(3x+1)(3x-1)}$ $= \frac{-(3x-1)}{(3x+1)(3x-1)} = \frac{-1}{3x+1}$	M1 <i>Common denominator</i>  A1
7	Total mass = $2.5 \times 10^{-2} \times 2 = 5 \times 10^{-2}$ Mass of golf ball = $5 \times 10^{-2} - 2 \times 10^{-3} = 4.8 \times 10^{-2}$ $\frac{4.8 \times 10^{-2}}{2 \times 10^{-3}} = 24$	M1 A1
8	Total amount = $625000 \left(1 + \frac{1}{100}\right)^{24}$ $= 650485.0747 \approx \$650485.10$	M1 A1
9	Misleading feature – The baseline is missing in this graph and it starts at some arbitrary number (that is 50 in this case instead of zero).  Accept – The vertical scale did not start from zero  Effect- It looks like the percentage of students who went overseas from Class A is <b>2 times more</b> than that for Class B & C.	B1 B1
10 (a)	$\left(\frac{125}{a^6}\right)^{\frac{1}{3}} = \left(\frac{a^6}{125}\right)^{\frac{1}{3}} = \frac{a^2}{5}$	B1
10 (b)	$\frac{a^4 \times a^{-1}}{\sqrt[3]{a^2}} = a^k$ $a^{4-1-\frac{2}{3}} = a^k$ $a^k = a^{\frac{7}{3}} \Rightarrow k = \frac{7}{3}$ or $2\frac{1}{3}$	B1
11 (a)	$14 - 6x + x^2 = (x-3)^2 + 5$	B1

11 (b)	<p>Answer</p>	<p>y-intercept &amp; shape correct B1</p> <p>Min point correctly indicated B1</p>
11 (c)	$x = 3$	B1
12 (a)	$\frac{35.5}{150} = 23\frac{2}{3} \text{ cm}$	B1
12 (b)	$1 \text{ cm}^2 - 2.25 \text{ m}^2$ $2.25 \times 60 = 135 \text{ cm}^2$	M1, A1
13 (a)	$28^\circ \text{C} \Rightarrow -12^\circ \text{C}$ Every 1 min, temperature will drop $8^\circ \text{C}$ Let $t$ be the time when the temperature is the same for X and Y $28 - 8t = -60 + 3t$ $88 = 11t$ $t = 8 \text{ mins}$ Time is 08 08	<p>M1 -use of <math>8^\circ \text{C}</math></p> <p>A1</p>
13 (b)	$-60 + 3(8) = -36^\circ \text{C}$	B1
14 (a)	$\angle CFG = 120^\circ - 82^\circ = 38^\circ$ (ext angle of a triangle) $\angle EFQ = 38^\circ$ (vert. opp angles)	B1

14 (b)	$\angle PBC = \angle BFC = 22^\circ + 38^\circ = 60^\circ$ (corr angles) $\angle PCB = \angle PGF$ (corr angles) $\angle PGF = 180^\circ - 120^\circ$ (angles on str line) $\therefore \angle PCB = 60^\circ$ Since $\angle PBC = \angle PCB = 60^\circ$ , $\Delta PBC$ is a isos/equilateral triangle $\Rightarrow PB = PC$	<p>B1</p> <p>B1</p> <p>B1 - mention isos/equilateral triangle &amp; <math>PB = PC</math></p>
15 (a)	Area of triangle $ABC$ $= \frac{1}{2}(12.5)^2 \sin \angle ABC$ $= 27\frac{7}{8} \text{ cm}^2$ M1 - use $\sin \angle ABC = \sin \angle PBC$	<p>M1</p> <p>A1</p>
15 (b)	Let $x$ be the perpendicular height $\frac{1}{2} \times AB \times x = 27\frac{7}{8} \Rightarrow x = 3.5 \text{ cm}$ $d^2 + 3.5^2 = 12.5^2$ $d = 12 \text{ cm}$	<p>M1- for <math>x</math></p> <p>A1</p>
16 (a)	$ax + by + 1 = 0$ $y = -\frac{a}{b}x - \frac{1}{b}$ Parallel to $x$ -axis $\Rightarrow$ gradient = 0 $y = -\frac{1}{b}$ $-\frac{1}{b} = 4 \Rightarrow b = -\frac{1}{4}$ and $a = 0$	B1, B1
16 (b)	Gradient = $\frac{5+1}{0-2} = -3$ $y = -3x + 5$ Sub $x = 3$ $y = -3(3) + 5 = -4$ $C(3, -4)$	<p>M1</p> <p>A1</p>

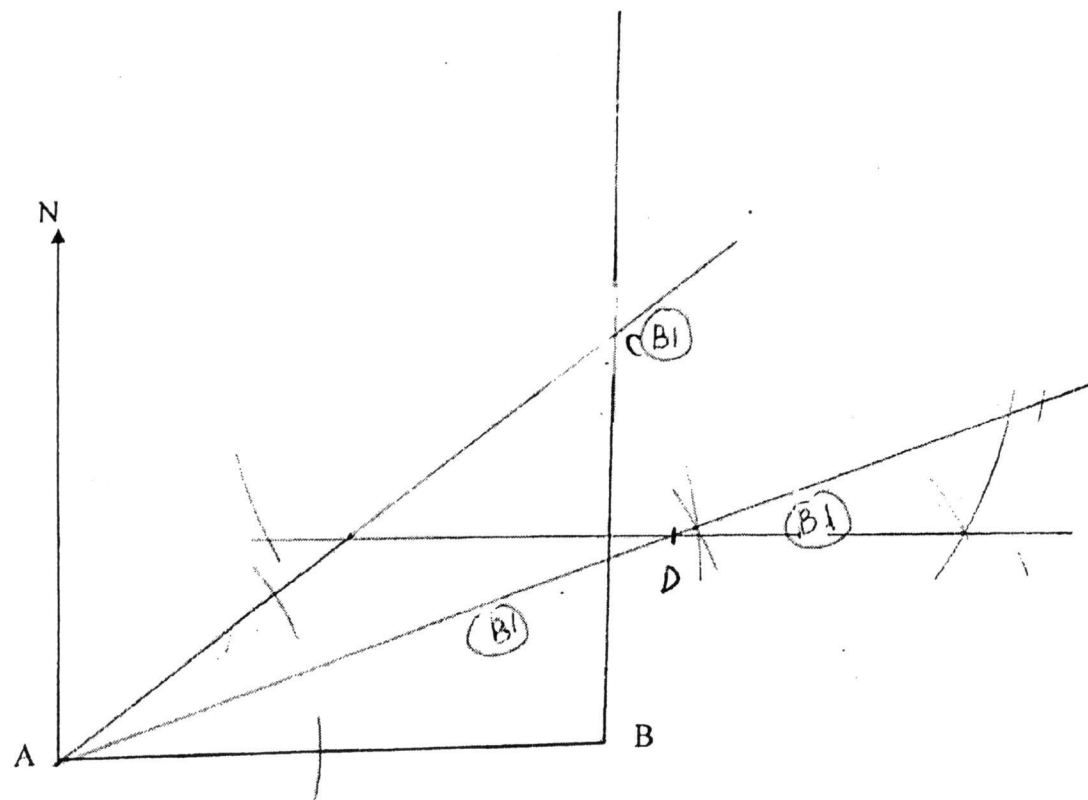
17	$\angle ADC = 180^\circ - 92^\circ = 88^\circ$ (angles in opp segment) $\angle EDG = 88^\circ$ (vert opp angles) $\angle DEG = \frac{180^\circ - 88^\circ}{2} = 46^\circ$ (isos triangle)	B1 – any 2 correct reasons B1 - answer
17 (b)	Reflex $\angle EDG = 360^\circ - 88^\circ = 272^\circ$ (angle at a point) $\angle EFG = \frac{272^\circ}{2} = 136^\circ$ (angle at the centre = twice angle at circumference) $\angle EFG + \angle EDG = 136^\circ + 88^\circ = 224^\circ \neq 180^\circ$ angle in opp segment does not apply $\Rightarrow$ Not possible for a circle to pass through E, F, G and D	B1 B1 – Use angles in the opp segment B1
18 (a)	$p = 3^2 \times 5, q = 3^2 \times 7$ $LCM = 3^2 \times 5 \times 7$	B1
18 (b)	$\frac{pq}{k} = 3^4 \times 5 \times 7 \times \frac{1}{k}$ is a perfect cube $k = 3 \times 5 \times 7 = 105$	B1
18 (c)	$p = 3^2 \times 5, q = 3^2 \times 7$ $HCF = 3^2 = 9$	B1
18 (d)	$pqr = 3^{4+m} \times 5^{1+n} \times 7^2$ $p, q$ and $r$ are expressed as a product of prime factors that are odd numbers, thus $p, q$ and $r$ are odd numbers. The product of 3 odd numbers is still an odd number	B1

19	$\angle AOB = \cos^{-1} \frac{4}{6}$ $\angle AOC = \cos^{-1} \frac{4}{6} \times 2 = 96.379$ or $\angle AOC = 83.621$ Area of sector = $\frac{96.3793}{360} \times \pi \times 6^2 = 30.278$ or Area of sector = 26.270 Area of triangle = $\frac{1}{2} 6^2 \sin 96.3793 = 17.888$ or Area of triangle = $4\sqrt{20}$ Area of segment = $30.278 - 17.888 = 12.4cm^2$ or Area of segment = $8.38cm^2$	B1 B1 B1 B1 B1	Many left the question blank. For students who assume angle $AOC$ as $90^\circ$ or $60^\circ$ no marks are awarded. However, for wrong value of angle $AOC$ , ecf is awarded for correct use of formula for area of triangle and area of sector
20(a)		B1	A few students obtained the correct answer.
20(b)	$3 = \frac{k}{6^2} \Rightarrow k = 108$ $F = \frac{108}{d^2}$	M1 A1	Common error $3 = \frac{k}{6} \Rightarrow k = 18$ $F = \frac{18}{d}$ Weaker students left the question blank
20(c)	$F = \frac{108}{d^2}$ $12 = \frac{108}{d^2}$ $d^2 = \frac{108}{12} \Rightarrow d = \pm 3, d = 3$	B1	For those who did not obtain correct answer for (b) also did not obtain the correct answer for (c) Weaker students left the question blank
21(a)	$\sqrt{12^2 + (-16)^2} = 20units$	B1	Majority obtain the correct answer

21(b)	$\overline{OB} = \overline{OA} - \overline{AB}$ $= \begin{pmatrix} 2 \\ -7 \end{pmatrix} - \begin{pmatrix} 12 \\ -16 \end{pmatrix} = \begin{pmatrix} -10 \\ 9 \end{pmatrix}$ $B(-10, 9)$	B1	<p>Majority obtain the correct answer. Few students did not put bracket. No marks awarded There are some who obtain answer</p> $\begin{pmatrix} 12 \\ -16 \end{pmatrix} - \begin{pmatrix} 2 \\ -7 \end{pmatrix} = \begin{pmatrix} 10 \\ -9 \end{pmatrix}$ $B(10, -9)$
21(c)	$\overline{CD} = k\overline{BA}$ $\begin{pmatrix} -3 \\ m \end{pmatrix} = k \begin{pmatrix} 12 \\ -16 \end{pmatrix}$ $12k = -3 \Rightarrow k = -\frac{1}{4}$ $m = -16k \Rightarrow m = -16\left(-\frac{1}{4}\right) = 4$	M1  A1	Well answered. Except for a few students in weaker classes.
22(a)	$75 - 70 = 5$	B1	Well answered. Common wrong answer $90 - 30 = 60$
22(b)	$\frac{80}{100} \times 120 = 96$ <p>From graph, least marks = 75.5</p>	B1	A number of students gave 76. No marks awarded as the marking is clear at 75.5
22(c)	$\frac{10}{120} \times \frac{10}{119} \times 2 = \frac{5}{357}$	M1 for first pair A1	Badly done
23	See the attached  950 m	B4	Badly done. Many students did not understand the meaning of bearing of C from A.
24(a)	$\overline{DC} = \overline{DA} + \overline{AB} + \overline{BC}$ $\overline{DC} = 3p + 4q$	B1	
24(b)	$\overline{AB} = 2(3p + 4q)$ $\overline{AB} = 2\overline{DC} \Rightarrow \overline{AB} \parallel \overline{DC}$ <p>DA is not parallel to BC Since it has one pair of parallel sides <math>\Rightarrow ABCD</math> is a trapezium</p>	B1-ratio +parallel  B1	$AB = 2DC$ must mention parallel if not 1m will not be awarded
24(c)	$\overline{AB} = 2\overline{DC}$ $ \overline{DC}  = \frac{1}{2} \overline{AB} $ <p>Ratio = 1:2</p>	B1	Dependent if they have the correct answer in (b) Accepted 0.5 as answer
24(d)	(i) $\frac{\text{area of triangle } ADC}{\text{area of triangle } ABC} = \frac{DC}{AB} = \frac{1}{2}$	B1	Badly done

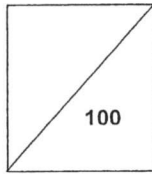
24(d)	(ii) $\frac{\text{area of triangle } ADC}{\text{area of triangle } ABCD} = \frac{1}{3}$	B1	Badly done
-------	---	----	------------

23)





NORTH VISTA SECONDARY SCHOOL  
MID-YEAR EXAMINATION 2017



NAME: \_\_\_\_\_ ( ) CLASS: \_\_\_\_\_

SUBJECT : MATHEMATICS (PAPER 2)

DATE : 3 MAY 2017

LEVEL/STREAM : SECONDARY 4 EXPRESS/

TIME : 2 HOURS 30 MINUTES

SECONDARY 5 NORMAL(ACADEMIC)

CODE : 4048/02

**READ THESE INSTRUCTIONS FIRST**

Write your register number and name on all the work you hand in.  
Write in dark blue or black pen on both sides of the paper.  
You may use a pencil for any diagrams or graphs.  
Do not use staples, paper clips, highlighters, glue or correction fluid.

**Answer all questions.**

If working is needed for any question it must be shown with the answer.  
Omission of essential working will result in loss of marks.  
The use of an approved scientific calculator is expected, where appropriate.  
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.  
For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

At the end of the examination, fasten all your work securely together.  
The number of marks is given in brackets [ ] at the end of each question or part question.  
The total of the marks for this paper is 100.

*Mathematical Formulae*

*Compound Interest*

$$\text{Total amount} = P \left( 1 + \frac{r}{100} \right)^n$$

*Mensuration*

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

*Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

*Statistics*

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2}$$

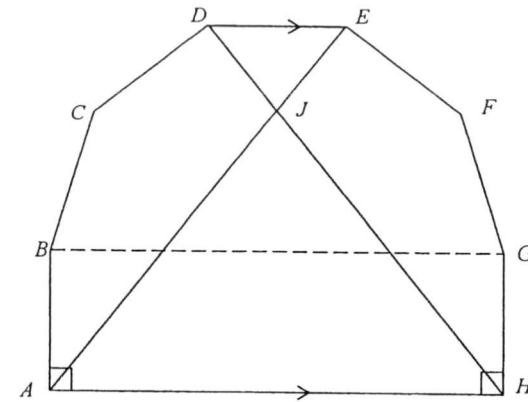
Answer all the questions.

- 1 (a) It is given that  $a = \frac{5b+c^2}{b-2c}$ .
- (i) Find  $a$  when  $b = 6$  and  $c = -2$ . [1]
- (ii) Express  $b$  in terms of  $a$  and  $c$ . [2]
- (b) Solve the equation  $\frac{3x+4}{5} - \frac{x}{7} = 4$ . [2]
- (c) Solve the simultaneous equations.
- $$3x + 2y = 4$$
- $$x - y = -7$$
- [3]
- (d) Simplify  $\frac{2x^2 + x - 3}{4x^2 - 9}$ . [3]

- 2 The table below shows the number of customers for different types of treatments offered by three salons, A, B and C on average for a one-week period.

	Straightening	Curling	Colouring
Salon A	6	4	9
Salon B	4	7	5
Salon C	8	2	3

- (a) Write down a  $3 \times 3$  matrix  $P$  to represent the number of customers in salon A, B and C on average in a week. [1]
- (b) One month is defined as four weeks.  
Evaluate the matrix  $M = 4P$ . [1]
- (c) Evaluate the matrix  $T = (1 \ 1 \ 1)M$ . Explain what the elements of  $T$  represent. [2]
- (d) The cost of straightening, curling and colouring is \$150, \$120 and \$55 respectively. It can be represented as a  $3 \times 1$  column matrix  $Q$ .  
Evaluate the matrix  $MQ$ . [2]
- (e) State what the elements of  $MQ$  represent. [1]



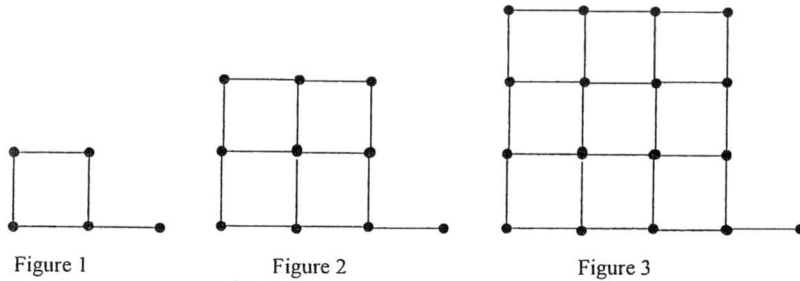
The diagram shows an octagon,  $ABCDEFGH$ .

$BCDEFG$  is part of a regular decagon and  $ABGH$  is a rectangle.

It is given that  $AH$  is parallel to  $DE$ .

- (a) Find
- (i) angle  $CDE$ , [2]
- (ii) angle  $ABC$ . [2]
- (b) Explain why triangles  $EFG$  and  $DCB$  are congruent. [2]
- (c) It is given that  $DJ = JE$  and that angle  $EDJ = 55^\circ$ .  
Jason claims that points  $A$ ,  $H$  and  $J$  lies on the circumference of a circle, and that  $AH$  is the diameter of the circle. Do you agree? Explain your answer. [2]

- 4 The diagram shows a sequence of figures made up of dots and lines.



The number of dots and lines can be written as sequences as shown in the table below.

Figure	Number of dots ( $D_n$ )	Number of lines ( $T_n$ )	$T_n - D_n$
1	5	$1 + 2(1 + 1^2) = 5$	0
2	10	$1 + 2(2 + 2^2) = 13$	3
3	17	$1 + 2(3 + 3^2) = 25$	8
4	$a$	$b$	$c$
...	...	...	...
$n$			

- (a) Find the values of  $a$ ,  $b$  and  $c$ . [3]  
 (b) Find an expression, in terms of  $n$ , for the  $n$ th term of the sequence  $T_n$ . [1]  
 (c) Explain why the number of lines must be odd for all values of  $n$ . [1]  
 (d) (i) Find an expression, in terms of  $n$ , for the  $n$ th term of the sequence  $D_n$ , and show that  $D_n = n^2 + 2n + 2$ . [1]  
 (ii) Hence, show that  $T_n - D_n = (n+1)(n-1)$ . [2]

- 5 Answer the whole of this question on a sheet of graph paper.

The variables  $x$  and  $y$  are connected by the equation

$$y = \frac{3}{x^2} + x.$$

Some of the corresponding values of  $x$  and  $y$  are given in the table below.

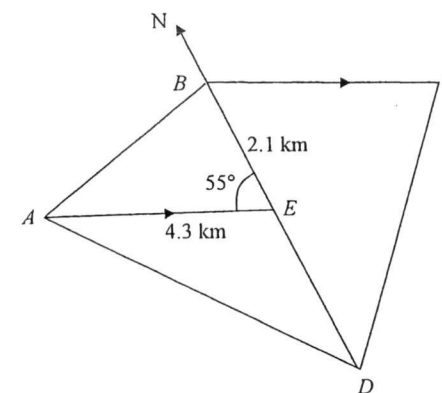
$x$	0.5	1	2	3	4	5	6
$y$	12.5	4	$p$	3.33	4.19	5.12	6.08

- (a) Find the value of  $p$ . [1]  
 (b) Using a scale of 2 cm to represent 1 unit, draw a horizontal  $x$ -axis for  $0 \leq x \leq 6$ .  
 Using a scale of 1 cm to represent 1 unit, draw a vertical  $y$ -axis for  $-1 \leq y \leq 13$ . [3]  
 (c) Use your graph to find the solutions to the equation,  $\frac{3}{x^2} + x = 3$  in the range  $0 \leq x \leq 6$ . [1]  
 (d) By drawing a tangent, find the gradient of the curve at (1, 4). [2]  
 (e) (i) On the same axes, draw the line  $y = -\frac{1}{2}x + 7$  for  $0 \leq x \leq 6$ . [2]  
 (ii) Write down the  $x$ -coordinate(s) of the point(s) for which the line intersects the curve. [1]  
 (iii) The value of  $x$  is a solution of the equation  $3x^3 + Ax^2 + Bx + 6 = 0$ .  
 Find the value of  $A$  and the value of  $B$ . [2]



- 6 A light aircraft flew from Singapore to Ho Chi Minh and returned to Singapore. The distance between Singapore and Ho Chi Minh is 1080 km.
- (a) On the outward flight, the average speed of the aircraft was  $x$  kilometers per hour. Write down an expression, in terms of  $x$ , for the time taken in hours. [1]
- (b) On the return flight, the average speed was 30 km/h greater than the average speed on the outward flight. Write down an expression, in terms of  $x$ , for the time taken, in hours, on the return flight. [1]
- (c) The time taken on the return flight was 30 minutes less than the time taken on the outward flight. Form an equation in  $x$  and show that it reduces to  $x^2 + 30x - 64800 = 0$ . [3]
- (d) Solve the equation  $x^2 + 30x - 64800 = 0$ . [3]
- (e) Calculate,
- (i) the time taken, in hours, on the outward flight, [2]
- (ii) the average speed for the whole flight from Singapore to Ho Chi Minh and back to Singapore, assuming that there is no rest time. [2]

7



The diagram shows five points  $A$ ,  $B$ ,  $C$ ,  $D$  and  $E$  on an empty piece of land where  $BED$  is a straight line,  $AE$  is parallel to  $BC$  and  $D$  is south of  $B$ .  $AE = 4.3$  km,  $BE = 2.1$  km and  $\angle AEB = 55^\circ$ .

- (a) Given that the bearing of  $D$  from  $C$  is  $200^\circ$ , find
- (i) the bearing of  $C$  from  $B$ , [1]
- (ii) the bearing of  $C$  from  $D$ . [1]
- (b) Calculate the length of  $AB$ . [3]
- (c) If the area of  $\triangle AED$  is  $7 \text{ km}^2$ , find the length of  $ED$ . [2]
- A man walked from  $A$ , along  $AE$ , until he reached a point  $X$  where  $BX$  is the shortest distance from  $B$  to  $AE$ .
- (d) Calculate the distance the man walked. [2]
- (e) A vertical tower  $TB$  stands at  $B$  such that  $T$  is the top of the tower. Given that the angle of elevation of  $T$  from  $E$  is  $19^\circ$ , calculate the angle of depression of  $D$  from  $T$ . [3]

- 8 (a) The time taken, in minutes, by 20 pupils to complete puzzle A was recorded below.

Time ( $x$ min)	$15 < x \leq 20$	$20 < x \leq 25$	$25 < x \leq 30$	$30 < x \leq 35$
Frequency	8	7	3	2

- (i) Hence, calculate the mean and standard deviation. [2]

The time taken by the same 20 pupils to complete puzzle B, was recorded. The mean and standard deviation of the time taken to complete puzzle B are 26.4 min and 4.89 min respectively.

- (ii) Which puzzle was easier? Explain. [2]

- (iii) Due to an error in the stopwatch used to record the time taken for the 20 pupils to complete puzzle B, the timing for each pupil should be 1 min more than the recorded timing. How will this affect the mean and standard deviation? [2]

- (b) A box contains 6 red marbles and 9 blue marbles. Two marbles are drawn from the box one after another without replacement.

- (i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

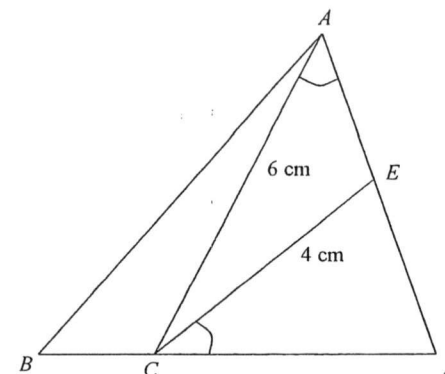
- (ii) Find, as a fraction in its simplest form, the probability that

(a) both marbles drawn are of different colours, [1]

(b) the second marble drawn is red. [1]

- (iii) A third marble is drawn. Find the probability that only the third marble drawn is red. [2]

- 9 In the diagram,  $E$  lies on  $AD$  and  $C$  lies on  $BD$  such that  $BC : CD = 1 : 3$ . It is given that  $AC = 6$  cm,  $CE = 4$  cm and  $\angle ECD = \angle CAD$ .



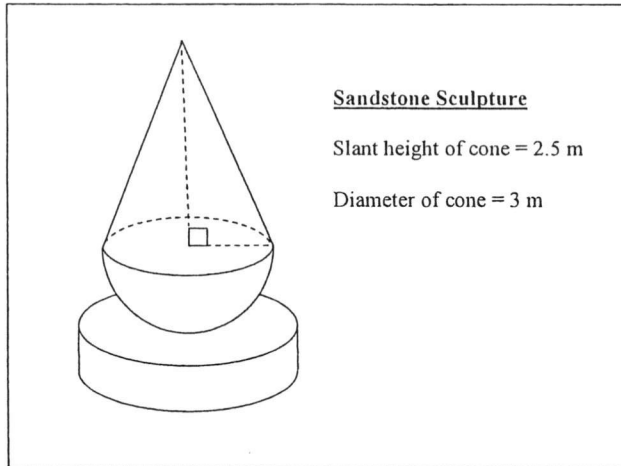
- (a) Prove that  $\triangle ECD$  and  $\triangle CAD$  are similar. [2]

- (b) Write down the ratio of  $CD : AD$ . [1]

- (c) Find the ratio of area of  $\triangle ECD$  : area of  $\triangle CAD$ . [1]

- (d) Given that the area of  $\triangle ACE$  is  $8 \text{ cm}^2$ , find the area of  $\triangle BAD$ . [4]

- 10 The diagram shows a sandstone sculpture consisting of a cone attached to a hemisphere.



- (a) Calculate the volume of the sculpture. [4]
- (b) The sandstone sculpture is to be mounted on a cylindrical concrete platform. The mass (kg) of the platform must be more than the mass (kg) of the sculpture so that it can provide sufficient support.

The recommended radius of the platform is 2 m.

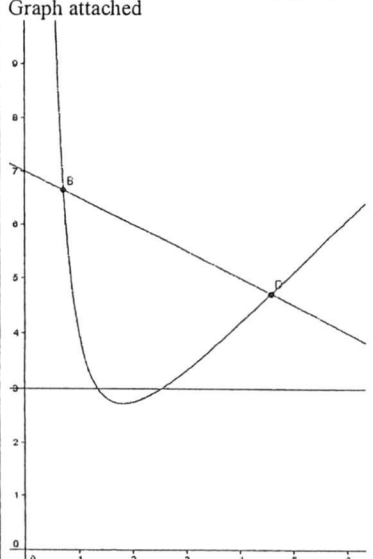
Recommend a suitable height for the platform. Justify your answer using mathematical reasoning. [6]

**Useful Information**

Density of concrete =  $2400 \text{ kg/m}^3$   
 Density of sandstone =  $1442 \text{ kg/m}^3$

Q	Answer	Marks / Remarks
1ai	$3\frac{2}{5}$ or 3.4	B1 (leave in mixed numbers)
1aii	$a = \frac{5b+c^2}{b-2c}$ $ab - 2ac = 5b + c^2$ $ab - 5b = c^2 + 2ac$ $b(a-5) = c^2 + 2ac$ $b = \frac{c^2 + 2ac}{a-5}$ $= \frac{c(c+2a)}{a-5}$	M1  A1
1b	$\frac{3x+4}{5} - \frac{x}{7} = 4$ $\frac{7(3x+4) - 5x}{35} = \frac{4}{1}$ $21x + 28 - 5x = 140$ $x = 7$	M1 (combine fractions)  A1
1c	$x = -2$ $y = 5$	M1 (substitution or elimination) B1 B1
1d	$\frac{2x^2 + x - 3}{4x^2 - 9} = \frac{(2x+3)(x-1)}{(2x+3)(2x-3)}$ $= \frac{x-1}{2x-3}$	M1 (factorise numerator) M1 (factorise denominator)  A1
2a	$P = \begin{pmatrix} 6 & 4 & 9 \\ 4 & 7 & 5 \\ 8 & 2 & 3 \end{pmatrix}$	B1
2b	$M = \begin{pmatrix} 24 & 16 & 36 \\ 16 & 28 & 20 \\ 32 & 8 & 12 \end{pmatrix}$	B1

2c	$T = \begin{pmatrix} 1 & 1 & 1 \\ 24 & 16 & 36 \\ 32 & 8 & 12 \end{pmatrix}$ $= \begin{pmatrix} 72 & 52 & 68 \end{pmatrix}$ <p>T represents the total number of customers who went for straightening, curling and colouring in a month respectively.</p>	M1  A1
2d	$Q = \begin{pmatrix} 150 \\ 120 \\ 55 \end{pmatrix}$ $MQ = \begin{pmatrix} 24 & 16 & 36 \\ 16 & 28 & 20 \\ 32 & 8 & 12 \end{pmatrix} \begin{pmatrix} 150 \\ 120 \\ 55 \end{pmatrix}$ $= \begin{pmatrix} 7500 \\ 6860 \\ 6420 \end{pmatrix}$	M1  A1
2e	<b>MQ</b> represents the amount of money received by shops A, B and C respectively in a month.	B1
3ai	<p>Since <i>BCDEFG</i> is part of a regular decagon,</p> <p>Sum of interior angles of a decagon  <math>= (10 - 2) \times 180</math>  <math>= 1440^\circ</math></p> $\angle CDE = \frac{1440}{10}$ $= 144^\circ$	M1  A1
3aii	<p>Since <i>ABCDEFGH</i> is an octagon,</p> <p>Sum of interior angles of an octagon  <math>= (8 - 2) \times 180</math>  <math>= 1080^\circ</math></p> $\angle ABC = \frac{1080 - 2(90) - 4(144)}{2}$ $= 162^\circ$ <p>Alternative:  <math display="block">\angle ABC = \frac{144}{2} + 90 = 162^\circ</math></p>	M1  A1
3b	<i>BC</i> = <i>CG</i> (sides of a regular polygon) <i>CD</i> = <i>EF</i> (sides of a regular polygon)	

	Angle $BCD = \text{angle } GFE$ (interior angle of a regular polygon) Therefore, by SAS, triangle $EFG$ and $DCB$ are congruent.	B1 B1
3c	$\angle DJE = 180 - 55 - 55$ $= 70^\circ$ (isos $\Delta$ ) Since $\angle DJE = 70^\circ \neq 90^\circ$ , by the property of angles in a semicircle, Jason's claim is untrue.	M1 A1
4a	$a = 26$ $b = 41$ $c = 15$	B1 B1 B1
4b	$T_n = 1 + 2(n + n^2)$ or $1 + 2n + 2n^2$	B1
4c	$2(n + n^2)$ will give an even number since any number multiplied by 2 will be an even number. By adding 1 to the product, it gives an odd number. Hence, the number of lines will always be an odd number.	B1
4e	$D_n = (n + 1)^2 + 1$	B1
4f	$T_n - D_n = 1 + 2(n + n^2) - ((n + 1)^2 + 1)$ $= 1 + 2n + 2n^2 - n^2 - 2n - 2$ $= n^2 - 1$ $= (n + 1)(n - 1)$	M1 M1
5a	$p = 2.75$ (exact)	B1
5b	Graph attached 	Axes – B1 Points – B1 Smooth curve – B1

5c	$x = 1.35, 2.53 \pm 0.5$	B1
5d	Tangent drawn Gradient = $-5 \pm 1$	M1 A1
5ei	Table of values Correct line with label	M1 A1
5eii	$x = 0.71, 4.57$	B1
5eiii	$\frac{3}{x^2} + x = -\frac{1}{2}x + 7$ $3 + x^2 = -\frac{1}{2}x^3 + 7x^2$ $3x^3 - 14x^2 + 6 = 0$ Therefore, $A = -14$ and $B = 0$	M1 (equate) A1 (values of $A$ and $B$ )
6a	$\frac{1080}{x}h$	B1
6b	$\frac{1080}{x+30}h$	B1
6c	$\frac{1080}{x} - \frac{1080}{x+30} = \frac{1}{2}$ $\frac{1080(x+30) - 1080x}{x(x+30)} = \frac{1}{2}$ $64800 = x^2 + 30x$ $x^2 + 30x - 64800 = 0$	M1 M1 M1
6d	$x = \frac{-30 \pm \sqrt{30^2 - 4(1)(-64800)}}{2(1)}$ $x = \frac{-30 \pm \sqrt{260100}}{2}$ $x = 240$ or $-270$	M1 M1 A1
6e(i)	Time = $\frac{1080}{240}$ Time = $4\frac{1}{2}$ h	M1 A1
6e(ii)	Average speed = $\frac{1080 \times 2}{4\frac{1}{2} + 4} = 254\frac{2}{17}$ km/h	M1, A1
7a(i)	$125^\circ$	B1
7a(ii)	$020^\circ$	B1
7b	$AB^2 = 2.1^2 + 4.3^2 - 2(2.1)(4.3)\cos 55^\circ$ $AB^2 = 12.541$ $AB = 3.5414 = 3.54$ km (3s.f.)	M1 M1 A1
7c	$\angle AED = 125^\circ$ $\frac{1}{2}(4.3)(ED)\sin 125^\circ = 7$	M1

